Two and Three-Dimensional Contact Element Implementation for Geotechnical Applications in OpenSees

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August 24, 2005
OpenSees Developer Symposium
Presentation Overview

- Background
- Contact Element & Interface Material Formulations
- OpenSees Implementation
- Element Features
- Examples
Objectives

1. **Realistic soil-pile interaction**
2. Consideration of complex soil models
3. **Alternative pile modeling approaches**
Background: Interface Behavior

Pile-soil interaction: stick, slip, debonding, and rebonding behavior

(Desai et al., 1988)
Background: Interface Behavior

Pile-soil interaction: stick, slip, debonding, and rebonding behavior

Finite Element approaches:
- Zero-length elements
- Joint and thin-layer elements
- Gap elements
Contact Element Model Development

Node-to-Segment Element

Node-to-Surface Element
Contact Element

Node-to-Segment Element

\[ t_n = (1-\alpha) t_n \]

Master body

Slave body

\[ g \]

\[ \alpha t_n \]
Contact Element Formulation

Contact element applies a geometric constraint to the system that relates a slave node to a master contact line segment or surface.

Using the method of Lagrange Multipliers, the element utilizes the Hertz-Signori-Moreau conditions for contact:

\[ g \geq 0 \quad t_n \geq 0 \quad t_n \cdot g = 0 \]
Contact Material Formulation

- The geometric constraints are related with an interface constitutive law:

  - Mohr-Coulomb Friction Law

\[ f = \left| t_s \right| - \mu \cdot t_n - c \leq 0 \]

- Can also use non-linear and history dependent material models, including specific models for concrete structures on soil
Contact Material Formulation

\[ f = |t_s| - \mu \cdot t_n - c \leq 0 \]

- **Sticking**
- **Sliding**
- **Elastic slip**
- **Plastic slip**
Variational Contact Formulation

- **Expression for Virtual Work:**

\[ \delta W = t_n \delta g + \delta t_n \ g - t_s \ \delta s \]

- **Linearization:**

\[
\begin{align*}
  d(\delta W) & = \delta g \ dt_n + \delta t_n \ dg - \delta s \ dt_s \\
  dt_s & = \frac{\partial t_s}{\partial s} \ ds + \frac{\partial t_s}{\partial t_n} \ dt_n =: C_{ss} \ ds + C_{sn} \ dt_n
\end{align*}
\]

Note: \( C_{ss} \) & \( C_{sn} \) depend on the state: sticking, sliding
2D Contact Formulation

Linearization and 2D Tangent Stiffness Matrix:

\[
d(\delta W) = \delta g \ dt_n + \delta t_n \ dg - \delta s \ dt_s
\]

\[
\delta g =: \delta q^T B_n \quad B_n = \begin{bmatrix}
\alpha n \\
(1 - \alpha) n \\
- n
\end{bmatrix} \quad \delta s =: \delta q^T B_s \quad B_s = \begin{bmatrix}
\alpha t \\
(1 - \alpha) t \\
- t
\end{bmatrix}
\]

\[
d(\delta W) = \left[ \delta q^T \quad \delta t_n \right] \cdot \begin{bmatrix}
-B_s C_{ss} B_s^T & B_n^T - B_s^T C_{sn}
\end{bmatrix} \cdot \begin{bmatrix}
dq \\
d t_n
\end{bmatrix} = K_T
\]
Implementation in OpenSees

New element and material classes
Implementation in OpenSees

Continuum Element

global fields $\Rightarrow$ ::update$(u)$ $\Rightarrow$ ::setTrialSr ain$(u)$

EQUILIBRIUM $\leftarrow$ ::getResisting Force() $\leftarrow$ ::getStress$(\varepsilon)$

$\leftarrow$ ::getTangentStiff() $\leftarrow$ ::getTangent()
Implementation in OpenSees

Contact Element

Interface Material

global fields $\rightarrow \text{::update()} (u) \rightarrow \text{::setTrialStain()} \varepsilon (g, t_n, s)

EQUILIBRIUM $\rightarrow \text{::getResistingForce()} \rightarrow \text{::getStress()} \sigma (s, t_s, t_n)$

$\text{::getTangentStiff()} \rightarrow \text{::getTangent()}$
3D Contact Element

2D Node-to-Line Element

3D Node-to-Surface Element
3D Geometric Pseudo-Nonlinearity

- Project $x_{s_n}$ on to master surface patch & determine tangent plane
- Slip, $s_n^{n+1}$, moves along tangent plane of step $n$
- Converges to nonlinear solution
Solution Strategy w/ Lag Step

- No contact search algorithm
- Contact Conditions:
  \[ g = (x_s - x_\xi) \cdot n \]
  \[ \begin{cases} 
  > 0 & \text{in contact} \\
  \leq 0 & \text{not in contact} \\
  < 0 & \text{should be released} 
\end{cases} \]
  \[ t_n \begin{cases} 
  \geq 0 & \text{stability near boundary of in and out of contact} \\
  < 0 & 
\end{cases} \]

- Added lag step for stability near boundary of in and out of contact
Solution Strategy w/ Lag Step

\[
\begin{align*}
\text{should_release} &= \text{true} \\
\text{was_in}\_\text{Contact} &= \text{true} \\
\text{to_be}\_\text{released} &= \text{false}
\end{align*}
\]

\[
\begin{array}{|c|}
\hline
\text{Variable} \\
\hline
\text{should_release} \\
\hline
\text{was_in}\_\text{Contact} \\
\hline
\text{to_be}\_\text{released} \\
\hline
\text{in}\_\text{Contact} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|}
\hline
\text{true} \\
\hline
\text{false} \\
\hline
\end{array}
\]

\[
\begin{align*}
t_n < 0 \\
g \leq 0
\end{align*}
\]

\[
\begin{align*}
\text{should_release} &= \text{true} \\
\text{was_in}\_\text{Contact} &= \text{true} \\
\text{to_be}\_\text{released} &= \text{false}
\end{align*}
\]
GiD Development

Developed pre- and post-processing tools using commercial software GiD

- Model creation
- Mesh generation
- Results visualization
GiD Contact Element Generation

- No native support for this type of element
- GiD creates contact pairs for all nodes within range that can go in and out of contact.
Example 1: Simple 3D Blocks

Block moving across surface:
Example 2: 3D Friction Pile
Example 2: 3D Friction Pile

Parameter Testing and Calibration: Evaluate frictional forces developed in contact element

\[ Q_{contact} = \sum_{i=1}^{N} f_{s_i} \]

And compare with conventional \( \beta \)-method used for pile analysis:

\[ Q_s = \pi B \int_{0}^{D} \bar{\sigma}_h(z) \tan \delta \, dz \]
Example 2: 3D Friction Pile

Frictional contact element gives good approximation:

\[ Q_{contact} = \sum_{i=1}^{N} f_{s_i} \]

\[ Q_s = \pi B \int_{0}^{D} \overline{\sigma_h}(z) \tan \delta \, dz \]

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( G )</th>
<th>( Q_{contact} )</th>
<th>( Q_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1000</td>
<td>63</td>
<td>55</td>
</tr>
<tr>
<td>0.5</td>
<td>1000</td>
<td>128</td>
<td>110</td>
</tr>
<tr>
<td>1</td>
<td>1000</td>
<td>462</td>
<td>440</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>918</td>
<td>880</td>
</tr>
</tbody>
</table>

Typical Values of \( \mu = \tan \delta \):

\[ \delta / \phi = 1.0 - 0.5 \]

Concrete-sand: \( \mu = 0.35 - 0.6 \)
Example 3: 3D Pushover Analysis
Example 3: 3D Pushover Analysis
Example 3: 3D Pushover Analysis with Cohesive Contact Material
Example 3: 3D Pushover Analysis with Plastic Soil
Summary

- Contact elements are implemented in OpenSees using a stable, pseudo-nonlinear approach.

- Examples demonstrate element capability to describe interface behavior for pile analysis.

- Further validation and testing is underway prior to submission to OpenSees repository.