Deterioration modeling of steel components in support of collapse prediction of steel moment frames under earthquake loading

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Abstract: Reliable collapse assessment of structural systems under earthquake loading requires analytical models that are able to capture component deterioration in strength and stiffness. For calibration and validation of these models a large set of experimental data is needed. This paper discusses the development of a database on experimental data of steel components and the use of this database for quantification of important parameters that affect the cyclic moment-rotation relationship at plastic hinge regions in beams. Based on information deduced from the steel component database, empirical relationships for modeling of pre-capping plastic rotation, post-capping rotation and cyclic deterioration for beams with reduced beam section (RBS) and beams other than RBS are proposed. Quantitative information is also provided for modeling of the effective yield strength, post-yield strength ratio, residual strength, and ductile tearing of steel components subjected to cyclic loading.

Keywords: Component deterioration; steel database; steel moment connections; sidesway collapse; moment-rotation relationships; residual strength; deterioration models; reduced beam section

Introduction

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Significant progress has been made in recent years in methods to predict collapse under earthquake loading (e.g., Ibarra et al. 2002; Vamvatsikos and Cornell 2002; Ibarra and Krawinkler 2005; Haselton and Deierlein 2007; Zareian and Krawinkler 2009) and to develop engineering approaches for collapse protection (FEMA P695 2009; ATC-76-1 2009; Zareian et al. 2010). The collapse mode addressed in these studies is associated with sidesway instability in which P-Delta effects accelerated by component deterioration fully offset the first order story shear resistance and dynamic instability occurs. One of the main challenges has been, and still is, the ability to reliably predict deterioration properties of structural components and to incorporate these properties into analysis tools.

Experimental studies have shown that the hysteretic behavior of structural components depends upon numerous structural parameters that affect the deformation and energy dissipation characteristics, leading to the development of a wide range of versatile deterioration models. A summary of various hysteresis models developed during the 1960s and 70s for reinforced concrete elements is presented in Otani (1981). More recently, Baber and Noori (1985), Casciati (1989), and Reinhorn et al. (1995) modified the widely known Bouc-Wen model (1967, 1980) to incorporate component deterioration. Song and Pincheira (2000) developed a model that simulates post-capping behavior but does not incorporate cyclic strength deterioration. Sivaselvan and Reinhorn (2000, 2006), based on earlier models by Iwan (1966) and Mostaghel (1999), developed a smooth hysteretic model with stiffness and strength degradation and pinching characteristics, derived from inelastic material behavior. More recently, Ibarra et al. (2005) developed an energy-based phenomenological deterioration model that captures most important modes of component deterioration.

Reliable deterioration modeling of structural components requires validation of analytical models described earlier with experimental data from components that have been subjected to various loading histories. Specific databases of experimental data are available for this purpose for reinforced concrete components [e.g. PEER database, (Berry et al. 2004), http://nisee.berkeley.edu/spd] and in part for steel components (SAC database, http://www.sacsteel.org/connections/). The latter database does not include
cyclic moment-rotation hysteresis diagrams, which are much needed for the development of deterioration parameters of steel components.

In this paper the primary focus is to provide information for the missing aspects of comprehensive modeling of the deterioration characteristics of structural steel components based on a recently developed database that includes comprehensive data of more than 300 experiments on steel wide flange beams (Lignos and Krawinkler 2007, 2009). The experimental data is used to calibrate deterioration parameters of the phenomenological deterioration model summarized in the next section, and to develop relationships that associate parameters of this deterioration model with geometric and material properties that control deterioration in structural steel components.

**Deterioration model**

The deterioration model developed by Ibarra et al. (2005), referred to as Ibarra-Krawinkler (IK) model, forms the basis of the deterioration modeling discussed in this paper. This model was modified by Lignos and Krawinkler (2009) to address asymmetric component hysteretic behavior including different rates of cyclic deterioration in the two loading directions, residual strength, and incorporation of an ultimate deformation $\theta_u$ at which the strength of a component drops to zero because of unstable crack growth and fracture.

The modified IK model establishes strength bounds based on a monotonic backbone curve (see Fig. 1a) and a set of rules that define the characteristics of hysteretic behavior between the bounds (see Fig. 1b). For a bilinear hysteretic response three modes of cyclic deterioration are defined with respect to the backbone curve (basic strength, post-capping strength, and unloading/reloading stiffness deterioration) as illustrated in Fig. 1b. The model can be applied to any force-deformation relationship, but in this discussion is described in terms of moment and rotation quantities as defined in Fig.1. The backbone curve is defined by three strength parameters $M_y = \text{effective yield moment}, M_c = \text{capping moment}$.
strength (or post yield strength ratio $M_c/M_y$), and residual moment $M_y = \kappa \cdot M_y$] and four deformation parameters ($\theta_y = \text{yield rotation}$, $\theta_p = \text{pre-capping plastic rotation for monotonic loading (difference between yield rotation and rotation at maximum moment)}$, $\theta_{pc} = \text{post-capping plastic rotation (difference between rotation at maximum moment and rotation at complete loss of strength)}$, and $\theta_u = \text{ultimate rotation capacity}$.

The rates of cyclic deterioration are controlled by a rule developed by Rahnama and Krawinkler (1993). This rule is based on the hysteretic energy dissipated when the component is subjected to cyclic loading. It is assumed that every component has a reference hysteretic energy dissipation capacity $E_i$, which is an inherent property of the components regardless of the loading history applied to the component. The reference hysteretic energy dissipation capacity is expressed as a multiple of $M_y \cdot \theta_p$, i.e.,

$$ E_i = \lambda \cdot \theta_p \cdot M_y \text{ or } E_i = \Lambda \cdot M_y $$

where $\lambda = \lambda \cdot \theta_p$ is a reference cumulative rotation capacity, and $\theta_p$ and $M_y$ are the pre-capping plastic rotation and effective yield strength of the component, respectively.

Cyclic strength deterioration (basic strength deterioration and post-capping strength deterioration) is modeled by translating the two strength bounds (the lines intersecting at the capping point) towards the origin at the rate,

$$ M_i = (1 - \beta_i) \cdot M_{i-1} $$

after every excursion $i$ in which energy is dissipated. The moment $M_i$ is any reference strength value on each strength bound line (the intersection of the strength bound with the y-axis may be used for convenience), and $\beta_i$ is an energy based deterioration parameter given by

$$ \beta_i = \left( \frac{E_i}{\sum_{j=1}^{i-1} E_j} \right)^c $$

where $E_i$ is the energy dissipated during excursion $i$.
where $E_i$ is the hysteretic energy dissipated in excursion $i$, $\Sigma E_j$ is the total energy dissipated in past excursions, $E_r$ is the reference energy dissipation capacity from Eq. (1), and $c$ is an empirical parameter, usually taken as 1.0. Different rates of deterioration in the positive and negative direction, such as in the case of a beam with a composite slab, can be accommodated by multiplying the right hand side of Eq. (3) by a parameter $0 < D^{+/-} \leq 1$, which slows down the rate of deterioration in one direction and results in two different values of $\beta$ in each direction (see Lignos and Krawinkler 2009).

The same concepts apply to modeling of unloading stiffness deterioration, i.e., the deteriorated stiffness after excursion $i$ is given by

$$K_i = \left(1 - \beta_i\right) K_{i-1} \quad (4)$$

Different rates of deterioration for each cyclic deterioration mode can be incorporated by using different $\Lambda$ values for each mode. Extensive calibration studies (Lignos and Krawinkler 2007, 2009) have shown that for steel components this refinement does not lead to significant model improvements. For more details on this deterioration model the reader is referred to Ibarra et al. (2005) and Lignos and Krawinkler (2009).

For each experiment of the database discussed in the next section, parameters of the modified IK model were determined by matching the digitized moment-rotation response to a hysteretic response controlled by the backbone curve shown in Fig. 1 and a cyclic deterioration parameter $\Lambda$. A combination of engineering mechanics concepts and visual observations is employed to select appropriate parameters and pass judgment on satisfactory matching. For this purpose an interactive Matlab based tool was developed to automate the calibration process (Lignos and Krawinkler 2009). An example of a satisfactory calibration of the modified IK deterioration model is shown in Fig. 2 for two steel beams with and without composite action. Ma et al. (2006), Yun et al. (2007) have used system identification and self learning simulation for calibration of degrading systems with respect to experimental data. However, the use of visual observation and judgment (in addition to mechanics concepts) was found to be preferable to attempts to use rigorous approaches such as a nonlinear least square optimization technique (Dennis
1977) and neural networks (Medsker and Jain 2000). The former was partially unsuccessful because of the large number of variables, and the latter was found to be unreliable because the size of the steel database was too small to train the network.

The modified Ibarra Krawinkler deterioration model has been implemented in DRAIN-2DX (Prakash et al. 1993) and Open System for Earthquake Engineering Simulation (OpenSees 2010) analysis software. Collapse prediction of steel moment frames, which accounts for component deterioration based on the model parameters discussed in this publication, has been validated through comparisons with recent small and full scale shaking table collapse tests (Lignos and Krawinkler 2009; Lignos et al. 2010a, 2010b).

A new database for deterioration modeling of steel components

The missing aspect of comprehensive modeling of deterioration characteristics of structural components is the availability of relationships that associate parameters of deterioration models, such as the ones discussed in the previous section, with geometric and material properties and detailing criteria that control deterioration in actual structural elements. In order to provide information for deterioration model parameters in support of collapse assessment of steel moment resisting frames, a data collection of component tests is needed in a consistent format that permits validation and calibration of deterioration models. For this purpose, three databases have been developed [(1) wide flange beams, (2) steel tubular sections and (3) concrete beams. The focus of this paper is in the first one. More information about the other two databases can be found in Lignos and Krawinkler (2009, 2010).

The steel database includes steel W-sections (mostly beams but also a few columns from Newell and Uang 2006). At this stage of development the steel W-section database includes more than 300 specimens. The complete set of data together with comprehensive documentation can be downloaded through the Network for Earthquake Engineering Simulation (NEES) Central repository (available from https://nees.org/warehouse/project/84).
The database contains data in the following three categories: (1) metadata [includes (a) distinction based on configuration of beam-to-column subassembly and test setup; (b) connection type, (c) measured material properties of beam and column components, (d) slab details, (e) report excerpts that contain a qualitative summary for the individual tests]; (2) reported results (measurements and observations as reported in test documentation, including digitized hysteretic load displacement response, moment-rotation response and panel zone shear force—distortion response (if reported); and (3) deduced data (information deduced from metadata and reported data for the purpose of calibration of deterioration models).

The steel W-section database documents experimental data from tests that have been conducted on beam-to-column subassemblies in which inelastic deformations are primarily concentrated in flexural plastic hinge regions of W (or H) sections. The primary deterioration mode of the steel components that develop a plastic hinge is local and/or lateral torsional buckling. Several cases in which components fail in a brittle mode (e.g., fracture around weldments), and are referred as non-ductile, are included in the database but are not part of any regression analysis discussed later in this paper since emphasis is on modern connections that are currently used in engineering practice. Various types of beam-to-column connections are employed in the test specimens, with the connection type clearly identified in each entry of the database. About eighty of the specimens have “reduced beam sections” (RBS) in which plastic hinges develop away from the beam-to-column connection.

Cyclic response data of many of the more recent experiments were received from researchers in digitized format. However, more than 40 percent of the cyclic response data, primarily from older experiments, were received in paper format. Force-deformation responses of these tests had to be manually digitized from research reports. To facilitate this effort an object-oriented digitization software called Digitizer was developed by Lignos and Krawinkler (2009) that provides all the digital data of interest.

In the evaluation of modeling parameters presented in the subsequent sections, the data of the W-section database are subdivided into RBS data and other-than-RBS data. The latter contained results from
tests of various beam-to-column connections in which a plastic hinge in the beam developed at or near the column face and the pertinent model parameters could be quantified with confidence. Tests in which the connection type clearly affected plastic hinge behavior, such as fracture at beam-to-column weldments or at welded flange plates, were eliminated from consideration. Thus, the “other-than-RBS” connection types used in the evaluation reflect general plastic hinge behavior in beams and not behavior of individual connection types. For individual connection types the number of tests is relatively small and the trends are not sufficiently clear to justify parameter quantification based on connection type. For the same reason only beams without a slab are considered in this evaluation. With these limitations the focus is on quantification of modeling parameters for moment-rotation relationships with symmetric hysteretic response characteristics. The emphasis in the following discussion is on the deformation parameters $\theta_p$, $\theta_{pc}$, and $\Lambda$, followed by a brief discussion on the modeling parameters $M_y$, $M_{cf}/M_y$, $\kappa$, and $\theta_u$.

Trends for deformation modeling parameters

This section illustrates trends that show the dependence of modeling parameters ($\theta_p$, $\theta_{pc}$, and $\Lambda$) on selected geometric properties of steel W sections. Trends are illustrated by plotting data points of a single model parameter against a pertinent geometric parameter. The information presented in the plots (see Fig. 3 to 6) is obtained from calibrations in which the parameters of the modified IK deterioration model are matched to the experimental moment-rotation relationships of the W-sections steel database (e.g. see Fig. 2). A regression line is included in the individual plots to illustrate the overall trends for the modeling parameter, whenever the coefficient of determination, $R^2$, is larger than 0.1. The parameter $R^2$ provides insight into the “goodness” of linear fit assuming that each one of the geometric parameters can be treated as an independent random variable ignoring the correlation between various geometric parameters. The linear regression lines serve only to illustrate trends; they are not presented for quantitative evaluation of data. The development of multivariate regression equations that account for correlations of geometric and
material parameters in the quantification of modeling parameters is discussed later in this paper. Trends for the following four data sets are evaluated:

1. Beams with other-than-RBS connections and depth $102\text{mm (4”)} \leq d \leq 914\text{mm (36“)}$ (data set 1)

2. Beams with RBS connections and depth $457\text{mm (18“)} \leq d \leq 914\text{mm (36“)}$ (data set 2)

3. Beams with other-than-RBS connections and depth $d \geq 533\text{mm (21“)}$ (data set 3)

4. Beams with RBS connections and depth $d \geq 533\text{mm (21“)}$ (data set 4)

Data set (1) contains experiments on small sections, which are useful to observe trends but conceivably de-emphasize trends for the sizes of sections used in engineering practice to design a steel moment resisting frame in a seismic region. This is why we have generated data sets (3) and (4), which are subsets that contain only beams with $d \geq 533\text{mm (21“)}$. However, for beams with RBS there are no tests available with $d \leq 457\text{mm (18“)}$ hence data sets (2) and (4) do not differ by much. Only a few selected plots are presented in this paper. A detailed discussion of trends of component deterioration parameters with respect to geometric and material parameters is presented in Lignos and Krawinkler (2009).

**Statistical information on parameters $\theta_p$, $\theta_{pc}$ and $\Lambda$**

Cumulative distribution functions, CDFs, for $\theta_p$, $\theta_{pc}$ and $\Lambda$ as obtained from the four data sets are shown in Fig. 3. Each plot shows CDFs for other-than-RBS and RBS sections. The CDFs reveal general statistical characteristics but do not display dependencies on individual properties. This information is relevant for detailed studies concerned with quantification of modeling uncertainties and their effect on the collapse capacity of structural systems subjected to earthquake excitations, since so far there was not available a systematic collection of experimental data that could be used to document statistical information (median and standard deviation) on deterioration parameters of components. The log-normally distributed CDFs for the four data sets shown in Fig. 3 are comparable, but in general the median value of the modeling parameters for beams with other-than-RBS connections is smaller than that
for beams with RBS connections. The dispersion is larger for beams other-than RBS compared to beams with RBS, partially because this set includes experimental data from different connection types.

**Dependence of modeling parameters on beam depth $d$**

An increase in beam depth $d$ is associated with a clear decrease in modeling parameters. This is supported by Fig. 4a, which shows data and a linear regression line for the pre-capping plastic rotation $\theta_p$ for data set (1) (full data set for beams other-than-RBS). This data set includes beams with a depth varying from 102mm (4”) to 914mm (36”). Others (FEMA 2000; FEMA 2000a) have pointed out the strong dependence of plastic rotation capacity on beam depth. This strong dependence is driven in part by the incorporation of small sections in the database and is not confirmed for the range of primary interest for tall buildings [$d \geq 533mm (21’’)$] based on Fig. 4b.

**Dependence of modeling parameters on shear span to depth ratio $L/d$**

Based on simple curvature analysis with disregard of local instabilities, $\theta_p$ of a given beam section is perceived to be linearly proportional to the beam shear span $L$ (distance from plastic hinge location to point of inflection). This perception is supported by the plot in Fig. 5a, which shows the dependence of $\theta_p$ on $L/d$ for the full other-than-RBS data set [beams with $102mm (4’’) \leq d \leq 914mm (36’’)$]. But the strong dependence on $L/d$ is not evident when only beams of depth $\geq 533mm (21’’)$ are considered (see Fig. 5b). The reason is that most deep beams are susceptible to a predominance of web buckling and lateral torsional buckling, and both of these susceptibilities increase with a decrease in the moment gradient (more uniform moment, as implied by an increase in the $L/d$ ratio). This phenomenon offsets much of the curvature integration effect for a larger plastic hinge length. Based on this information it is concluded that for beams with depth $\geq 533mm (21’’)$ a description of beam plastic deformation capacity in terms
of a ductility ratio $\frac{\theta_p}{\theta_y}$ is often misleading because $\theta_y$ increases linearly with $L$ (for a given beam section) but $\theta_p$ does not. Assume two cantilever beams made of the same W section. One beam has length $L$ and the other $L/2$. The yield rotation $\theta_y$ for the first beam with length $L$ will be $\frac{M_y}{6EI/L}$ and for the second beam with length $L/2$ will be $\frac{M_y}{(6EI/L/2)}$, i.e. $\theta_y$ linearly increases with length. But the experimental data for set with $d \geq 533\text{mm}$ show that $\theta_p$ does not depend strongly on $L/d$ (see Fig. 5b). In other words, the ratio $\frac{\theta_p}{\theta_y}$ depends strongly on beam span $L$. Similar observations are made for the parameters $\theta_{pc}$ and $\Lambda$.

**Dependence of modeling parameters on $L_n/r_y$**

This ratio is associated with sensitivity to lateral torsional buckling. The parameter $L_n$ is defined here as the distance from the column face to the nearest lateral brace and $r_y$ is the radius of gyration about the $y$-axis of the beam. AISC (2005) requires that this ratio be less than $2500/F_y$. Results from the steel beam database indicate that $\theta_p$ is somewhat but not greatly affected by $L_n/r_y$, provided that the ratio is close to or smaller than the value required by seismic codes. A decrease of $L_n/r_y$ to 50% of the value required by AISC (2005) increases in average $\theta_p$ by 2.5% and 10% for beams other than RBS and beams with RBS, respectively. Providing lateral bracing close to the RBS portion of a beam decreases the rate of cyclic deterioration since twisting of the RBS region is delayed. Uang et al. (2000) reached to the same conclusion for beams with RBS.

**Dependence of modeling parameters on the width/thickness ratio of the beam flange $b_f/2t_f$**

When the effect of $b_f/2t_f$ ratio on $\theta_p$ is viewed in isolation, a small $b_f/2t_f$ ratio has a negligible effect on $\theta_p$. For most of the deeper beams in the database a small $b_f/2t_f$ implies a narrow wide flange beam
with small radius of gyration $r_y$ and large fillet to fillet web depth over web thickness ratio $h/t_w$, both of which have a detrimental effect on $\theta_p$ since (1) a larger $h/t_w$ ratio makes a beam more susceptible to web local buckling and (2) a small $r_y$ makes a beam more susceptible to lateral torsional buckling. On the other hand, the data show a clear benefit of a smaller $b_f/2t_f$ ratio for the parameters $\theta_{pc}$ and $\Lambda$ since a beam with smaller $b_f/2t_f$ ratio does not develop a large flange local buckle, i.e. post-capping strength deterioration and cyclic deterioration occur at a slower rate.

**Dependence of modeling parameters on the depth to thickness ratio of the beam web $h/t_w$**

This geometric parameter is found to be very important for all three modeling parameters (see Fig. 6). The reason is that a beam with large $h/t_w$ ratio is more susceptible to web local buckling. This triggers flange local buckling and at larger inelastic cycles also triggers lateral torsional buckling (Lay 1965; Lay and Galambos 1966) Figure 6 indicates also that the trends for all three modeling parameters are similar for RBS and other-than-RBS sections.

**Regression equations for $\theta_p$, $\theta_{pc}$, and $\Lambda$, accounting for geometric and material properties**

In this section regression equations are proposed in order to predict deterioration modeling parameters discussed previously. The primary focus is on $\theta_p$, $\theta_{pc}$, and $\Lambda$. Recommendations for modeling of effective bending strength $M_e$, post yield strength ratio $M_e/M_y$, residual bending strength $\kappa$, and ultimate rotation capacity $\theta_u$ parameters are also presented.

Lay (1965) and Lay and Galambos (1966) showed that web local buckling is coupled with flange local buckling and lateral torsional buckling. Hence a nonlinear regression model is used to evaluate the
contribution of each important property identified previously to the selected response parameter (RP). The general nonlinear model used is

$$RP = a_1 \cdot (X_1)^{a_2} \cdot (X_2)^{a_3} \cdots (X_n)^{a_{n+1}}$$  \hspace{1cm} (5)$$

in which $a_1, a_2, \ldots, a_{n+1}$ are constants known as regression coefficients and $X_1, X_2, \ldots, X_n$ are the predictor variables. Based on an evaluation of steel database information and observations on trends discussed partially in the previous section, six parameters are found to primarily affect the deterioration parameters of steel components. Using these six parameters Eq. (5) becomes

$$RP = a_1 \left( \frac{h}{t_w} \right)^{a_2} \left( \frac{b_f}{2 \cdot t_f} \right)^{a_3} \left( \frac{L_b}{R_y} \right)^{a_4} \left( \frac{L}{d} \right)^{a_5} \left( \frac{c_{\text{unit}}^1 \cdot d}{533} \right)^{a_6} \left( \frac{c_{\text{unit}}^2 \cdot F_y}{355} \right)^{a_7}$$  \hspace{1cm} (6)$$

in which, $F_y$ is the expected yield strength of the flange of the beam in MPa, which is normalized by 355MPa (typical nominal yield strength of European structural steel and equivalent with nominal yield strength of about 50ksi US steel), and $c_{\text{unit}}^1$ and $c_{\text{unit}}^2$ are coefficients for units conversion. They both are 1.0 if mm and MPa are used, and they are $c_{\text{unit}}^1 = 25.4$ and $c_{\text{unit}}^2 = 6.895$ if $d$ is in inches and $F_y$ is in ksi, respectively.

Stepwise regression analysis (Chatterjee et al. 2000) is used to develop regression equations for the three model parameters $\theta_p$, $\theta_{pc}$, and $\Lambda$. Only variables that are statistically significant at the 95% level using a standard $t$-test and $F$-test (see Chatterjee et al. 2000) are included in the regression equations presented in the subsequent sections. Variables with insignificant impact are not included in the regression equations. Equations are presented for beams other-than-RBS and beams with RBS. For beams other than RBS two sets of equations are proposed; one for the entire range of data and the other for the dataset with $d \geq 533$mm (21”). For beams with RBS the regression equations are based on the full set of tests since there are no beams with RBS with $d < 457$mm (18”) in the W-sections database.

**Pre-capping plastic rotation $\theta_p$**
For the full data set for beams other-than-RBS (data set 1) the equation for $\theta_p$ obtained from multivariate regression analysis using 107 specimens is

$$
\theta_p = 0.0865 \cdot \left( \frac{h}{t_w} \right)^{-0.365} \cdot \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.140} \cdot \left( \frac{L}{d} \right)^{0.340} \cdot \left( \frac{c_{unit} \cdot d}{533} \right)^{-0.721} \cdot \left( \frac{c_{unit} ^2 \cdot F_y}{355} \right)^{-0.230}
$$

(7)

$$
R^2 = 0.505 \ , \ \sigma_{ln} = 0.32
$$

The large values of regression coefficients for web depth over thickness ratio $h/t_w$, beam depth $d$, and span to depth ratio $L/d$ confirm trends pointed out previously. Figure 7a shows data points for predictions obtained from Eq. (7) plotted against the data points obtained from experimental results based on the calibration process described earlier in this paper.

For the data set of beams with $d \geq 533$mm (21") the regressed equation for pre-capping plastic rotation $\theta_p$ based on 78 specimens is given by

$$
\theta_p = 0.130 \cdot \left( \frac{h}{t_w} \right)^{-0.550} \cdot \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.0230} \cdot \left( \frac{L}{d} \right)^{0.090} \cdot \left( \frac{c_{unit} \cdot d}{533} \right)^{-0.330} \cdot \left( \frac{c_{unit} ^2 \cdot F_y}{355} \right)^{-0.130}
$$

(8)

$$
R^2 = 0.457 \ , \ \sigma_{ln} = 0.351
$$

In Eq. (8) the effects of $d$ and $L/d$ on $\theta_p$ are not as significant as in Eq. (7) for the entire range of data, as concluded from the trends plots discussed in the previous section of this paper.

Based on 72 test specimens, with beams with RBS with $d \geq 533$mm (21") the regressed equation for pre-capping plastic rotation $\theta_p$ is given by

$$
\theta_p = 0.070 \cdot \left( \frac{h}{t_w} \right)^{-0.314} \cdot \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.100} \cdot \left( \frac{L}{r_y} \right)^{-0.185} \cdot \left( \frac{L}{d} \right)^{0.113} \cdot \left( \frac{c_{unit} \cdot d}{533} \right)^{-0.760} \cdot \left( \frac{c_{unit} ^2 \cdot F_y}{355} \right)^{-0.0700}
$$

(9)

$$
R^2 = 0.56 \ , \ \sigma_{ln} = 0.24
$$
Equation (9) indicates that the effect of $h/t_w$ and $d$ dominates on plastic rotation capacity $\theta_p$ of beams with RBS. Uang and Fan (1999) came to similar conclusions regarding the effect of $h/t_w$ on $\theta_p$, based on a data set of 55 RBS specimens and using the difference between the rotations at 80% of the ultimate strength and at yield strength as a definition of plastic rotation capacity.

**Post-capping plastic rotation $\theta_{pc}$**

For the development of predictive equations for $\theta_{pc}$ only specimens with clear indication of post-capping behavior are considered from the W-section database. For beams other-than-RBS 104 specimens were used. The empirical equation for $\theta_{pc}$, obtained from multivariate regression analysis of the full set of other-than-RBS beams, is given by

$$
\theta_{pc} = 5.63 \cdot \left( \frac{h}{t_w} \right)^{-0.565} \cdot \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.800} \cdot \left( \frac{c_{unit} \cdot d}{533} \right)^{-0.280} \cdot \left( \frac{c_{unit}^2 \cdot F_y}{355} \right)^{-0.430}
$$

(10)

$$R^2 = 0.48, \ \sigma_{ln} = 0.25$$

Predicted versus calibrated $\theta_{pc}$ values for the total range of data set 1 are presented in Fig. 7b.

After eliminating specimens with $d < 533mm$ (21") (data set 3) the proposed empirical equation for $\theta_{pc}$ based on 72 specimens is given by

$$
\theta_{pc} = 7.50 \cdot \left( \frac{h}{t_w} \right)^{-0.610} \cdot \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.710} \cdot \left( \frac{L_b}{r_y} \right)^{-0.110} \cdot \left( \frac{c_{unit} \cdot d}{533} \right)^{-0.161} \cdot \left( \frac{c_{unit}^2 \cdot F_y}{355} \right)^{-0.320}
$$

(11)

$$R^2 = 0.49, \ \sigma_{ln} = 0.24$$

The regression equation for $\theta_{pc}$ for beams with RBS based on 61 specimens is,

$$
\theta_{pc} = 9.52 \cdot \left( \frac{h}{t_w} \right)^{-0.513} \cdot \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.863} \cdot \left( \frac{L_b}{r_y} \right)^{-0.108} \cdot \left( \frac{c_{unit}^2 \cdot F_y}{355} \right)^{-0.360}
$$

(12)

$$R^2 = 0.48, \ \sigma_{ln} = 0.26$$
Patterns reflected in Eqs. (10) to (12) agree with the ones from earlier studies by Axhag (1995) and White and Barth (1998). These researchers proposed empirical equations for predicting the descending slope of the moment-rotation curve of beams and concluded that flange and web local buckling are the primary contributors to the descending slope of the beams.

**Reference cumulative plastic rotation** \( \Lambda \)

As discussed earlier, the reference cumulative plastic rotation \( \Lambda \) is a parameter that defines the rate of cyclic deterioration. The specimens considered for the development of predictive equations for \( \Lambda \) are the ones that fail in a ductile manner and for which cyclic deterioration is clearly observed. All modes of cyclic deterioration are assumed to be defined by the same \( \Lambda \). The exponent \( c \) of Eq. (3) is kept equal to 1.0 for the sake of simplicity.

Equation (13) is the best-fit multivariate regression equation for predicting the cumulative rotation capacity \( \Lambda \) for the full set of other-than-RBS beams based on 85 specimens with clear indication of cyclic deterioration,

\[
\Lambda = \frac{E_y}{M_y} = 495 \cdot \left( \frac{h}{t_w} \right)^{-1.34} \cdot \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.595} \cdot \left( \frac{c_{\text{unit}} \cdot F_y}{355} \right)^{-0.360}
\]  

\[ R^2 = 0.484, \sigma_{\text{in}} = 0.35 \]

Equation (13) indicates that the geometric parameter \( d, L/d \), and \( L_b/r_y \) become statistically insignificant. The reason why \( L_b/r_y \) ratio has a small effect on \( \Lambda \) is that all the specimens included in the multivariate regression analysis satisfy the AISC (2005) lateral bracing requirements. The small effect of \( L_b/r_y \) was pointed out also by Roeder, (2002).

For the data set of beams with nominal depth larger than 533mm (21") the following equation is derived to predict \( \Lambda \) (66 specimens showed clear indication of cyclic deterioration):

\[
\Lambda = \frac{E_y}{M_y} = 536 \cdot \left( \frac{h}{t_w} \right)^{-1.26} \cdot \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.525} \cdot \left( \frac{L_b}{r_y} \right)^{0.130} \cdot \left( \frac{c_{\text{unit}} \cdot F_y}{355} \right)^{-0.291}
\]
The proposed equation for predicting the cumulative rotation capacity $\Lambda$ for beams with RBS based on 55 specimens is

$$\Lambda = \frac{E_y}{M_y} = 585 \left( \frac{h}{t_w} \right)^{-1.14} \left( \frac{b_f}{2 \cdot t_f} \right)^{-0.632} \left( \frac{L_b}{r_y} \right)^{-0.205} \left( \frac{c_{unit} \cdot F_y}{355} \right)^{-0.391}$$  

$$R^2 = 0.496, \sigma_{in} = 0.34$$

Uang et al. (2000) have shown that beams with RBS are susceptible to twisting at the RBS region because of the reduced flanges, and additional lateral bracing reduces the rate of strength deterioration at large deformation levels because it reduces the lateral buckling amplitude near the RBS location. Roeder (2002) came to the same conclusion. This is reflected in the exponent of the $L_b/r_y$ term in Eq. (15).

Experimental data with the following range of parameters are used in deriving Eqs (7) to (15):

- $20 \leq h/t_w \leq 55$ for other-than-RBS; $21 \leq h/t_w \leq 55$ for RBS.
- $20 \leq L_b/r_y \leq 80$ for other-than-RBS; $20 \leq L_b/r_y \leq 65$ for RBS.
- $4 \leq b_f/2t_f \leq 8$ for other-than-RBS; $4.5 \leq b_f/2t_f \leq 7.5$ for RBS.
- $2.5 \leq L/d \leq 7$ for other-than-RBS; $2.3 \leq L/d \leq 6.3$ for RBS.
- $102\text{mm}(4\text{\"}) \leq d \leq 914\text{mm}(36\text{\"})$ for other-than-RBS; $533\text{mm}(21\text{\"}) \leq d \leq 914\text{mm}(36\text{\"})$ for RBS.
- $240\text{MPa}(35\text{ksi}) \leq F_y \leq 450\text{MPa}(65\text{ksi})$ for other-than-RBS; $262\text{MPa}(38\text{ksi}) \leq F_y \leq 435\text{MPa}(63\text{ksi})$ for RBS. The specimens included in the steel database were fabricated from three main types of steel material; A36, A572, Grade 50 and A992, Grade 50. The yield strength values reported here are the ones obtained from actual coupon tests conducted by the experimentalists.

The range of validity of the regression equations is only as good as the experimental data allows it to be. The data does not include heavy W14 sections (heavier than W14x370) and heavy (heavier than W36x150) and deep (e.g. deeper than W36) beam sections. The predictions from the regression equations have been compared with data from the only series of experiments found in the literature on heavy W14
sections (Newell and Uang 2006) and have been found to provide reasonably close values of experimentally obtained modeling parameters. Until more tests on columns become available, the above equations provide the best estimates that can be offered for columns.

Tables 1 and 2 summarize the variation of deterioration parameters for beams other-than RBS and with RBS, respectively, for a range of sections (W21 to W36) that satisfy seismic compactness criteria. The range of deterioration parameter values is also reflected in the cumulative distribution functions of these parameters presented in Fig. 3. Sections whose geometric or material properties are slightly outside the range of properties, on which the predictive equations are based, are marked with an asterisk.

**Effective yield strength** $M_y$

As mentioned previously, the modified IK deterioration model does not account for cyclic hardening. But the effect of isotropic hardening is incorporated approximately by increasing the yield moment (bending strength) to an effective value $M_y$ that accounts for isotropic hardening in average. The effective yield strength typically is larger by a small amount than the predicted bending strength $M_{y,p}$, which is defined as the plastic section modulus $Z$ times the measured material yield strength obtained from coupon tests. Table 3 summarizes the mean and standard deviation of $M_y / M_{y,p}$ ratios for beams other than RBS and beams with RBS. For the latter, $M_{y,p}$ is defined based on the reduced section properties.

Options exist for more refined modeling that account explicitly for combined isotropic and kinematic hardening (e.g., Sivaselvan and Reinhorn 2002; Jin and El-Tawil 2003). Such options were not incorporated to keep the model relatively simple for engineering implementation.

**Post-yield strength ratio** $M_c / M_y$

Post-yield hardening, and subsequently $M_c$, is described by the ratio of the maximum moment on the backbone curve shown in Fig. 1a over the effective yield bending strength, $M_y$, discussed earlier. The
M_c/M_y and \( \theta_c/\theta_y \) ratios define the strain hardening stiffness of the backbone curve shown in Fig. 1a. This stiffness is important because of its effect on the \( P-\Delta \) stability of a structural system (Medina and Krawinkler 2003). Table 3 summarizes statistics (mean and standard deviation) of \( M_c/M_y \) for RBS and other-than-RBS connections based on information extracted from the database of steel components. In general, \( M_c/M_y \) is a more stable parameter to describe post-yield strength increase than the traditional strain hardening ratio because the latter depends strongly on yield rotation, which in turn depends strongly on the beam span selected in the experiment, i.e., on the moment gradient. Experimental data used in this study have shown that the strength increase beyond yielding is much less sensitive to the moment gradient than the yield rotation, which is linearly proportional to the beam span.

**Residual strength ratio \( \kappa \)**

Low cycle fatigue experimental studies (Krawinkler et al. 1983; Ricles et al. 2004) indicate four ranges of cyclic deterioration: a range of negligible deterioration in which local instabilities have not yet occurred or are insignificant. The second range involves an almost constant rate of cyclic deterioration due to continuous growth of local buckles. In the third range, deterioration proceeds at a very slow rate due to the stabilization in buckle size. This range is associated with the residual strength of a steel component. These three ranges are followed by a range of very rapid deterioration, which is caused by crack propagation at local buckles (ductile tearing). From the data sets for W-sections a residual strength ratio \( \kappa = M_r/M_y \) of about 0.4 is suggested for sets (3) and (4). This value is based on a relatively small set of data points from which an estimate of \( \kappa \) could be made with confidence. In order to assess residual strength more reliably, more experiments with very large deformation cycles need to be conducted.

**Ultimate rotation capacity \( \theta_u \)**

At very large inelastic rotations cracks may develop in the steel base material close to the apex of the most severe local buckle, and rapid crack propagation will then occur followed by ductile tearing and
essentially complete loss of strength (see Ricles et al. 2004 for illustrations and end of last loading cycle of the experimental data shown in Fig. 2b). The modified IK deterioration model captures this failure mode with the ultimate rotation capacity $\theta_u$. This rotation depends on the loading history and may be very large for cases in which only a few very large cycles are executed (e.g. near fault loading history or ratcheting type of global behavior) as discussed in Uang et al. (2000) and Lignos and Krawinkler (2009). Estimates of $\theta_u$ are made here only for experiments with step-wise increasing cycles of the type required in the AISC (2005) seismic specifications. For beams other-than-RBS an estimate of $\theta_u$ is 0.05 to 0.06 radians based on available data from various researchers (Allen et al. 1996; Ricles et al. 2000). For beams with RBS, an estimate of $\theta_u$ is 0.06 to 0.07 radians (Engelhardt et al. 2000; Ricles et al. 2004). For monotonic type of loading $\theta_u$ is on the order of three times as large as the $\theta_u$ values reported above for symmetric cyclic loading protocols. Ductile tearing is not found to be critical in analytical studies in which the collapse capacity of a steel moment resisting frame has been evaluated because steel frame structures approach their collapse capacity usually before ductile tearing occurs (Ibarra and Krawinkler 2005; Lignos and Krawinkler 2009; ATC-76-1 2009; Lignos et al. 2010a, 2010b).

Conclusions

This paper is concerned with deterioration modeling of steel components based on a recently developed database on experimental studies of wide flange beams. The database of more than 300 specimens contains, in consistent format, extensive information of worldwide experimental data on components that have been subjected to monotonic and cyclic loading. The steel database can serve for validation and improvement of deterioration models used for collapse assessment of steel moment resisting frames. Based on statistical evaluation of calibrated moment rotation diagrams obtained from tests included in this database, and with the use of multivariate regression analysis, empirical equations are proposed that predict the deterioration modeling parameters $\theta_p$, $\theta_{pc}$ and $\Lambda$ of beams with reduced beam sections (RBS)
and beams other-than RBS. Quantitative information for modeling of effective yield moment $M_y$, post-yield strength ratio $M_y / M_y$, residual strength ratio $\kappa$, and ultimate rotation capacity $\theta_u$ is also provided.

From available trend plots, cumulative distribution functions on deterioration parameters, and predictive equations the main conclusions are the following:

- The median value of the pre-capping plastic rotation $\theta_p$ is on the order of 0.02 rad, the median of post-capping rotation capacity $\theta_{pc}$ is on the order of 0.20 rad, and the median of the reference cumulative rotation capacity $\Lambda$ is on the order of 1.0 rad.
- For all the connection types evaluated, the primary contributor to the deterioration parameters $\theta_p$, $\theta_{pc}$ and $\Lambda$ is the beam web depth $h$ over thickness ratio $h/w_t$. Of some importance is the effect of flange width to thickness ratio $b_f/2t_f$, beam depth $d$ and shear span over beam depth ratio $L/d$.
- For sections used commonly in modern steel moment resisting frames ($d \geq 533$ mm (21 in)) a description of beam deformation capacity in terms of a ductility capacity ratio $\theta_p/\theta_y$ is misleading because $\theta_y$ increases linearly with $L$ (for a given beam section) but $\theta_p$ does not.
- Experimental data indicate that deterioration modeling parameters are not very sensitive to the beam span (i.e., the length of the plastic hinge regions.
- Closely spaced lateral bracing (small $L_b/r_y$ ratio) increases $\theta_p$, $\theta_{pc}$ and $\Lambda$, but not by a large amount (provided that the $L_b/r_y$ ratio does not exceed an upper limit on the order of 70). The effect of $L_b/r_y$ on $\Lambda$ of beams with RBS is somewhat more important compared to beams other than RBS, particularly when additional bracing is installed near the RBS location.
- The effective yield strength $M_y$ used in the modified Ibarra–Krawinkler model, which accounts in average for cyclic hardening, is about 1.10 times the plastic moment $M_{y,p}$ obtained from
plastic section modulus times actual material yield strength for both beams other-than-RBS and beams with RBS.

- The post yield strength ratio $M_c/M_y$ is in average 1.10 for both beams other-than-RBS and beams with RBS. It is found that the ratio $M_c/M_y$ together with the ratio $\theta_p/\theta_y$ provide a much better definition of the post-yield stiffness than the traditional strain hardening ratio.

- A reasonable estimate of residual strength is 0.4 times the effective yield strength $M_y$. More experiments with very large deformation cycles are needed in order to assess residual strength with high confidence.

- Ultimate rotation capacity $\theta_u$ of steel components that fail in a ductile manner is strongly dependent on loading history. For components subjected to symmetric cyclic loading histories $\theta_u$ is on the order of 0.06rad, but it is about three times as large when the component is subjected to a near fault loading protocol or to monotonic loading.

The conclusion drawn here are based on interpretation of experimental data. Detailed analytical validation studies have not been performed. The data are available for such studies at https://nees.org/warehouse/project/84.

Acknowledgements

This study is based on work supported by the United States National Science Foundation (NSF) under Grant No. CMS-0421551 within the George E. Brown, Jr. Network for Earthquake Engineering Simulation (NEES) Consortium, and by a grant from the CUREE-Kajima Phase VI joint research program. This financial support is gratefully acknowledged. The authors would like to thank graduate students Yash Ahuja, Guillermo Soriano, Richard Weiner and Yavor Yotov for their invaluable assistance.
in the steel database development. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of sponsors.
References


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Figure 1. Modified Ibarra – Krawinkler (IK) deterioration model; (a) monotonic curve; (b) basic modes of cyclic deterioration and associated definitions.
Figure 2. Calibration examples of modified IK deterioration model; (a) beam with RBS (no slab, data from Uang et al. 2000); (b) asymmetric hysteretic response considering composite action (data from Ricles et al. 2004)
Figure 3. Cumulative distribution functions (CDFs) for: (a) $\theta_p$, (b) $\theta_{pc}$ and (c) $\Lambda$; left = full data sets 1 and 2; right = data sets 3 and 4, $d \geq 533$mm (21")
Figure 4. Dependence of plastic rotation $\theta_p$ on beam depth $d$ for beams other-than-RBS; (a) full set of data; (b) $d \geq 533$mm (21”)
Figure 5. Dependence of plastic rotation $\theta_p$ on shear span to depth ratio $L/d$ for beams other-than-RBS;

(a) full data set; (b) $d \geq 533\text{mm (21''})$
Figure 6. Dependence of modeling parameters on $h/w$ ratio of beam web, $d \geq 533$mm (21”)

- (a) $\theta_p$; other-than-RBS
- (b) $\theta_p$; RBS
- (c) $\theta_{pc}$; other-than-RBS
- (d) $\theta_{pc}$; beams with RBS
- (e) Cumulative plastic rotation, $\Lambda$; other-than-RBS
- (f) Cumulative plastic rotation, $\Lambda$; RBS
(a) Predicted versus calibrated values of $\theta_p$

(b) Predicted versus calibrated values of $\theta_{pc}$
Table 1. Modeling parameters for various beam sizes (other-than-RBS) based on regression equations:

assumed beam shear span $L=3810$mm (150"), $L_b/r_y= 50$, expected yield strength $F_y=379$MPa (55ksi)

<table>
<thead>
<tr>
<th>Section Size</th>
<th>$\theta_p$ (rad)</th>
<th>$\theta_{pc}$ (rad)</th>
<th>$\Lambda$</th>
<th>$h/t_w$</th>
<th>$b_f/2t_f$</th>
<th>$L_b/r_y$</th>
<th>$L/d$</th>
<th>$d$ (mm)</th>
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Table 2. Modeling parameters for various beam sizes (beams with RBS) based on regression equations; assumed beam shear span $L=3810$mm (150"), $L_b/r_y=50$, expected yield strength $F_y=379$MPa (55ksi)

<table>
<thead>
<tr>
<th>Section Size</th>
<th>$\theta_p$ (rad)</th>
<th>$\theta_{pc}$ (rad)</th>
<th>$\Lambda$</th>
<th>$h/t_w$</th>
<th>$b_f/2t_f$</th>
<th>$L_b/r_y$</th>
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*Values slightly outside the range of experimental data
Table 3. Statistics of ratios of effective to predicted component yield strength and capping strength to effective yield strength

<table>
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<tr>
<th>Connection Type</th>
<th>Mean $M_e / M_{y,p}$</th>
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<th>Mean $M_c / M_y$</th>
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