Nonlinear Stochastic Dynamic Analysis
by OpenSees

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Motivation – PEER PBEE Framework

- Mean annual frequency of occurrence of events \( \{ DV \geq dv \} \)

\[
v(DV) = \int \int \int G(dv \mid dm) \, dG(dm \mid edp) \, dG(edp \mid im) \, dv(im)\]

\( dv \): decision variable (e.g. dollar loss, duration of down time)
\( dm \): damage measure (e.g. accumulated plastic rotation at a joint)
\( edp \): engineering demand parameter (e.g. inter-story drift)
\( im \): intensity measure (e.g. PGA, SA at a given period)

A necessary ingredient from structural analysis

\( G(edp \mid im) = \Pr(EDP > edp \mid im) \):

Probability distribution of \( EDP \) given the intensity measure \( im \),
e.g., probability distribution of an inter-story drift given the SA at the
natural period of the structure.
Motivation – PEER PBEE Framework

- **Current PEER Methodology**
  - Determine M and R by disaggregation of hazard curve,
  - Select recorded ground motions from (M,R) bin,
  - Scale records to given $im$,
  - Perform nonlinear dynamic analysis and compute peak $edp$ values,
  - Fit lognormal distribution to the sample of computed $edp$’s.

Scaled spectra of recorded motions
Motivation – PEER PBEE Framework

- Potential shortcomings of the current methodology
  - Robustness: results may depend on the selected set of records.
  - Site effect: recorded motions may not correctly characterize the local site.
  - Site variability: may overestimate variability in EDP due to mix of records from different sites.
  - Scaling: scaled versions of recorded motions may not be realistic.

- Is there an alternative?
An Alternative

- **Stochastic characterization of ground motion**
  Define the ground motion as a stochastic process, consistent with the characteristics of the local site and given \( im \).

- **Nonlinear random vibration analysis**
  Compute the statistics of \( EDP \) by nonlinear random vibration analysis and first-passage probability.

\[
\begin{align*}
\text{random } f(t) & \rightarrow \text{Nonlinear system} \\
\text{random } x(t) & \rightarrow \text{mean, variance, CDF, PDF, crossing rate, first-passage probability, etc.}
\end{align*}
\]
An Alternative

Methods for nonlinear random vibration analysis

- **Fokker-Planck equation**
- **Stochastic averaging**
- **Moment/cumulant closure**
- **Perturbation**
- **Monte Carlo Simulation (MCS):** accurate but inefficient.
- **Equivalent Linearization Method (ELM):** efficient but not accurate for distribution tail and first-passage probability.

New method being implemented in OpenSees:

- **Tail-Equivalent Linearization Method (TELM):**
  Based on FORM, is more efficient than MCS and more accurate than ELM for distribution tails and first-passage probability.
Basic elements of TELM

- Discrete representation of stochastic input and definition in terms of standard normal random variables.
- Geometric characteristics of linear system in the standard normal space and determination of the unit-impulse response function (URF).
- Definition of nonlinear response in standard normal space and determination of the tail-equivalent linear system (TELS).
- Random vibration analysis with TELS for each response threshold.
- Determination of first-passage probability and distribution of peak response.
Discrete representation of stochastic input

- **General form for zero-mean Gaussian process**

\[ f(t, \mathbf{u}) = \sum_{i=1}^{n} s_i(t) u_i \]

\[ \mathbf{u} = (u_1, u_2, \ldots, u_n) \quad \text{vector of standard normal random variables} \]

\[ s_i(t), i = 1, 2, \ldots, n, \quad \text{deterministic shape functions} \]

- **White noise**

\[ s_i(t) = \sigma \delta(t - t_i) \]
Discrete representation of stochastic input

- General form for zero-mean Gaussian process

\[ f(t, \mathbf{u}) = \sum_{i=1}^{n} s_i(t) u_i \]

\[ \mathbf{u} = (u_1, u_2, \ldots, u_n) \quad \text{vector of standard normal random variables} \]

\[ s_i(t), i = 1, 2, \ldots, n, \quad \text{deterministic shape functions} \]

Filtered white noise

\[ s_i(t) = \sigma h_f(t - t_i) \]

\[ h_f(t) \quad \text{filter URF} \]

Sample realization
Discrete representation of stochastic input

- **General form for zero-mean Gaussian process**
  \[ f(t, \mathbf{u}) = \sum_{i=1}^{n} s_i(t) u_i \]
  \[ \mathbf{u} = (u_1, u_2, \ldots, u_n) \] vector of standard normal random variables
  \[ s_i(t), i = 1, 2, \ldots, n, \] deterministic shape functions

- **Nonstationary filtered white noise**
  \[ s_i(t) = \sum q_j(t) h_{fj}(t-t_i) \]
  \[ q_j(t) \] modulating functions

Sample realization
Characterization of a Linear System

- **Generic response**

\[ x(T, \mathbf{u}) = \int_0^T h(T - \tau) f(\tau, \mathbf{u}) d\tau = \sum_i \int_0^T h(T - \tau) s_i(\tau) u_i d\tau = \sum_i a_i(T) u_i \quad h(t) = \text{URF of linear system} \]

- **Hyper-plane for given threshold**

\[ x_0 - \sum_i a_i(T) u_i = 0 \]

\[ \mathbf{u}^* = \frac{\mathbf{a}(T)}{\|\mathbf{a}(T)\|^2} x_0 \quad \text{origin projection point} \]

- **Inverse relations for URF**

\[ \mathbf{a}(T) = \frac{x_0}{\|\mathbf{u}^*\|^2} \frac{\mathbf{u}^*}{\|\mathbf{u}^*\|} \]

\[ \sum_{j=1}^n h(T - t_j) s_i(t_j) \Delta t \equiv a_i(T), \quad i = 1, \ldots, n \]
Response of a nonlinear system

- **Generic response of a nonlinear system**
  \[ x(T, f(t,u)) = x(T,u) \] \text{ implicit function of } u

- **Limit-state surface for given threshold**
  \[ x_0 - x(T,u) = 0 \]

- **Design point for event** \( x(T,u) \geq x_0 \)
  \[ u^* = \min \left[ \|u\|^2 \middle| x_0 - x(T,u) = 0 \right] \]

- **First-Order approximation**
  \[ \Pr[x(T) \geq x_0] \approx \Phi(-\beta) \]

- **TELS**: A linear system that has the same design point and the tail probability \( \Phi(-\beta) \).
Characteristics of the TELS

- **Influences on TELS:**
  - Excitation $f(t)$ – no dependence on scale, mild dependence on frequency content.
  - Threshold $x_0$ – strong dependence.
  - Time $T$ – no dependence for stationary processes.
  - Nature of system nonlinearity – strong dependence.

- To determine distributions, TELS for a sequence of thresholds must be determined. A special algorithm for this purpose is being implemented.
Random vibration analysis with TELS

- **Statistics at given time:**
  - Mean response
  - Standard deviation
  - CDF, PDF
  - Mean up-crossing rate

- **First-passage probability for specified duration**
  - Mean of peak response
  - Standard deviation of peak response
  - CDF, PDF of peak response
Sample application – 6dof structure

- **6-story shear building model**

<table>
<thead>
<tr>
<th>Node</th>
<th>Stiffness ($k_0$, kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 6</td>
<td>$2.0 \times 10^4$</td>
</tr>
<tr>
<td>Node 5</td>
<td>$4.0 \times 10^4$</td>
</tr>
<tr>
<td>Node 4</td>
<td>$5.5 \times 10^4$</td>
</tr>
<tr>
<td>Node 3</td>
<td>$6.5 \times 10^4$</td>
</tr>
<tr>
<td>Node 2</td>
<td>$7.0 \times 10^4$</td>
</tr>
<tr>
<td>Node 1</td>
<td>$7.5 \times 10^4$</td>
</tr>
</tbody>
</table>

- **Limit state – inter-story drifts**

$$g(x_0, \tau) = x_0 - |d_i(\tau)| \leq 0$$

- **Stochastic acceleration**

1. White noise
   - Rock site
   - Firm ground: $T_f = 0.4s$, $\zeta_f = 0.6$
   - Soft ground: $T_f = 0.67s$, $\zeta_f = 0.3$

2. Filtered White Noise
   - Rock site

3% Rayleigh damping is assumed for both $1^{st}$ and $2^{nd}$ mode.
Sample application – Results

- First-passage probability for the duration 10s

- PDF of the maximum response for the duration 10s
PEER methodology

SA of selected ground motions

- Natural period of the objective structure
- Average of recorded motion
- Maximum response
- Mean and standard deviation

Maximum response
Comparisons of the results

Mean and variance of the maximum response

<table>
<thead>
<tr>
<th>EDP</th>
<th>Input motion</th>
<th>Mean, m</th>
<th>C.o.v.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; inter-story drift</td>
<td>Recorded motions</td>
<td>0.0263</td>
<td>0.390</td>
</tr>
<tr>
<td></td>
<td>Stochastic:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rock site</td>
<td>0.0252</td>
<td>0.206</td>
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</tr>
<tr>
<td>firm site</td>
<td>0.0215</td>
<td>0.203</td>
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<tr>
<td>soft site</td>
<td>0.0210</td>
<td>0.226</td>
<td></td>
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<tr>
<td>6&lt;sup&gt;th&lt;/sup&gt; inter-story drift</td>
<td>Recorded motions</td>
<td>0.0238</td>
<td>0.334</td>
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<tr>
<td></td>
<td>Stochastic:</td>
<td></td>
<td></td>
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<tr>
<td>rock site</td>
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<tr>
<td>firm site</td>
<td>0.0307</td>
<td>0.179</td>
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<tr>
<td>soft site</td>
<td>0.0199</td>
<td>0.176</td>
<td></td>
</tr>
</tbody>
</table>

Variations in the mean edp values depending on the stochastic model reflect the effect of the site condition.

The c.o.v. of the edp estimates based on recorded ground motions is about twice that estimated by the stochastic models. This is partly due to mixing of records from different sites.
Application of TELM to the I880 Bridge model

Finite element model

Limit-state
\[ G(x_0) = x_0 - d_{15005} \leq 0 \]

First-passage probability
(white noise excitation)

Finite element model

Concrete

Steel
Summary

- The tail-equivalent linearization method (TELM) for nonlinear random vibration analysis is developed.
- The first-passage probability and the distribution of the maximum response can be computed efficiently and accurately using the tail-equivalent linearized system.
- The method can be used to compute the conditional distribution of an EDP for a given im.
- One advantage of the approach within the context of PEER’s PBEE framework is that it does not require selection and scaling of recorded ground motions.
- Alternative models for discrete representation of stochastic ground motions can be developed and used.
- Full implementation and testing in OpenSees is on-going.