set ORIGIN =1

$ORIGIN := 1$

Note: Appendix D in the following text corresponds to Appendix D in Kadysiewski and Mosalam, PEER Report, 2009

Set the units

$kip := 1$

$in := 1$

$sec := 1$

$$m := \left(\frac{1}{0.0254} \cdot in\right)$$

$kN := 0.224808943 \cdot kip$

$$N := \left(\frac{1}{1000}\right) \cdot kN$$

$cm := 0.01 \cdot m$

$mm := 0.001 \cdot m$

$foot := 0.3048 \cdot m$

$$MPa := 1000 \cdot \frac{kN}{m^2}$$

$$psi := \frac{MPa}{145.037738}$$

$lbf := psi \cdot in \cdot in$

$ksi := 1000 \cdot psi$

$$g := 9.81 \cdot \frac{m}{sec^2}$$

Calculation of Infill Properties

$fme := 1.00 \cdot ksi$ Masonry expected compressive strength

$tinf := 6.00 \cdot in$ Thickness of masonry infill wall

$hinf := 140.0 \cdot in$ Height of masonry infill wall

$Linf := 120.0 \cdot in$ Length of masonry infill wall

$hcol := 140 \cdot in$ Floor-to-floor height

$Lcol := 120.0 \cdot in$ Centerline distance between columns

$Em := 550 \cdot fme$ FEMA 356 formula for masonry elastic modulus
(expected and lower bound, unit: psi, Tables 7-1 and 7-2
You can input Em seperately as well

$Em := 500 \cdot ksi$

$Efe := 3122 \cdot ksi$ Expected elastic modulus of frame concrete

$Ig := 8856 \cdot in^4$ Gross moment of inertia of the concrete columns

$Icol := 0.5 \cdot Ig$ Effective cracked moment of inertia of the concrete columns

$Icol = 4.428 \times 10^3$

$rinf := \sqrt{hinf^2 + Linf^2}$ Diagonal length of the infill

$rinf = 184.391$

$\theta inf := atan\left(\frac{hinf}{Linf}\right)$ Angle of the diagonal for the infill

$\theta inf = 0.862$

$Ldiag := \sqrt{hcol^2 + Lcol^2}$ Diagonal length between column centerlines and floor centerlines

$Ldiag = 184.391$

$\theta diag := atan\left(\frac{hcol}{Lcol}\right)$ Angle of the diagonal between beam-column workpoints

$\theta diag = 0.862$

$\Gamma := 1.2732$ PEER 2008/102, D-5

Calculate the axial stiffness of the infill strut:

Calculate the width of the compresiion strut which represents the infill, based on the method given in FEMA 356, Section 7.5.2

$$\lambda1 := \left(\frac{Em \cdot tinf \cdot \sin(2 \cdot \theta inf)}{4 \cdot Efe \cdot Icol \cdot hinf}\right)^{\frac{1}{4}}$$

$$\lambda1 = 0.025$$

$$a := 0.175 \cdot (\lambda1 \cdot hcol)^{-0.4} \cdot rinf$$

$$a = 19.589$$

$$kinf := \frac{a \cdot tinf \cdot Em}{rinf}$$

$$kinf = 318.708$$

Calculate the required area of the equivalent element, which will span between workpoints and will have an elastic modulus equal to Em

$$Aelem := \frac{kinf \cdot Ldiag}{Em}$$

$$Aelem = 117.534$$

Calculate the axial strength of the infill strut (Based on FEMA356, Section 7.5.2.2)

$Pce := 41.4 \cdot kip$ Expected gravity compressive force applied to infill panel

$vte := 900 \cdot psi$ Average bed joint strength

$An := tinf \cdot Linf$ Net bedded area of the infill

$$An = 720$$

$vme := \dfrac{0.75 \cdot \left(vte + \dfrac{Pce}{An}\right)}{1.5}$ Expected masonry shear strength

$$vme = 0.479$$

$$fvie := 0.05 \cdot ksi$$

$$vshear := \min(vme, fvie)$$

$$vshear = 0.05$$

$Qce := vshear \cdot An$ Expected horizontal shear capacity of infill

$Qce = 36$

$$Pn0 := \frac{Qce}{\cos(\theta diag)}$$

Axial capacity of the equivalent compression strut

$Pn0 = 55.317$

Calculate the in-plane displacement properties of the infill strut:

Calculate the "yield point", i.e., the axial deformation in the equivalent strut at the point where the initial tangent stiffness line intersects the element capacity:

$$\delta Ay0 := \frac{Pn0}{kinf}$$

assumes no OOP load

$\delta Ay0 = 0.174$

Calculate the IP horizontal deflection of the panel at the yield point:

$$uHy0 := \frac{\delta Ay0}{\cos(\theta diag)}$$

$uHy0 = 0.267$ Note : assumes that the vertical deflections of the endpoints are zero

Calculate the lateral deflection of the panel at the collpase prevention (CP) limit state:
Based on FEMA 356, Section 7.5.3.2.4, including Table 7-9:

1) Estimate 0.7< β < 1.3, where β is defined as Vfre/Vine, the ratio of frame to infill expected strengths

2) $\frac{Linf}{hinf} = 0.857$

3) $$table79 := \begin{pmatrix} 0.5 & 1\% \\ 1 & 0.8\% \\ 2 & 0.6\% \end{pmatrix}$$

Interpolated in Table 7-9. It is assumed that the CP limit state is reached when the element drift reaches point "d" as shown in Figure 7.1 of FEMA356

$$d := \text{linterp}\left(table79^{\langle 1 \rangle}, table79^{\langle 2 \rangle}, \frac{Linf}{hinf}\right)$$

$d = 0.00857$

$uHcp0 := d \cdot hinf$ Displacement of the panel at the limit state

$uHcp0 = 1.2$

$$\mu H0 := \frac{uHcp0}{uHy0}$$ Implied ductility at the collapse prevention level

$\mu H0 = 4.499$

Calculate the Out-of-Plane (OOP) parameters of the infill:

$\gamma inf := 15 \cdot \frac{kN}{m^3}$ Weight density of the infill bricks (assumed).

Calculate the OOP frequency of the infill, assuming that it spans vertically, with simply-supported ends:

$Iinf\_g := \frac{Linf \cdot tinf^3}{12}$ Moment of inertia of the uncracked infill (gross moment)

$Iinf := \frac{1}{2} \cdot Iinf\_g$ Estimated moment of inertia of the cracked infill

$Iinf = 1.08 \times 10^3$

$winf := Linf \cdot tinf \cdot \gamma inf$ Weight per unit of length (measured vertically) of the infill.

$winf = 0.04$

$fss := \frac{\pi}{2 \cdot hinf^2} \cdot \sqrt{\frac{Em \cdot Iinf \cdot g}{winf}}$ First natural frequency of the infill, spanning in the vertical direction, with top and bottom ends simply supported. (Blevins, 1979, Table 8-1).

$fss = 5.802$ $per := \frac{1}{fss}$

$per = 0.1723$

Calculate the OOP effective weight:

The OOP effective weight is based on the modal effective mass of the vertically spanning, simply supported (assumed) infill wall. For simple-simple conditions, the modal effective weight is equal to 81% of the total infill weight. See Appendix D for a derivation of this value.

$Winf := \gamma inf \cdot tinf \cdot hinf \cdot Linf$ Total weight of the infill.

$Winf = 5.57$

$MEW := 0.81 \cdot Winf$

$MEW = 4.512$

Modal effective weight, assuming that the wall spans vertically, is simply supported top and bottom.
(First mode). See Appendix D.
This value divide by g is the OOP mass value that should be assigned to the center node

----

Calculate the equivalent OOP spring which will provide the identical frequency.

$$k_{eq\_N} := (2\cdot\pi\cdot fss)^2\cdot\frac{MEW}{g}$$

$$k_{eq\_N} = 15.527$$

Calculate the moment of inertia of the equivalent beam element, such that it will provide the correct value of k_{eq_N}:

$$I_{eq} := \frac{k_{eq\_N}\cdot\left(Ldiag^3\right)}{48\cdot Em}$$

$I_{eq} = 4056.03129$ $\qquad$ $I_{elem} := I_{eq}$ $\qquad$ $\frac{a\cdot tinf^3}{12} = 352.601$

Using Equation D.28 from Appendix D:

$1.644\cdot\left(\frac{Ldiag}{hinf}\right)^3\cdot Iinf = 4056.6$ $\qquad$ (Same results)

Calculate the OOP Capacity of the infill:

The OOP capacity is based on FEMA 356, Section 7.5.3.2.

$$\frac{hinf}{tinf} = 23.333$$

Since this value is outside the range used in FEMA 356, Table 7-11, for determining λ, perform an extrapolation:

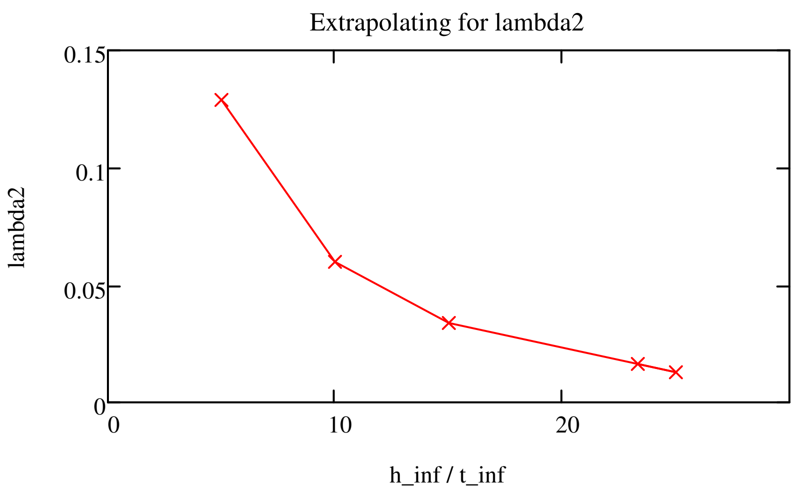
Array of values from Table 7-11:

$$FEMA\_Array := \begin{pmatrix} 5 & 0.129 \\ 10 & 0.060 \\ 15 & 0.034 \\ 25 & 0.013 \end{pmatrix}$$

$$\lambda_2 := \text{linterp}\left(\text{FEMA\_Array}^{\langle 1\rangle}, \text{FEMA\_Array}^{\langle 2\rangle}, \frac{\text{hinf}}{\text{tinf}}\right) \qquad \lambda_2 = 0.01650$$

Check by graphing the values:

$$\text{Array} := \begin{pmatrix} 5 & 0.129 \\ 10 & 0.060 \\ 15 & 0.034 \\ 25 & 0.013 \\ \frac{\text{hinf}}{\text{tinf}} & \lambda_2 \end{pmatrix} \qquad k := 1 .. \text{rows(Array)}$$



$$q_{in} := \frac{0.7 \cdot \text{fme} \cdot \lambda_2}{\frac{\text{hinf}}{\text{tinf}}}$$

Note: the expected, rather than the lower bound value, of masonry compressive strength is used here, since the expected OOP strength will be used in later calculations.

$q_{in} = 0.000495$ $\qquad$ $q_{in} = 0.495 \cdot \text{psi}$

$q_{in} \cdot \text{hinf} \cdot \text{Linf} = 8.316$ $\qquad$ Total OOP force on the wall at capacity.

Calculate the moment in the infill wall at the time that it reaches its capacity:

$$M_y := \frac{q_{in} \cdot Linf \cdot hinf^2}{8}$$

Assumes simple support at the top and bottom.

$$M_y = 145.53$$

Calculate the required yield moment for the equivalent element, such that the same base motion will bring it and the original wall to incipient yield:

$$M_{eq\_y} := 1.570 \cdot \frac{Ldiag}{hinf} \cdot M_y$$

Note: for derivation of this equation, see Appendix D.

$$M_{eq\_y} = 300.929 \cdot in \cdot kip$$

$$Mn0 := M_{eq\_y}$$

Defines the OOP "yield" moment for the equivalent member when the IP axial force is zero.

Determine the OOP point force, applied at the midspan of the equivalent element, to cause yielding:

$$F_{Ny0} := \frac{4 \cdot M_{eq\_y}}{Ldiag}$$

$$F_{Ny0} = 6.528$$

Calculate the displacement of the equivalent element at first yield and at the collapse prevention limit state, assuming no IP axial force:

$$u_{Ny0} := \frac{F_{Ny0}}{k_{eq\_N}}$$

OOP "yield" displacement, assuming no IP axial force.

$$u_{Ny0} = 0.42 \cdot in$$

The displacement at collapse prevention limit state:

FEMA 356, Section 7.5.3.3 gives a maximum OOP deflection based on an OOP story drift ratio of 5%.

$u_{Ncp0} := 0.05 \cdot hinf$

$u_{Ncp0} = 7$

This value seems too high, since it's larger than the thickness of the infill itself. Instead, define the CP displacement as equal to one half the thickness of the infill.

$u_{Ncp0} := \min\left(0.05 \cdot hinf, \frac{tinf}{2}\right)$

$u_{Ncp0} = 3 \cdot in$

The implied ductility ratio is:

$\mu_{Ncp0} := \frac{u_{Ncp0}}{u_{Ny0}}$

$\mu_{Ncp0} = 7.136$

This ductility still seems too high. Based on judgment, use a (conservative) ductility of 5:

$\mu_{Ncp0} := 5$

$u_{Ncp0} := u_{Ny0} \cdot \mu_{Ncp0}$

$u_{Ncp0} = 2.102 \cdot in$

Calculating the axial force - moment interaction curve for specific values of P_{n0} and M_{n0}:

Using the exponent relationship:

$$f\_P_n(M_n, Pn0, Mn0) := Pn0\cdot\left[1-\left(\frac{M_n}{Mn0}\right)^{\frac{3}{2}}\right]^{\frac{2}{3}}$$

This is the target P-M relationship for the equivalent member, located on the diagonal between structural workpoints.

$Pn0 = 55.317\cdot kip$ Axial capacity of the member under pure compression (calculated above).

$Mn0 = 300.929\cdot in\cdot kip$ Moment capacity of the member under pure bending (calculated above).

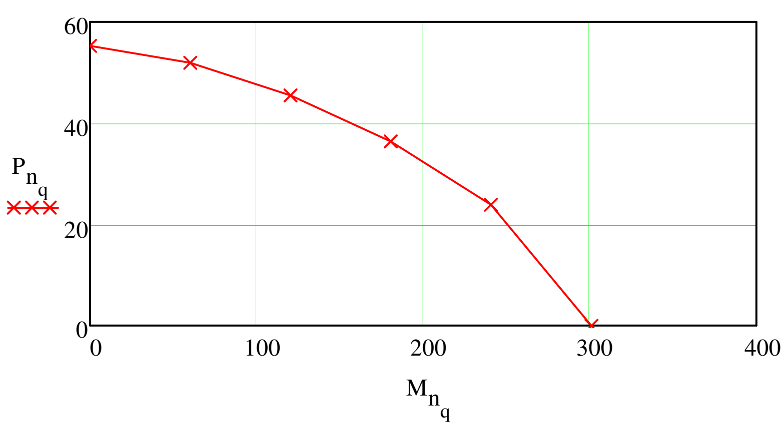
$N_{interaction} := 6$ Number of points on the interaction curve to be used for calculating fiber properties (should be an even number).

$N_{fiber} := 2\cdot(N_{interaction} - 1)$ $\quad N_{fiber} = 10$

$$M_n := \begin{array}{|l} \text{for } q \in 1..N_{interaction} \\ \quad M_{n_q} \leftarrow (q-1)\cdot\dfrac{Mn0}{N_{interaction}-1} \\ M_n \end{array} \qquad P_n := \begin{array}{|l} \text{for } q \in 1..N_{interaction} \\ \quad P_{n_q} \leftarrow f\_P_n(M_{n_q}, Pn0, Mn0) \\ P_n \end{array}$$

$M_n^T = (0\ \ 60.186\ \ 120.371\ \ 180.557\ \ 240.743\ \ 300.92$ $P_n^T = (55.317\ \ 51.968\ \ 45.542\ \ 36.467\ \ 23.926\ \ 0)$

$q := 1..N_{interaction}$



Calculate the required strength and location of the various fibers:

$$F_y := \begin{array}{|l} \text{for } p \in 1..N_{interaction} - 1 \\ \quad F_{y_p} \leftarrow \dfrac{P_{n_p} - P_{n_{p+1}}}{2} \\ \text{for } p \in N_{interaction}..2\cdot(N_{interaction} - 1) \\ \quad F_{y_p} \leftarrow F_{y_{2\cdot N_{interaction}-1-p}} \\ F_y \end{array}$$

$F_y =$

	1
1	1.675
2	3.213
3	4.538
4	6.27
5	11.963
6	11.963
7	6.27
8	4.538
9	3.213
10	1.675

$$\sum_{p=1}^{N_{fiber}} F_{y_p} = 55.317\cdot kip$$

$$z := \begin{array}{|l} \text{for } p \in 1..N_{interaction} - 1 \\ \quad z_p \leftarrow \dfrac{M_{n_{p+1}} - M_{n_p}}{2\cdot F_{y_p}} \\ \text{for } p \in N_{interaction}..2\cdot(N_{interaction} - 1) \\ \quad z_p \leftarrow -\left(z_{2\cdot N_{interaction}-1-p}\right) \\ z \end{array}$$

$z =$

	1
1	17.967
2	9.367
3	6.631
4	4.799
5	2.515
6	-2.515
7	-4.799
8	-6.631
9	-9.367
10	-17.967

$$abs(x) := if(x \geq 0.0, x, -x)$$

Absolute function (since the MathCad absolute function has some bugs).

Solve block for the determining the values of the parameters γ and η:

Estimate the values of the parameters: $\gamma := 30$ $\eta := -.2$

Given

$$\sum_{p=1}^{N_{fiber}} \left[\gamma\cdot\left(abs\left(z_p\right)\right)^{\eta}\right] = Aelem \qquad \sum_{p=1}^{N_{fiber}} \left[\left[\gamma\cdot\left(abs\left(z_p\right)\right)^{\eta}\right]\cdot\left(z_p\right)^2\right] = I_{elem}$$

$Result := Find(\gamma,\eta)$

$\gamma := Result_1\cdot in^2$ $\gamma = 102.789\cdot in^2$

$\eta := Result_2$ $\eta = -1.329$

$$A := \left| \begin{array}{l} \text{for } p \in 1..N_{fiber} \\ \quad A_p \leftarrow \gamma\cdot\left(abs\left(z_p\right)\right)^{\eta} \\ A \end{array} \right.$$

$A^T =$

	1	2	3	4	5	6	7	8
1	2.214	5.261	8.324	12.791	30.177	30.177	12.791	...

Check the results above:

$$\sum_{p=1}^{N_{fiber}} A_p = 117.534 \qquad \sum_{p=1}^{N_{fiber}} \left[A_p\cdot\left(z_p\right)^2\right] = 4056.0313$$

Determine the stress at yield:

$$\sigma_y := \left| \begin{array}{l} \text{for } p \in 1..N_{fiber} \\ \quad \sigma_p \leftarrow \dfrac{F_{y_p}}{A_p} \\ \sigma \end{array} \right.$$

$\sigma_y{}^T =$

	1	2	3	4	5	6	7	8	9	10
1	0.756	0.611	0.545	0.49	0.396	0.396	0.49	0.545	0.611	0.756

Calculate the strain at first yield:

$$\varepsilon_y := \left| \begin{array}{l} \text{for } p \in 1..N_{fiber} \\ \quad \varepsilon_{y_p} \leftarrow \dfrac{\sigma_{y_p}}{Em} \\ \varepsilon_y \end{array} \right.$$

$\varepsilon_y{}^T =$

	1	2	3	4	5
1	$1.513 \cdot 10^{-3}$	$1.221 \cdot 10^{-3}$	$1.09 \cdot 10^{-3}$	$9.804 \cdot 10^{-4}$	...

$$Ratio := \left| \begin{array}{l} \text{for } p \in 1..N_{fiber} \\ \quad Ratio_p \leftarrow \dfrac{\varepsilon_{y_p}}{z_p} \\ Ratio \end{array} \right.$$

$Ratio^T =$

	1	2	3	4	5	6
1	$8.42 \cdot 10^{-5}$	$1.304 \cdot 10^{-4}$	$1.644 \cdot 10^{-4}$	$2.043 \cdot 10^{-4}$	$3.152 \cdot 10^{-4}$	...

Summary of Fiber Properties:

Elastic Modulus: $Em = 500$

Fiber yield strength:

$F_y =$

	1
1	1.675
2	3.213
3	4.538
4	6.27
5	11.963
6	11.963
7	6.27
8	4.538
9	3.213
10	1.675

Fiber Area:

$A =$

	1
1	2.214145
2	5.260806
3	8.324044
4	12.79111
5	30.176721
6	30.176721
7	12.79111
8	8.324044
9	5.260806
10	2.214145

Fiber location (distance from CL):

$z =$

	1
1	17.967402
2	9.367025
3	6.63148
4	4.799369
5	2.515478
6	-2.515478
7	-4.799369
8	-6.63148
9	-9.367025
10	-17.967402

Fiber yield stress:

$\sigma_y =$

	1
1	0.756
2	0.611
3	0.545
4	0.49
5	0.396
6	0.396
7	0.49
8	0.545
9	0.611
10	0.756

Fiber yield strain:

$\varepsilon_y =$

	1
1	$1.513 \cdot 10^{-3}$
2	$1.221 \cdot 10^{-3}$
3	$1.09 \cdot 10^{-3}$
4	$9.804 \cdot 10^{-4}$
5	$7.929 \cdot 10^{-4}$
6	$7.929 \cdot 10^{-4}$
7	$9.804 \cdot 10^{-4}$
8	$1.09 \cdot 10^{-3}$
9	$1.221 \cdot 10^{-3}$
10	$1.513 \cdot 10^{-3}$

Verify that the given parameters will produce the desired section properties:

$$A_{calc} := \sum_{p=1}^{N_{fiber}} A_p \qquad A_{calc} = 117.534 \cdot in^2 \qquad \frac{A_{calc}}{Aelem} = 1.000$$

$$I_{calc} := \sum_{p=1}^{N_{fiber}} \left[A_p \cdot \left(z_p\right)^2\right] \qquad I_{calc} = 4056.0313 \cdot in^4 \qquad \frac{I_{calc}}{I_{elem}} = 1.000$$

$$P_{0\_calc} := \sum_{p=1}^{N_{fiber}} \left(A_p \cdot \sigma_{y_p}\right) \qquad P_{0\_calc} = 55.317 \cdot kip \qquad \frac{P_{0\_calc}}{Pn0} = 1.000$$

$$M_{0\_calc} := \sum_{p=1}^{N_{fiber}} \left(\sigma_{y_p} \cdot A_p \cdot abs\left(z_p\right)\right) \qquad M_{0\_calc} = 300.929 \cdot in \cdot kip \qquad \frac{M_{0\_calc}}{Mn0} = 1.000$$

Calculating the IP disp - OOP disp curve for specific values of OOP disp

Using the exponent relationship:

$$f\_OOP(OOP, IP0, OOP0) := IP0 \cdot \left[1 - \left(\frac{OOP}{OOP0}\right)^{\frac{3}{2}}\right]^{\frac{2}{3}}$$

This is the target P-M relationship for the equivalent member, located on the diagonal between structural workpoints.

$Pn0 = 55.317 \cdot kip$ Axial capacity of the member under pure compression (calculated above).

$Mn0 = 300.929 \cdot in \cdot kip$ Moment capacity of the member under pure bending (calculated above).

$N_{interaction} := 10$ Number of points on the interaction curve to be used for calculating fiber properties (should be an even number).

$$OOPv := \left| \begin{array}{l} \text{for } q \in 1..N_{interaction} \\ \quad OOPv_q \leftarrow (q-1) \cdot \dfrac{u_{Ncp0}}{N_{interaction} - 1} \\ OOPv \end{array} \right.$$

$$IIPv := \left| \begin{array}{l} \text{for } q \in 1..N_{interaction} \\ \quad IIPv_q \leftarrow f\_OOP\left(OOPv_q, uHcp0, u_{Ncp0}\right) \\ IIPv \end{array} \right.$$

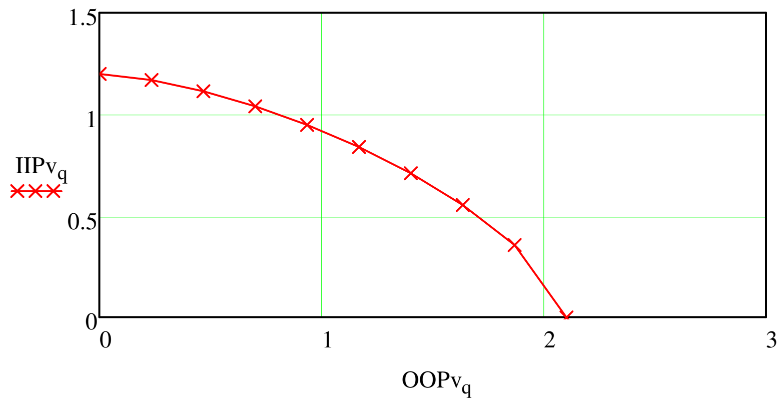
$OOPv^T =$

	1	2	3	4	5	6	7	8	9	10
1	0	0.234	0.467	0.701	0.934	1.168	1.401	1.635	1.869	2.102

$IIPv^T =$

	1	2	3	4	5	6	7	8	9	10
1	1.2	1.17	1.115	1.041	0.949	0.84	0.711	0.554	0.357	0

$q := 1..N_{interaction}$



1.5
1
0.5
0
0
1
2
3
$IIPv_q$
$OOPv_q$