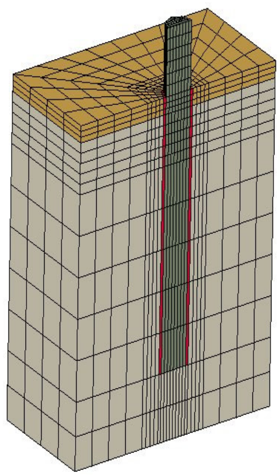
Frictional 3D Beam-to-Solid Contact Formulation for OpenSees

Peter Mackenzie-Helnwein
Pedro Arduino
Kathryn Ann Petek
http://www.ce.washington.edu

Overview

- **Introduction**
  - Background & Goal
- **3D Analysis of Soil-Pile Interaction**
  - Beam-Solid Approach
  - Contact Formulation & Implementation
  - Practical Applications
- **Summary and Conclusions**

3D Analysis of Soil-Pile Interaction

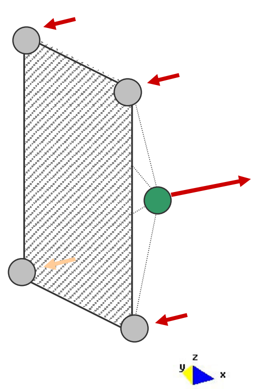


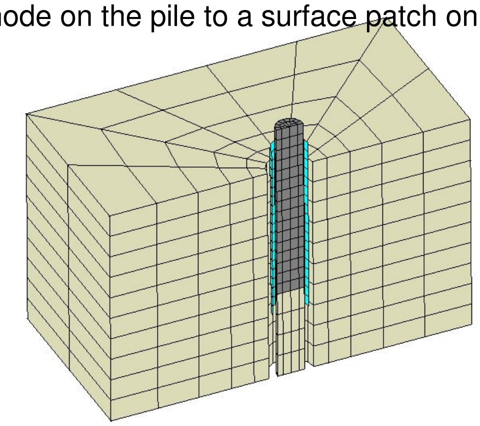
- Primary Modeling Issues
  - Characterization of soil
  - Pile structural modeling
  - Interface behavior
- Goal
  - Expand & improve capabilities for modeling soil-pile interaction
  - New beam-solid contact formulation

Soil-Pile Interaction – Continuum Approach

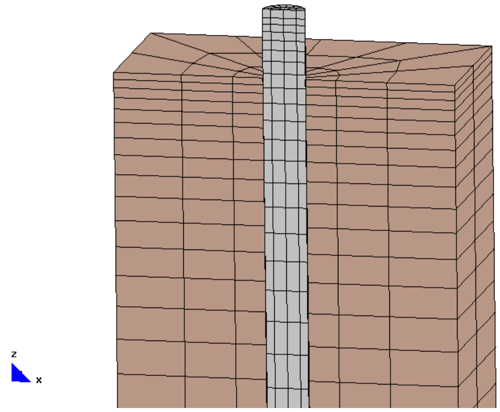
Constraint Based Interface (Solid-Solid)

- Contact element applies a geometric constraint to the system that relates a slave node on the pile to a surface patch on soil.

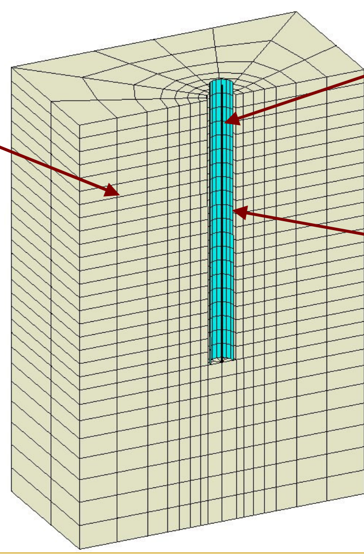




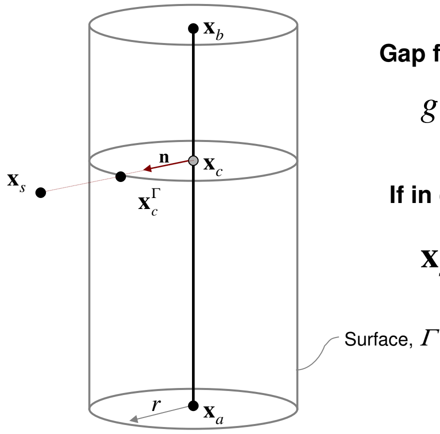
Soil-Pile Interaction – Continuum Approach



New approach: Beam-Solid Contact Element



Beam-Solid Contact Element



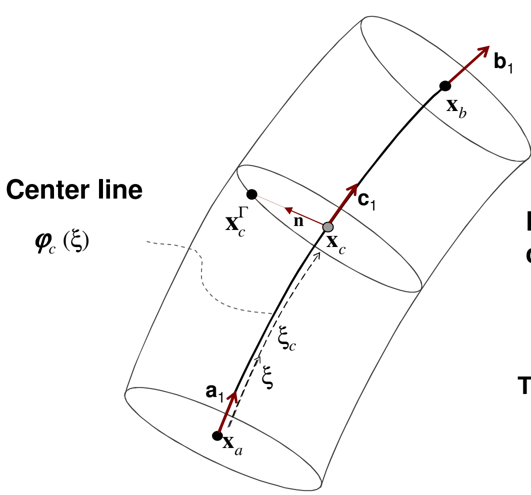
Gap function

$$g = \mathbf{n} \cdot (\mathbf{x}_s - \mathbf{x}_c) - r$$

If in contact

$$\mathbf{x}_s = \mathbf{x}_c^\Gamma$$

Beam-Solid Contact Element



Projected point on center line

$\mathbf{x}_c = \boldsymbol{\varphi}_c\,(\xi_c)$

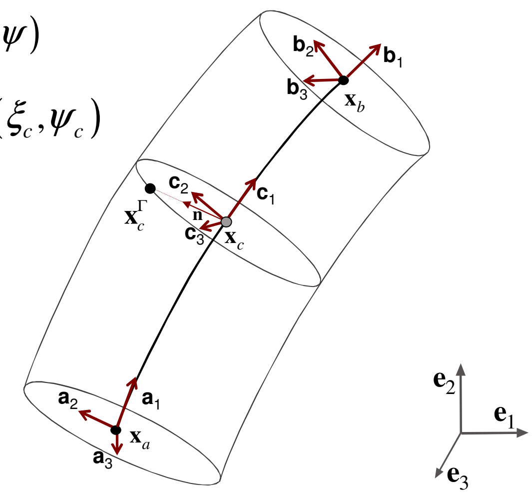
Tangent vector

$\mathbf{c}_1 = \boldsymbol{\varphi}_{c,\,\xi}\,(\xi_c)$

Beam-Solid Contact Element

$$\Gamma(\xi,\psi) =: \varphi_c\,(\xi) + \mathbf{r}(\xi,\psi)$$

$$\mathbf{x}_c^{\Gamma}(\xi_c,\psi_c) = \varphi_c\,(\xi_c) + \mathbf{r}(\xi_c,\psi_c)$$



Beam-Solid Contact Element

$$\Gamma(\xi,\psi) =: \varphi_c(\xi) + \mathbf{r}(\xi,\psi)$$

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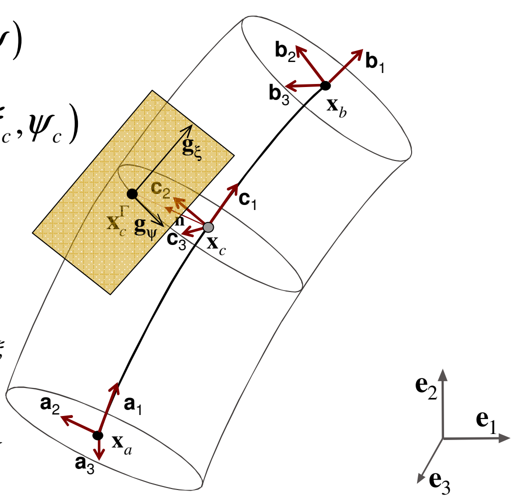
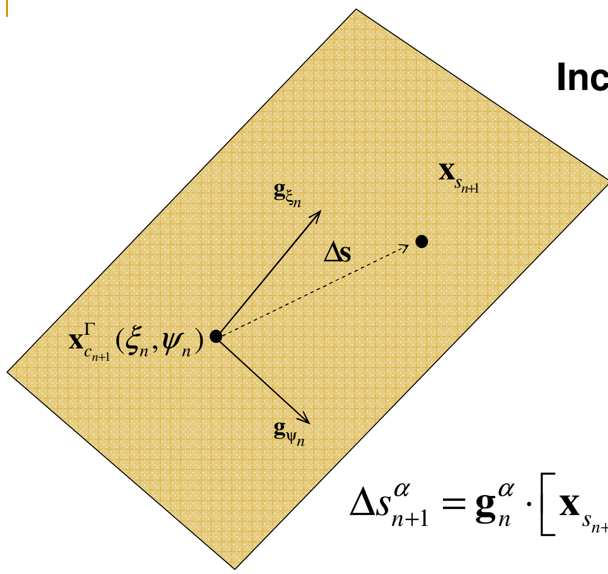


$$\mathbf{g}_{\xi}(\xi_c,\psi_c) = \Gamma_{,\xi} = \varphi_{c,\xi} + \mathbf{r}_{c,\xi}$$

$$\mathbf{g}_{\psi}(\xi_c,\psi_c) = \Gamma_{,\psi} = \varphi_{c,\psi} + \mathbf{r}_{c,\psi}$$

Beam-Solid Contact: Kinematics

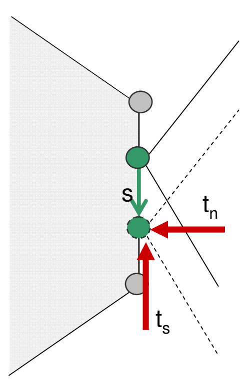


Incremental Slip:

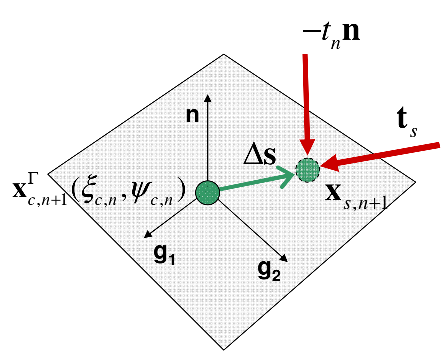
$$\Delta\mathbf{s} = \Delta s^{\alpha}_{n+1}\mathbf{g}_{\alpha n}$$

$$\Delta s^{\alpha}_{n+1} = \mathbf{g}^{\alpha}_{n} \cdot \left[\mathbf{x}_{s_{n+1}} - \mathbf{x}^{\Gamma}_{c_{n+1}}(\xi_n, \psi_n)\right]$$

Compare: 3D Contact Element (2005)



2D Node-to-Line Element

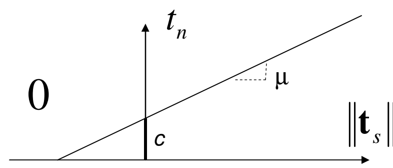


3D Node-to-Surface Element

Interface Law with friction and cohesion

- The geometric constraints are related with an interface constitutive law:
  - Mohr-Coulomb Friction Law

$$f = \|\mathbf{t}_s\| - \mu \cdot t_n - c \quad \leq \quad 0$$



  - Can also use non-linear and history dependent material models, including specific models for concrete structures on soil

Variational Formulation of Contact Constraints [Wriggers]

- Virtual Work expression: $\delta W = t_n \underline{\delta g} + \delta t_n g - \underline{\delta \mathbf{s}} \cdot \mathbf{t}_s$
- Linearization: $\Delta(\delta W) = \underline{\delta g} \Delta t_n + \delta t_n dg - \underline{\delta \mathbf{s}} \cdot \Delta \mathbf{t}_s$

$$\Delta \mathbf{t}_s = \frac{\partial \mathbf{t}_s}{\partial \mathbf{s}} \cdot \Delta \mathbf{s} + \frac{\partial \mathbf{t}_s}{\partial t_n} \Delta t_n =: \mathbf{C}_{ss} \Delta \mathbf{s} + \mathbf{C}_{sn} \Delta t_n$$

Note: $\mathbf{C}_{ss}$ & $\mathbf{C}_{sn}$ depend on the state: sticking, sliding

- Problem: $\delta \mathbf{s}$ requires $\delta \mathbf{x}_c^{\Gamma}$
- Solution: $\delta \mathbf{x}_c^{\Gamma} = \delta \varphi_c + \delta \mathbf{r} = \delta \varphi_c + \delta \phi_c \times \mathbf{r}$

Finite Element Implementation

Linearization and Tangent Stiffness Matrix:

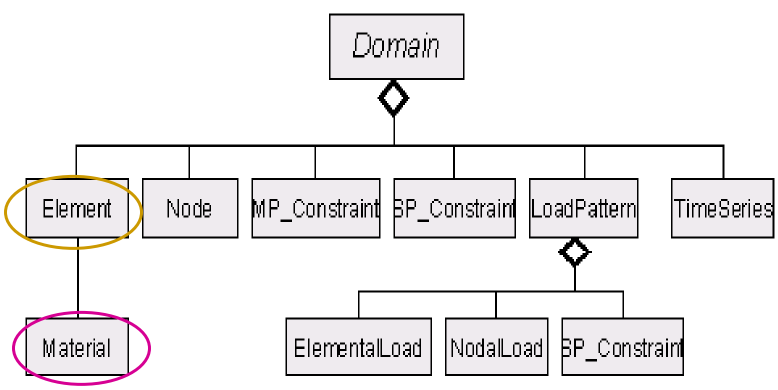
$$\Delta(\delta W) = \delta g \, \Delta t_n + \delta t_n \, \Delta g - \delta \mathbf{s} \cdot \Delta \mathbf{t}_s$$

$$\delta \mathbf{q} := \begin{Bmatrix} \delta \mathbf{u}_A \\ \delta \phi_A \\ \delta \mathbf{u}_B \\ \delta \phi_B \end{Bmatrix} \qquad \delta g =: \delta \mathbf{q}^T \mathbf{B}_n \qquad \delta \mathbf{s}^T =: \delta \mathbf{q}^T \mathbf{B}_s$$

$$\Delta(\delta W) = \begin{bmatrix} \delta \mathbf{q}^T & \delta t_n \end{bmatrix} \cdot \underbrace{\begin{bmatrix} -\mathbf{B}_s \mathbf{C}_{ss} \mathbf{B}_s^T & \mathbf{B}_n^T - \mathbf{B}_s^T \mathbf{C}_{sn} \\ \mathbf{B}_n^T & 0 \end{bmatrix}}_{\mathbf{K}_T} \cdot \begin{Bmatrix} \Delta \mathbf{q} \\ \Delta t_n \end{Bmatrix}$$

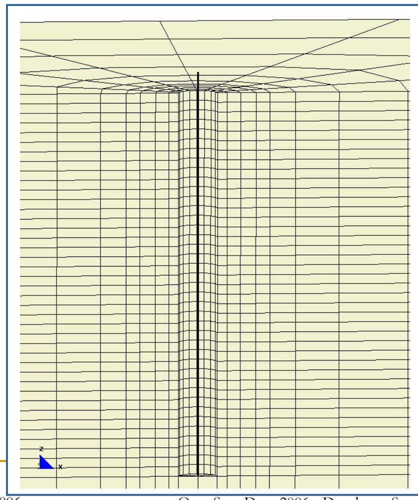
Implementation in OpenSees

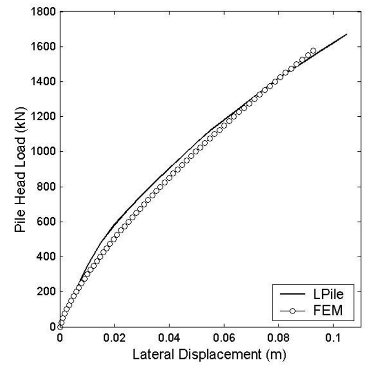
New element and material classes



Laterally Loaded Piles

- Perform numerical load test
- Compare results

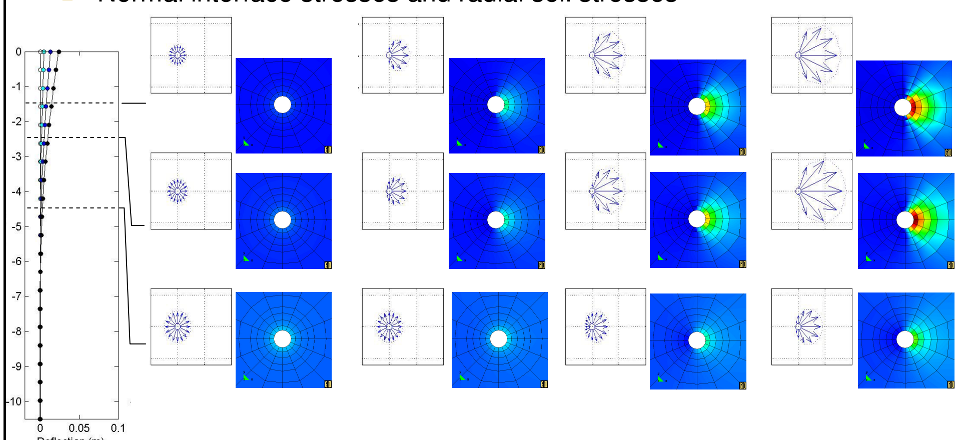




3x Magnification

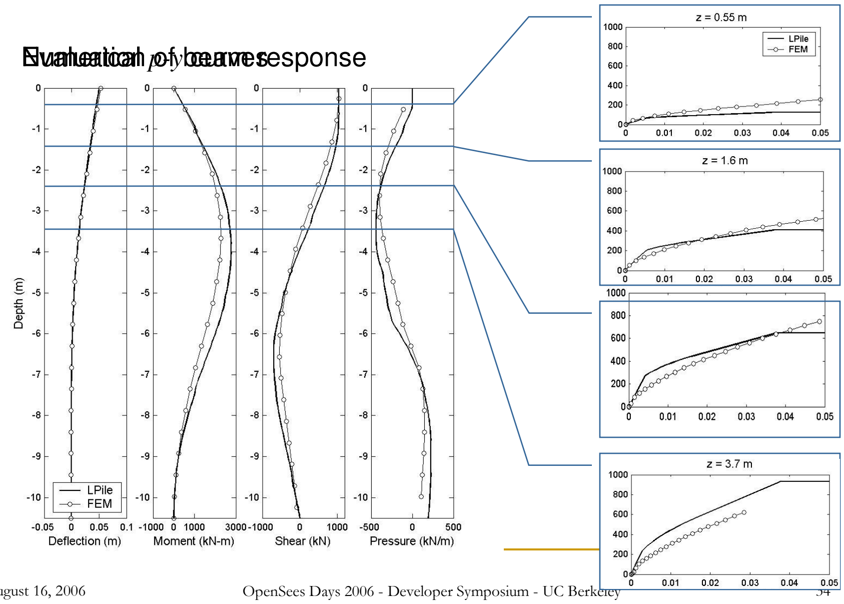
Laterally Loaded Piles

- Normal interface stresses and radial soil stresses

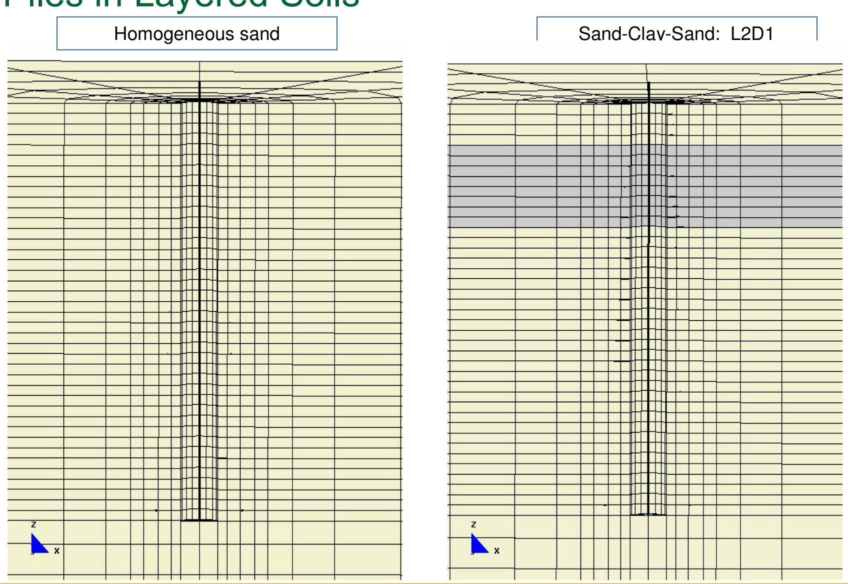


Laterally Loaded Piles

- Evaluation of p-y curves / Numerical beam response



Piles in Layered Soils



Conclusions

- Constraint based contact interface elements capture well pile-soil interface behavior
- A more sophisticated description of the kinematics of the contact interface allows for the use of efficient beam elements with fiber cross section to model pile behavior.
- The proposed beam-solid contact element significantly simplifies mesh generation without compromising accuracy.
- Validation simulations & practical application studies demonstrate usability of beam-solid contact elements


Thank you