Two and Three-Dimensional Contact Element Implementation for Geotechnical Applications in OpenSees

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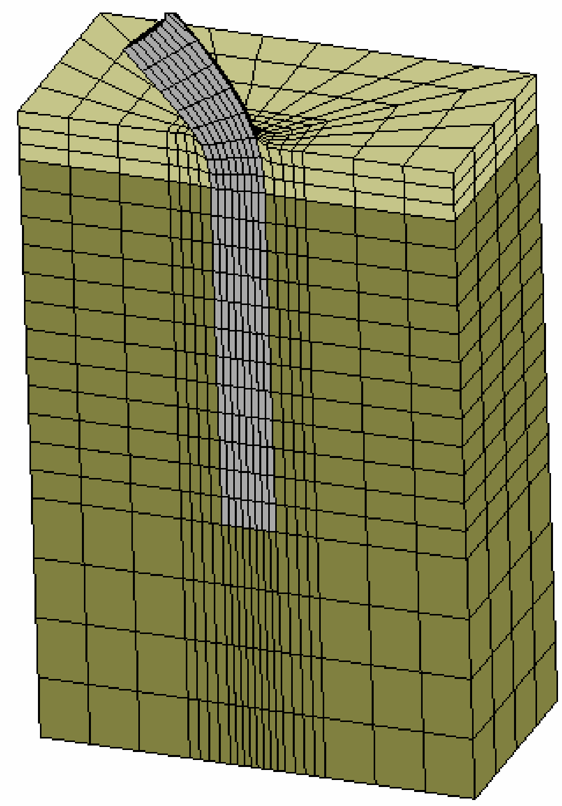
Presentation Overview

- Background
- Contact Element & Interface Material Formulations
- OpenSees Implementation
- Element Features
- Examples

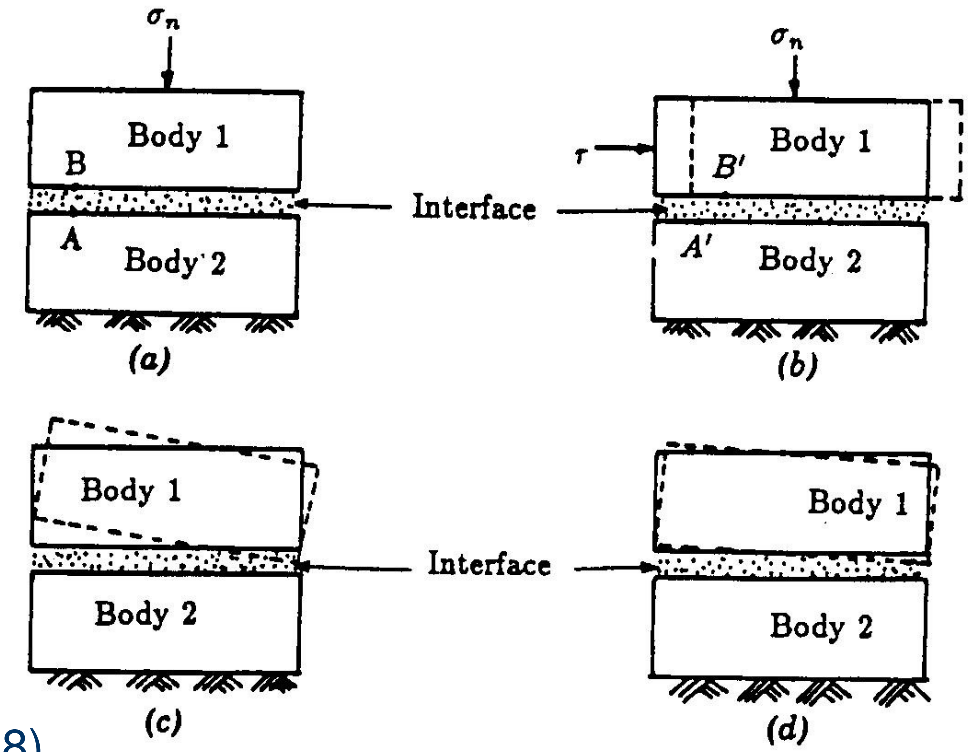
Objectives

1. Realistic soil-pile interaction
2. Consideration of complex soil models
3. Alternative pile modeling approaches



Background: Interface Behavior

Pile-soil interaction: stick, slip, debonding, and rebonding behavior



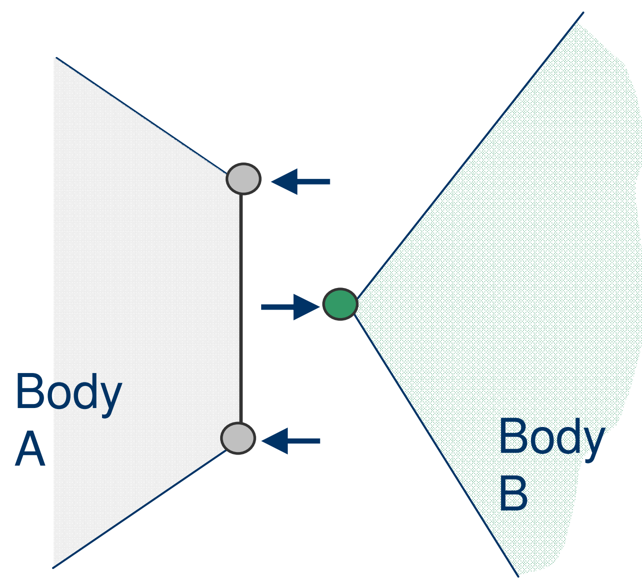
(Desai et al., 1988)

Background: Interface Behavior

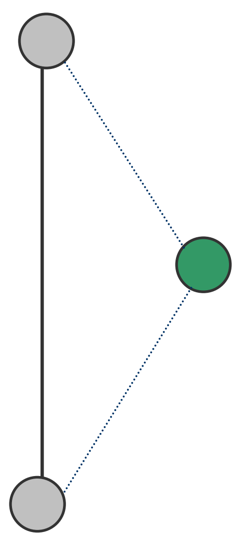
Pile-soil interaction: stick, slip, debonding, and rebonding behavior

Finite Element approaches:

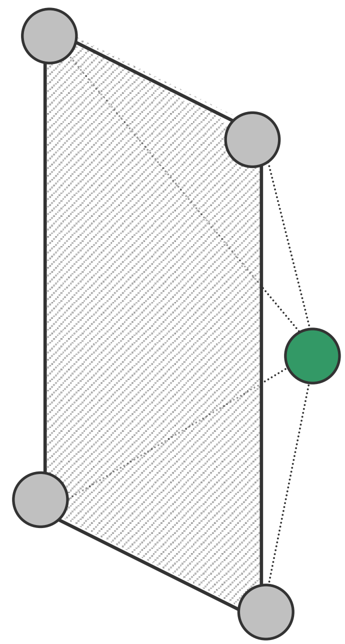
- Zero-length elements
- Joint and thin-layer elements
- Gap elements



Contact Element Model Development

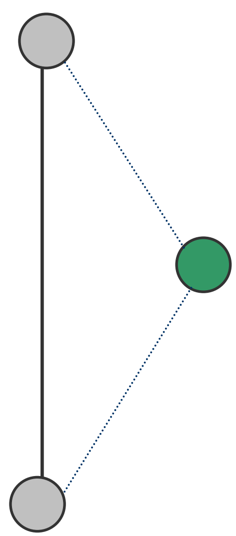


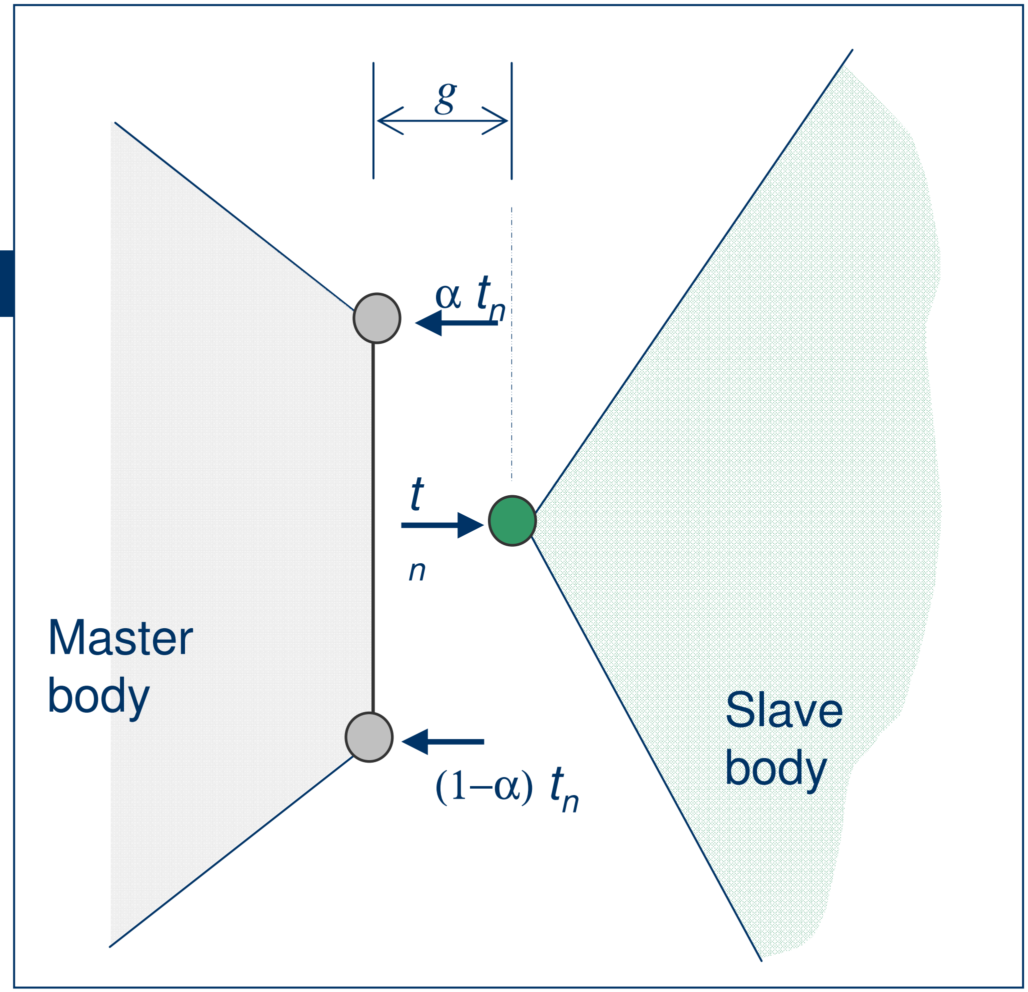
Node-to-Segment Element



Node-to-Surface Element

Contact Element





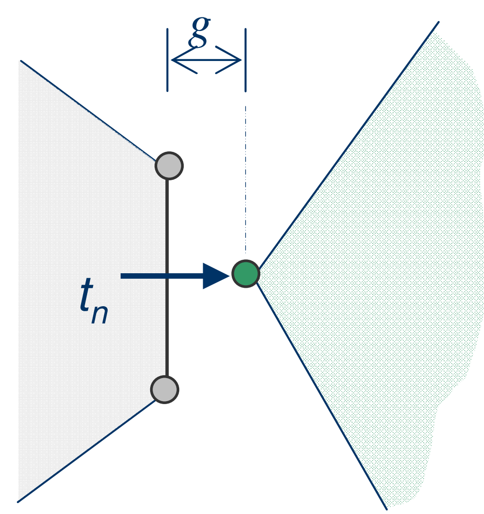
Node-to-Segment Element

Contact Element Formulation

Contact element applies a geometric constraint to the system that relates a slave node to a master contact line segment or surface.

Using the method of Lagrange Multipliers, the element utilizes the Hertz-Signori-Moreau conditions for contact:

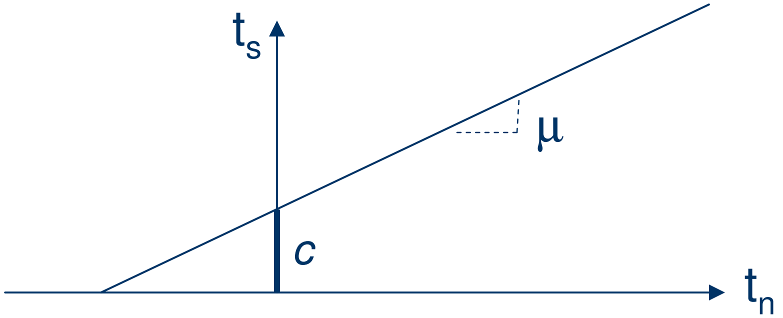
$$g \geq 0 \qquad t_n \geq 0 \qquad t_n \cdot g = 0$$



Contact Material Formulation

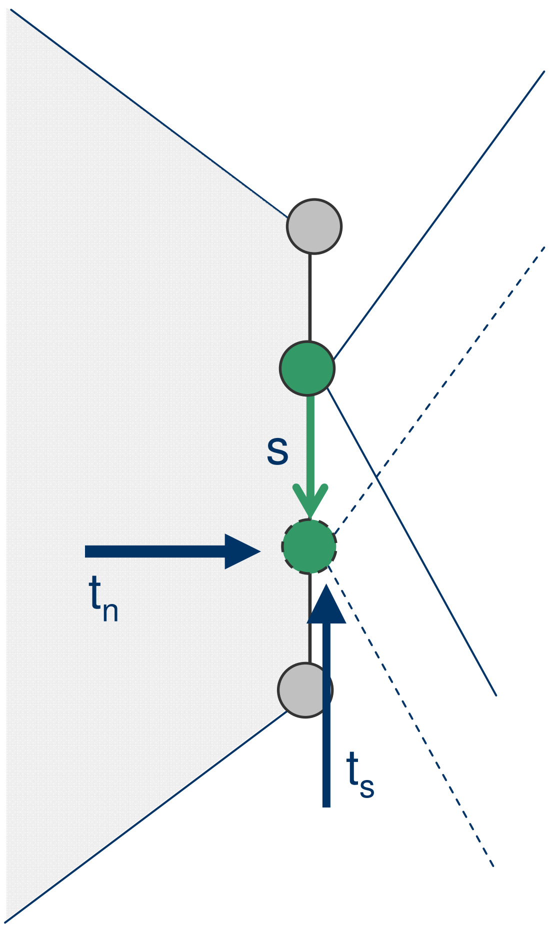
- The geometric constraints are related with an interface constitutive law:
  - Mohr-Coulomb Friction Law

$$f = |t_s| - \mu \cdot t_n - c \leq 0$$

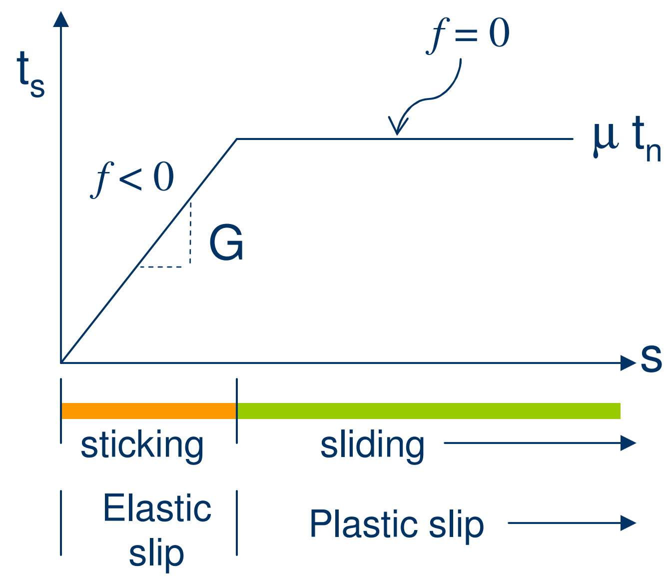


  - Can also use non-linear and history dependent material models, including specific models for concrete structures on soil

Contact Material Formulation



$$f = |t_s| - \mu \cdot t_n - c \leq 0$$



Variational Contact Formulation

- Expression for Virtual Work:

$$\delta W = t_n\, \delta g + \delta t_n\, g - t_s\, \delta s$$

- Linearization:

$$d(\delta W) = \delta g\, dt_n + \delta t_n\, dg - \delta s\, dt_s$$

$$g = g(\boldsymbol{q})$$

$$s = s(\boldsymbol{q})$$

$$dt_s = \frac{\partial t_s}{\partial s} ds + \frac{\partial t_s}{\partial t_n} dt_n =: C_{ss} ds + C_{sn} dt_n$$

Note: C_{ss} & C_{sn} depend on the state: sticking, sliding

2D Contact Formulation

Linearization and 2D Tangent Stiffness Matrix:

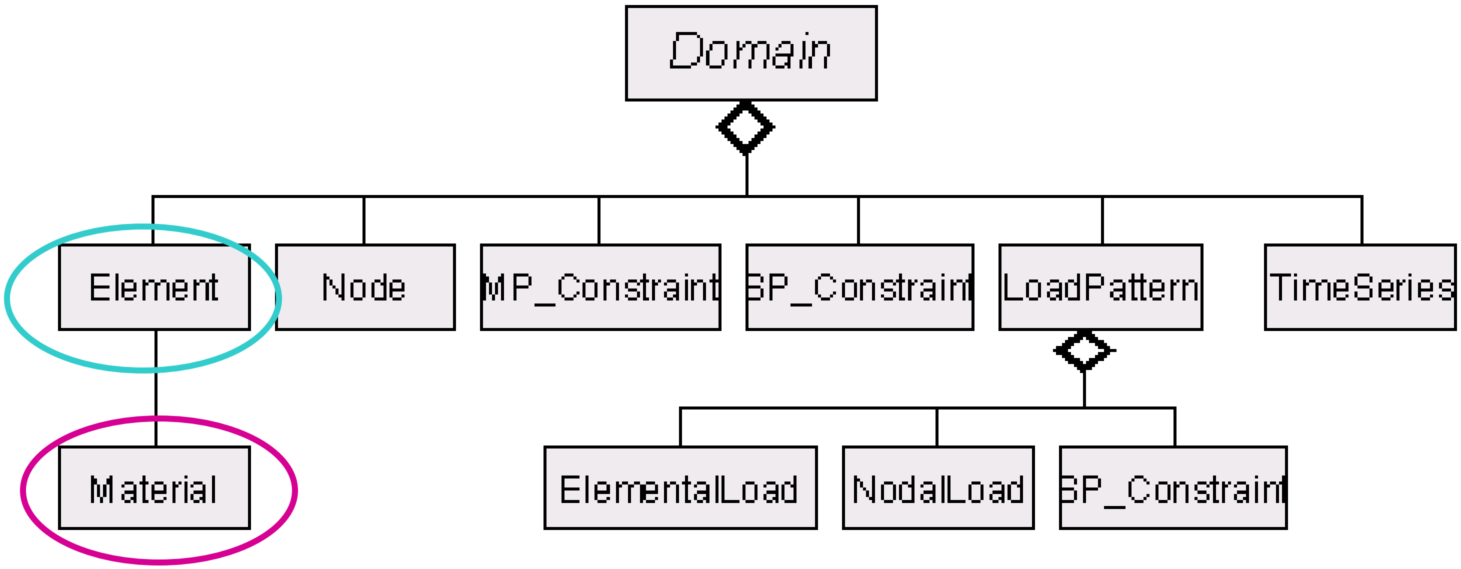
$$d(\delta W) = \delta g \; dt_n + \delta t_n \; dg - \delta s \; dt_s$$

$$\delta g =: \delta \boldsymbol{q}^T \boldsymbol{B}_n \qquad \boldsymbol{B}_n = \begin{Bmatrix} \alpha \, \boldsymbol{n} \\ (1-\alpha)\, \boldsymbol{n} \\ -\boldsymbol{n} \end{Bmatrix} \qquad \delta s =: \delta \boldsymbol{q}^T \boldsymbol{B}_s \qquad \boldsymbol{B}_s = \begin{Bmatrix} \alpha \, \boldsymbol{t} \\ (1-\alpha)\, \boldsymbol{t} \\ -\boldsymbol{t} \end{Bmatrix}$$

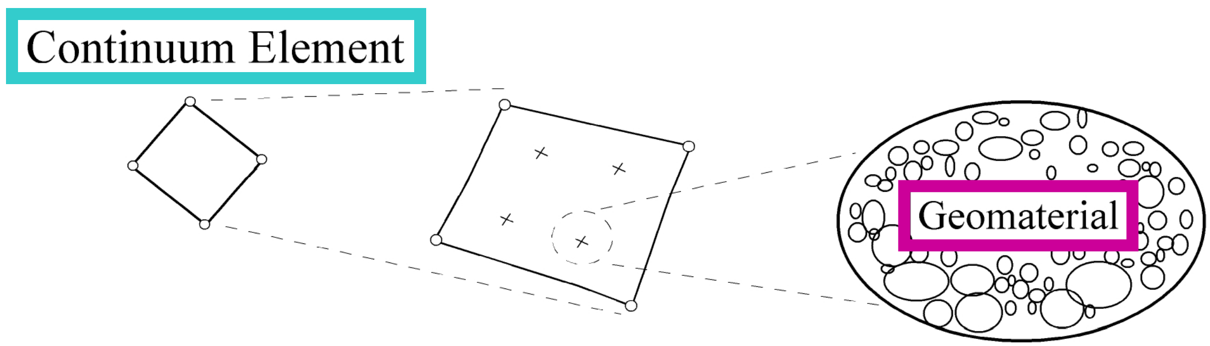
$$d(\delta W) = \begin{bmatrix} \delta \boldsymbol{q}^T & \delta \, t_n \end{bmatrix} \cdot \underbrace{\begin{bmatrix} -\boldsymbol{B}_s \boldsymbol{C}_{ss} \boldsymbol{B}_s^T & \boldsymbol{B}_n^T - \boldsymbol{B}_s^T \boldsymbol{C}_{sn} \\ \boldsymbol{B}_n^T & 0 \end{bmatrix}}_{\boldsymbol{K}_T} \cdot \begin{Bmatrix} d\boldsymbol{q} \\ d \, t_n \end{Bmatrix}$$

Implementation in OpenSees

New element and material classes



Implementation in OpenSees



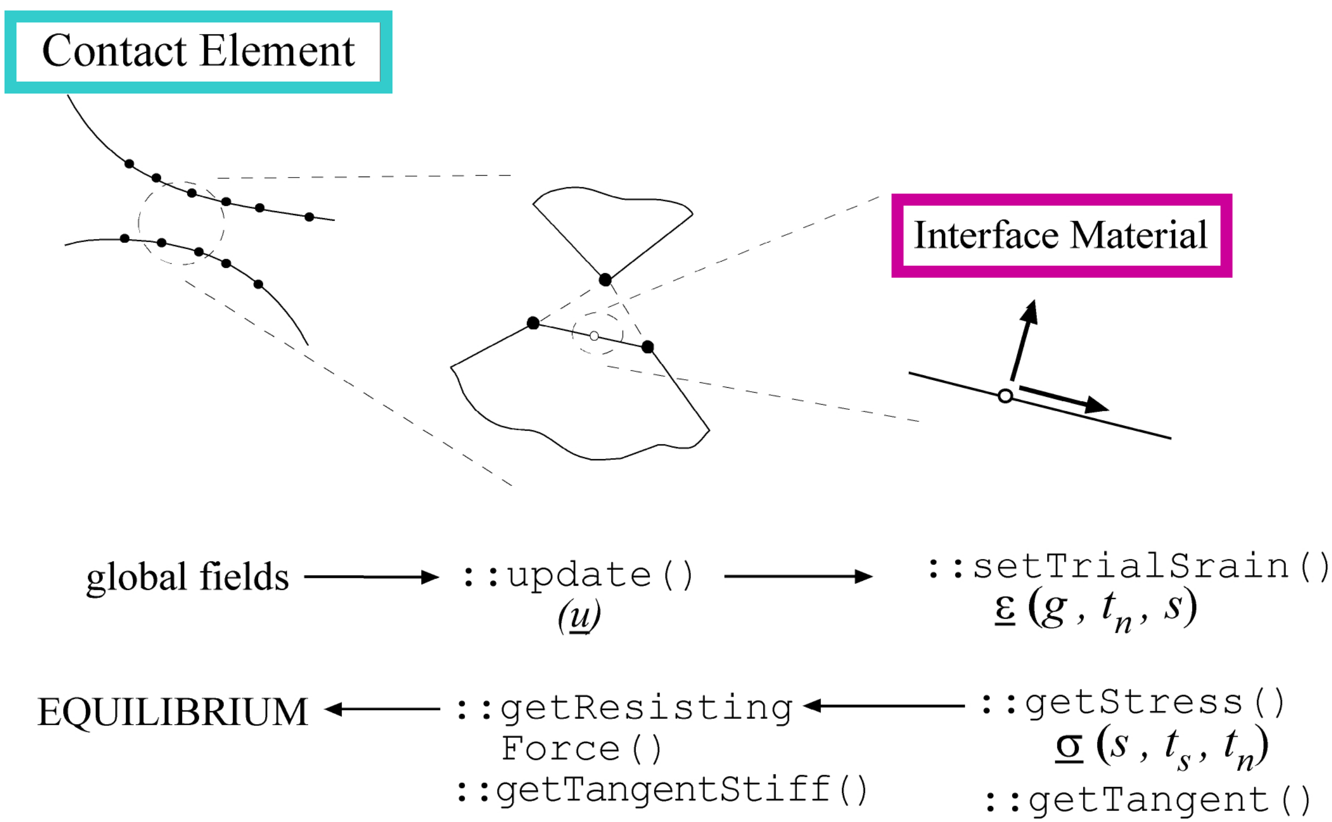
global fields → ::update() $(\underline{u})$ → ::setTrialSrain() $\underline{\varepsilon}\ (\underline{u})$

EQUILIBRIUM ← ::getResisting Force() ← ::getStress() $\underline{\sigma}\ (\underline{\varepsilon})$

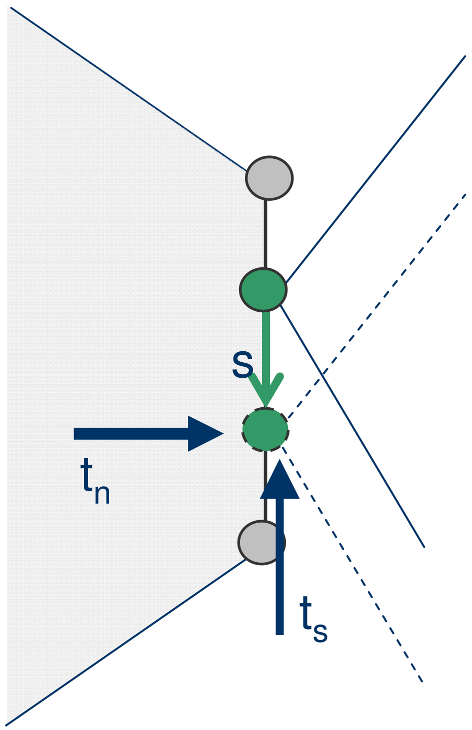
::getTangentStiff()

::getTangent()

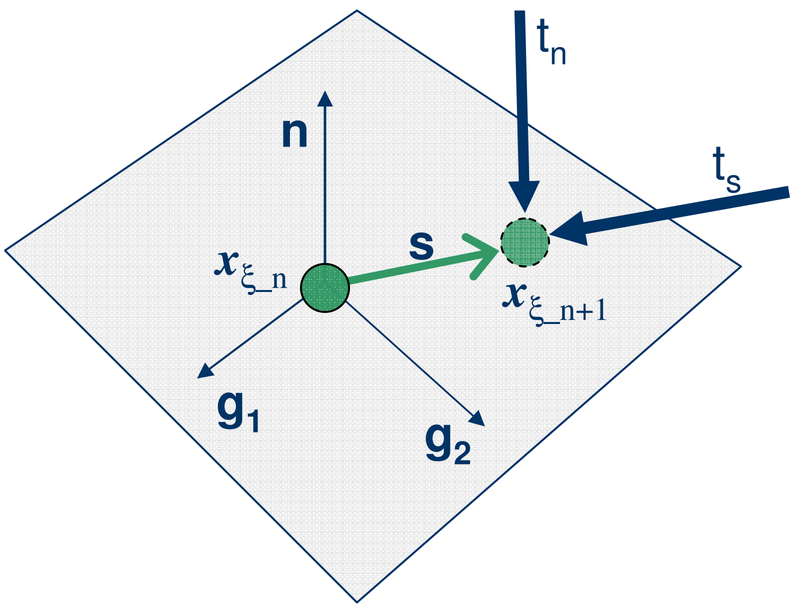
Implementation in OpenSees



3D Contact Element

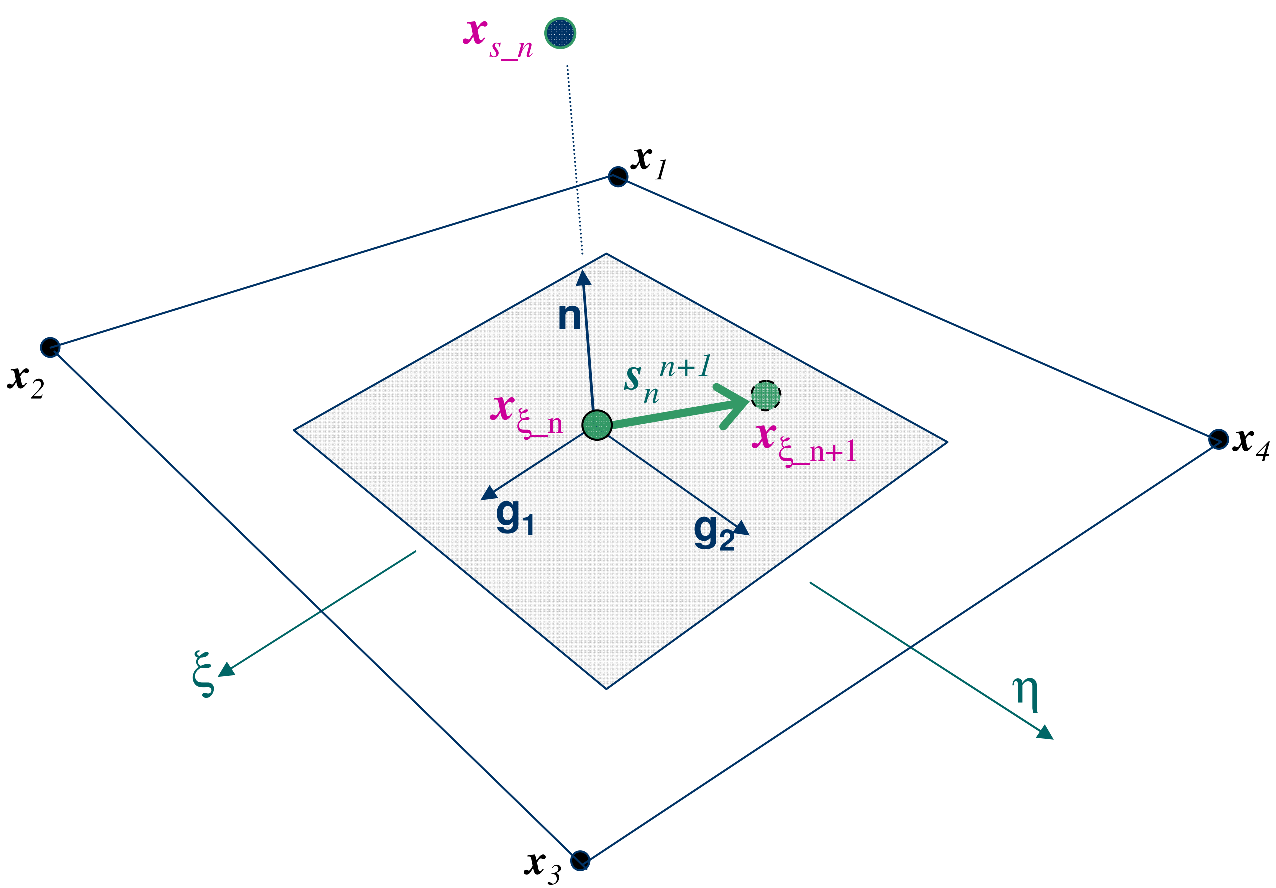


2D Node-to-Line Element



3D Node-to-Surface Element

3D Geometric Pseudo-Nonlinearity



- Project x_{s_n} *on to master surface patch & determine tangent plane*
- *Slip*, s_n^{n+1}, moves along tangent plane of step *n*
- Converges to nonlinear solution

Solution Strategy w/ Lag Step

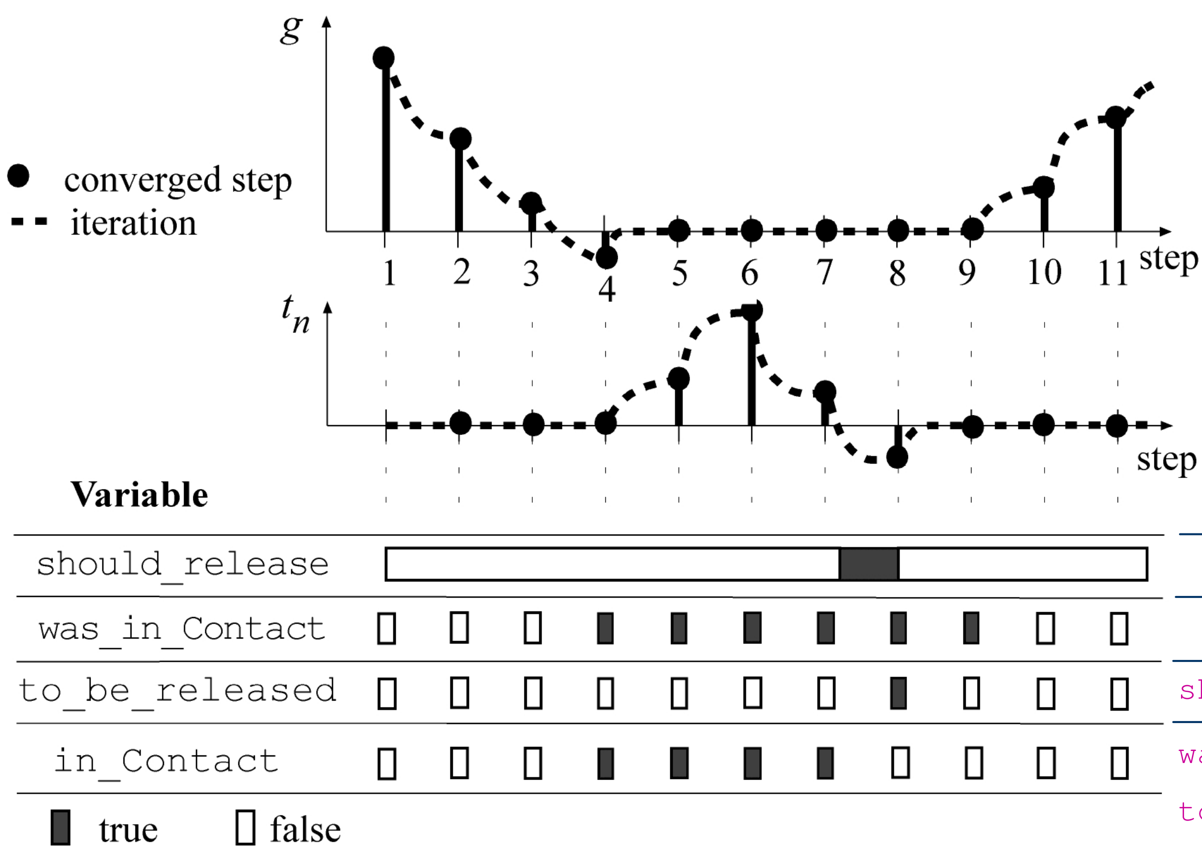
- No contact search algorithm
- Contact Conditions:

$$g = \left(\boldsymbol{x}_s - \boldsymbol{x}_\xi\right) \cdot \boldsymbol{n} \quad ? \begin{cases} > 0 \\ \leq 0 \end{cases} \qquad\qquad t_{\mathrm{n}} \quad ? \begin{cases} \geq 0 \\ < 0 \end{cases}$$

Determines: - in contact
- not in contact
- should be released

- Added lag step for stability near boundary of in and out of contact

Solution Strategy w/ Lag Step



$t_n < 0$

$g \leq 0$

should_release = true

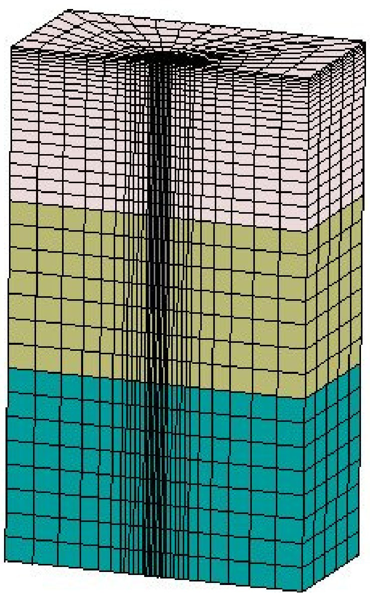
was_in_Contact = true

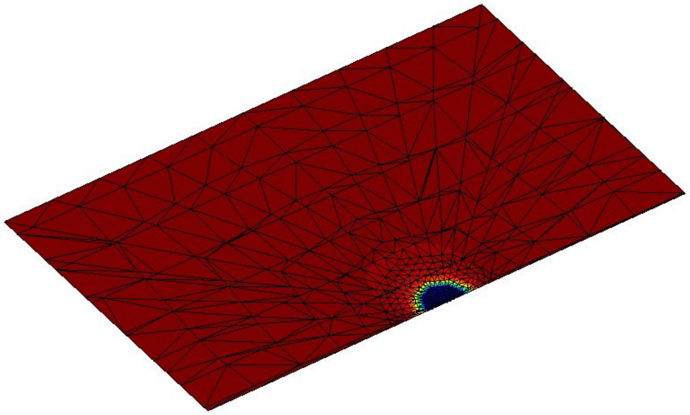
to_be_released = false

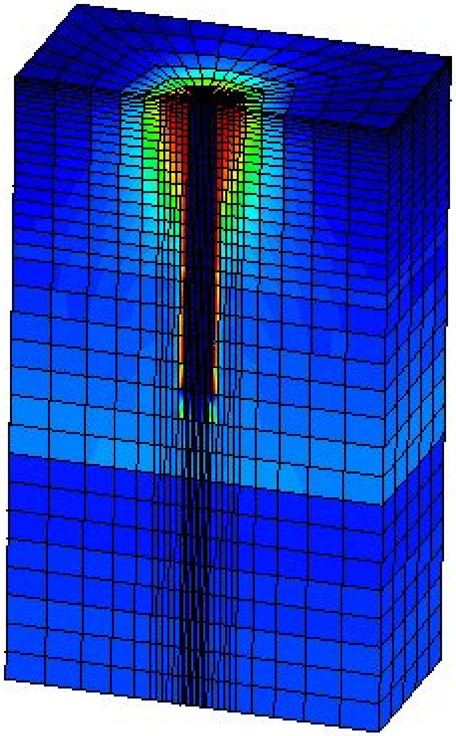
GiD Development

Developed pre- and post-processing tools using commercial software GiD

- Model creation
- Mesh generation
- Results visualization

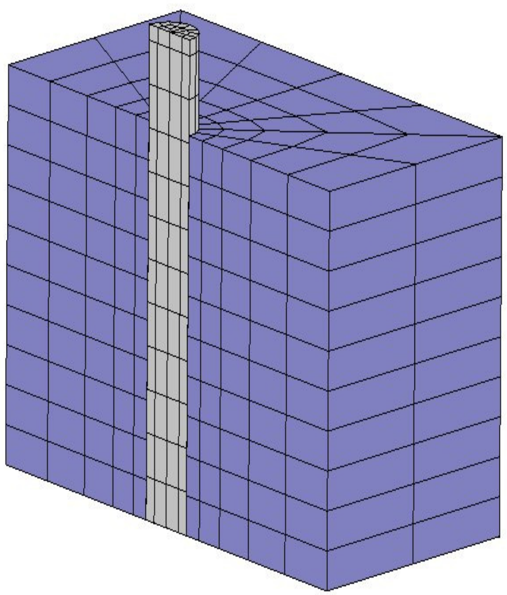


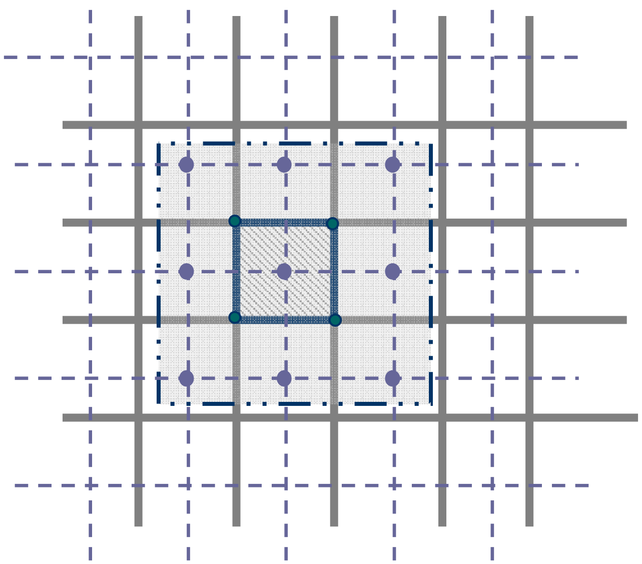




GiD Contact Element Generation

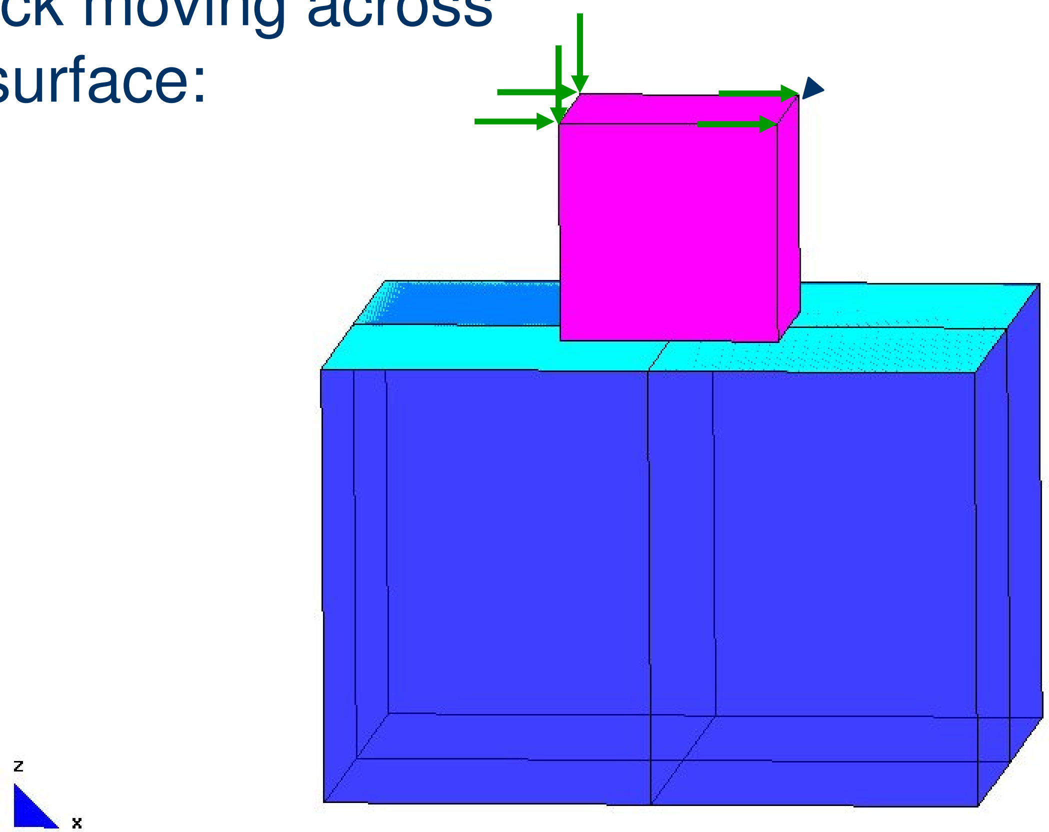
- No native support for this type of element
- GiD creates contact pairs for all nodes within range that can go in and out of contact.



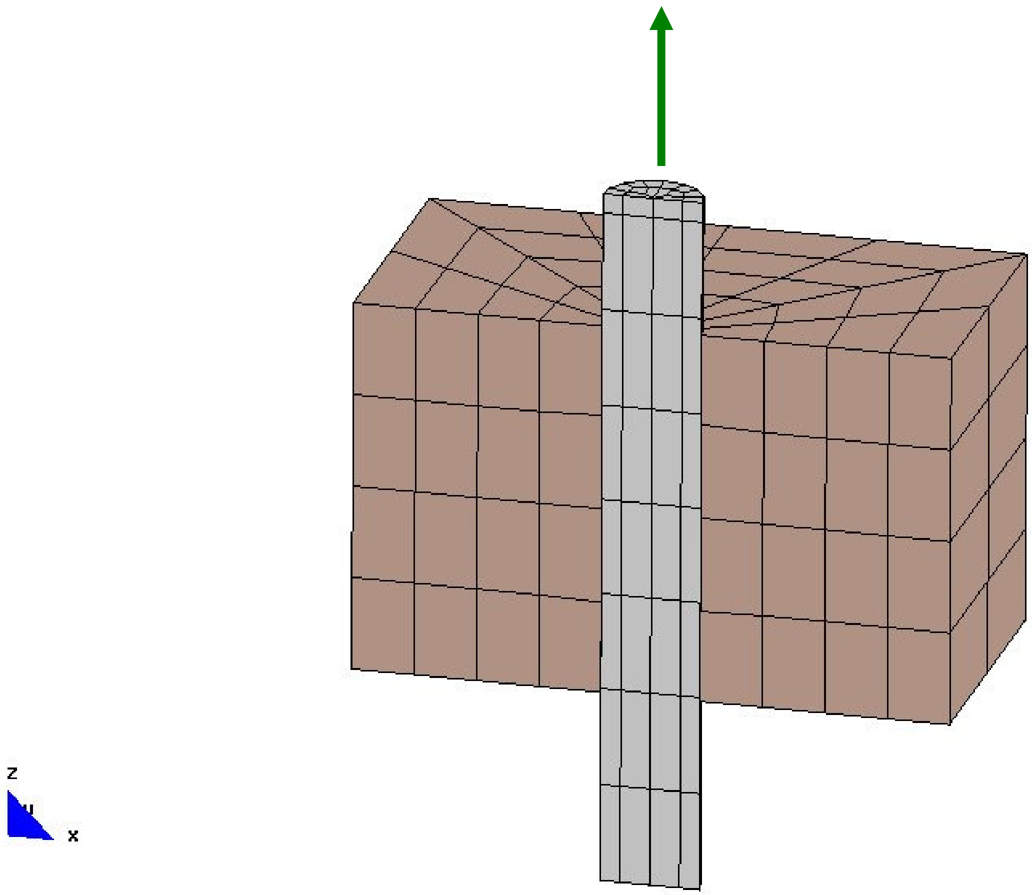


Example 1: Simple 3D Blocks

Block moving across surface:



Example 2: 3D Friction Pile



Example 2: 3D Friction Pile

Parameter Testing and Calibration: Evaluate frictional forces developed in contact element

$$Q_{contact} = \sum_{i=1}^{N} f_{s_i}$$

And compare with conventional β-method used for pile analysis :

$$Q_s = \pi \, \mathrm{B} \int_0^D \overline{\sigma}_h(z) \tan\delta \, \mathrm{d}z$$

Example 2: 3D Friction Pile

Frictional contact element gives good approximation:

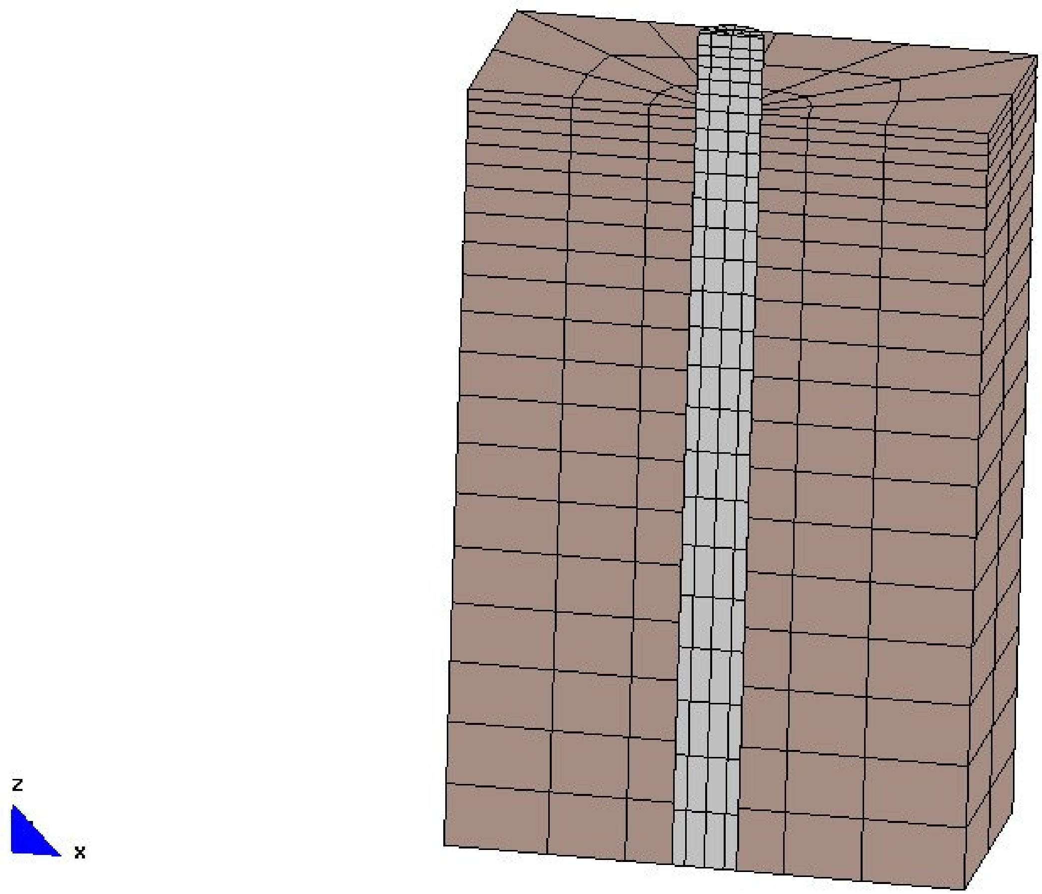
μ	G	$Q_{contact} = \sum_{i=1}^{N} f_{s_i}$	$Q_s = \pi \mathrm{B} \int_0^D \overline{\sigma}_h(z) \tan\delta \, dz$	
0.25	1000	63	55	13%
0.5	1000	128	110	14%
1	1000	462	440	5%
2	1000	918	880	4%

Typical Values of μ = tan δ :

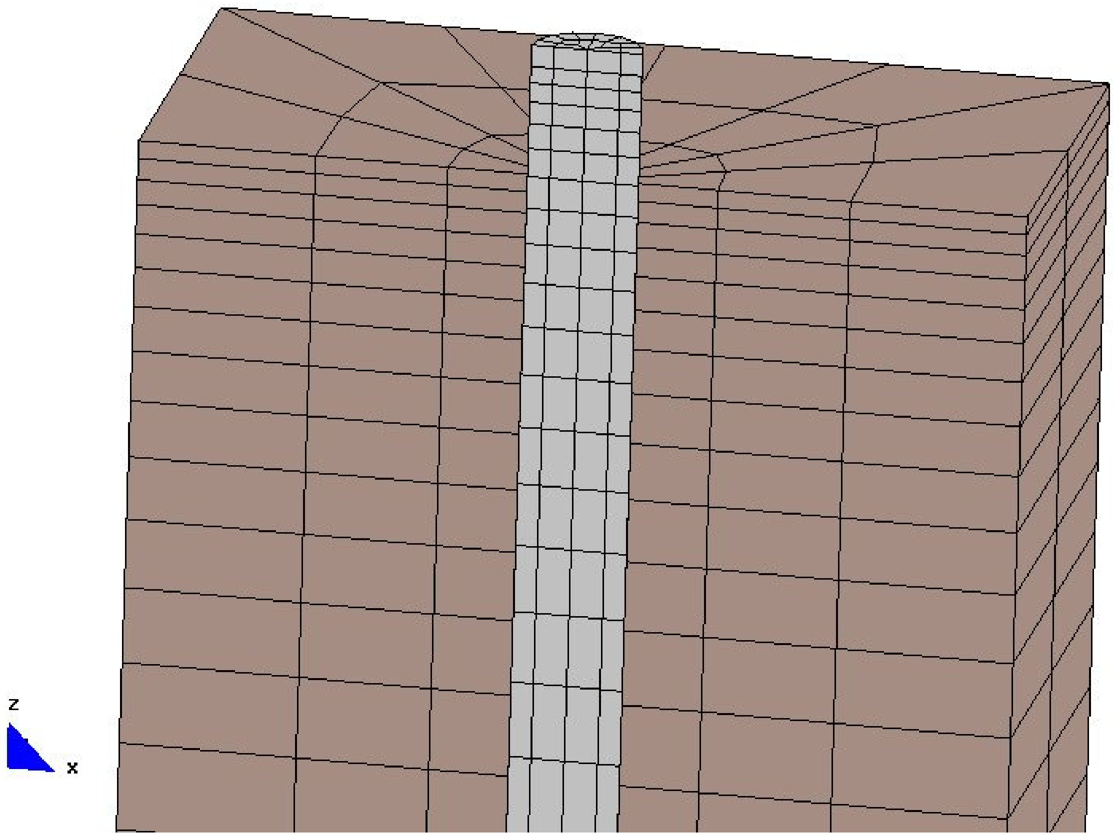
δ / ϕ = 1.0 – 0.5

Concrete-sand: μ = 0.35 – 0.6

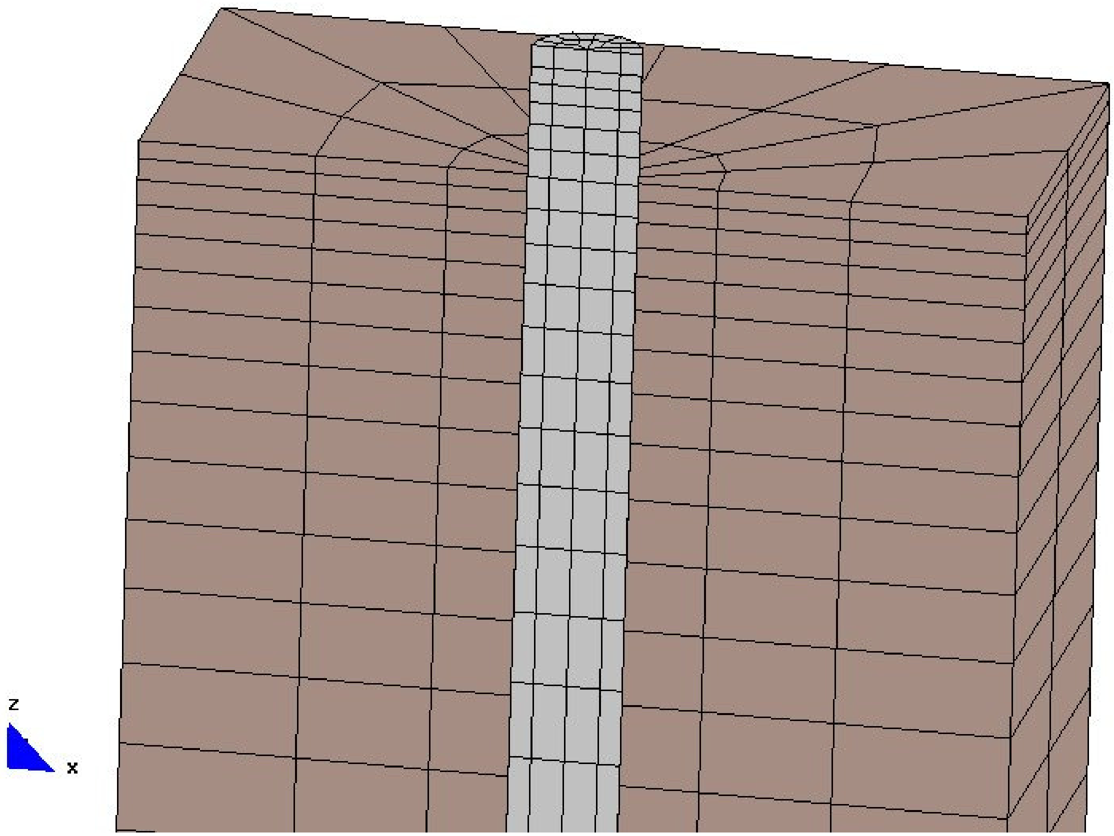
Example 3: 3D Pushover Analysis



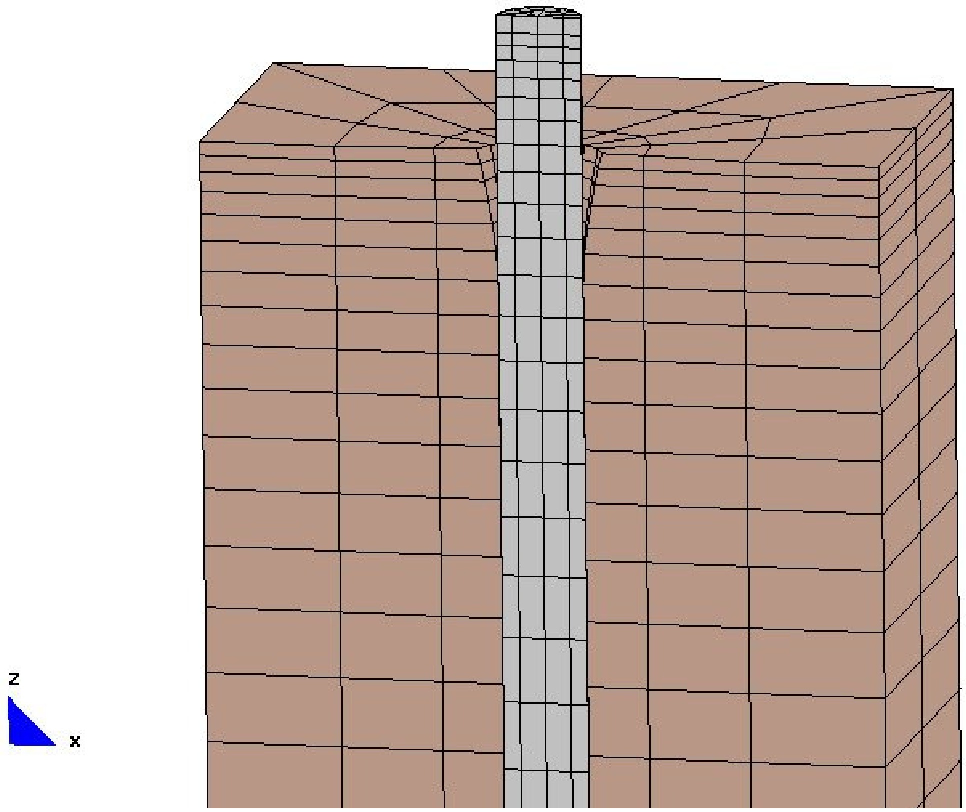
Example 3: 3D Pushover Analysis



Example 3: 3D Pushover Analysis with Cohesive Contact Material



Example 3: 3D Pushover Analysis with Plastic Soil



Summary

- Contact elements are implemented in OpenSees using a stable, pseudo-nonlinear approach
- Examples demonstrate element capability to describe interface behavior for pile analysis
- Further validation and testing is underway prior to submission to OpenSees repository.