OpenSees

Transient Integrators

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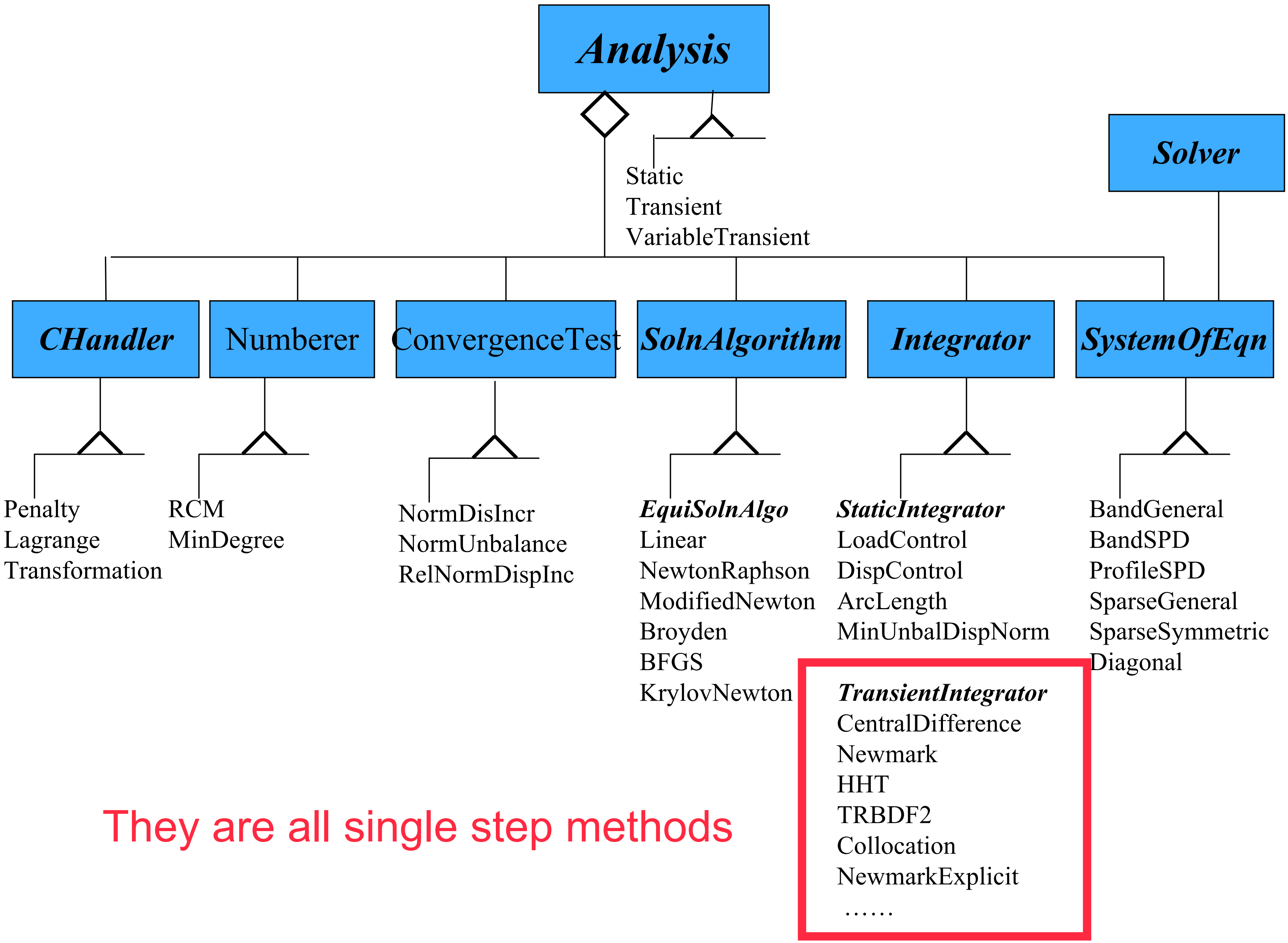


They are all single step methods

Single Step Methods Classifications

- Explicit

  CentralDifference

  NewmarkExplicit $gamma

  HHTExplicit $alpha

  ….

- Implicit

  Newmark $gamma $beta

  HHT $alpha

  TRBDF2

  …

1. all Need Linear Algorithm

2. in absence of damping all require full mass matrix.

Stability & Linear Systems

- Stability (bounded solution) and Accuracy are the most talked about properties of time integration schemes.
- For most integration schemes, the stability and accuracy provisions you read about are provided FOR LINEAR DYNAMICAL SYSTEMS.
- Conditionally Stable: numerical procedure leads to a BOUNDED solution if time step is smaller than some stability limit. Conditional stability requires time step to be inversely proportional to highest frequency.
- Unconditionally Stable: solution is bounded regardless of the time step.

Stability Limits Common Integrators

Central Difference is conditionally stable if:

$$\frac{\Delta t}{T_n} < \frac{1}{\pi} \quad (.318)$$

Newmark is unconditionally stable if:

$$\beta >= \frac{\gamma}{2} >= \frac{1}{4}$$

Average Acceleration $(\gamma = \frac{1}{2}, \beta = \frac{1}{4})$ ✓

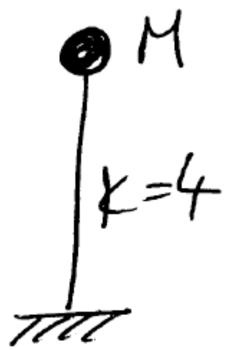
Linear Acceleration $(\gamma = \frac{1}{2}, \beta = \frac{1}{6})$ ✗

But Conditionally stable if: $\frac{\Delta t}{T_n} < 0.55$

Example

(see “Dynamics of Structures” A.K. Chopra, section 5.5)

Free Vibration (ex1.tcl) (ex2.tcl)



$u(0) = 1$

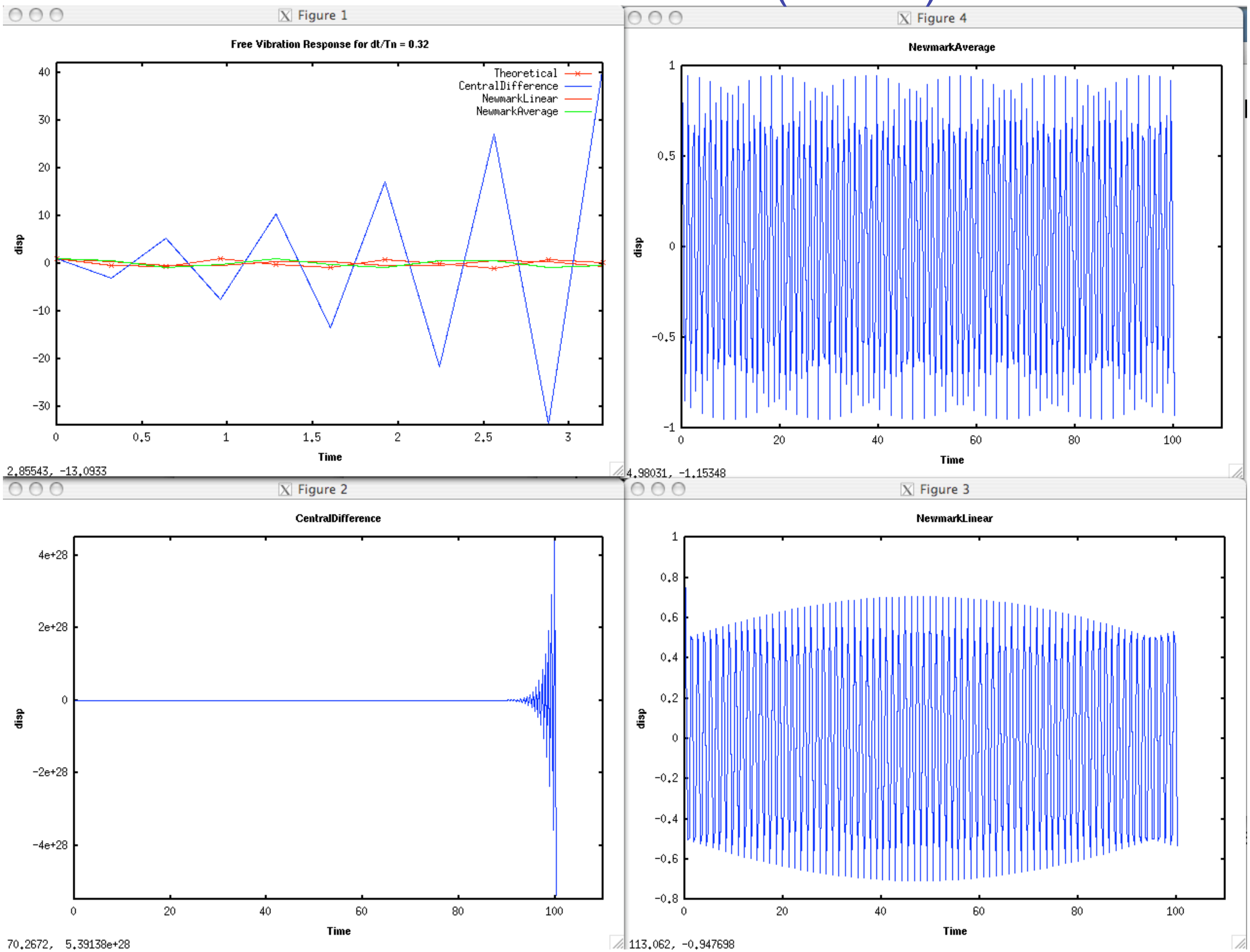
$u'(0) = 0$

$p(t) = \phi$

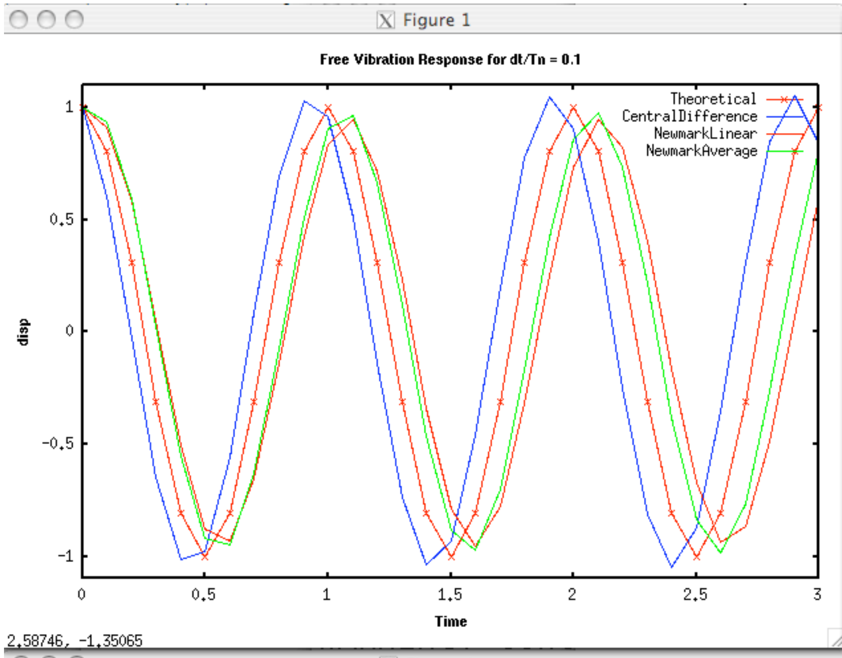
M is chosen to give desired period.

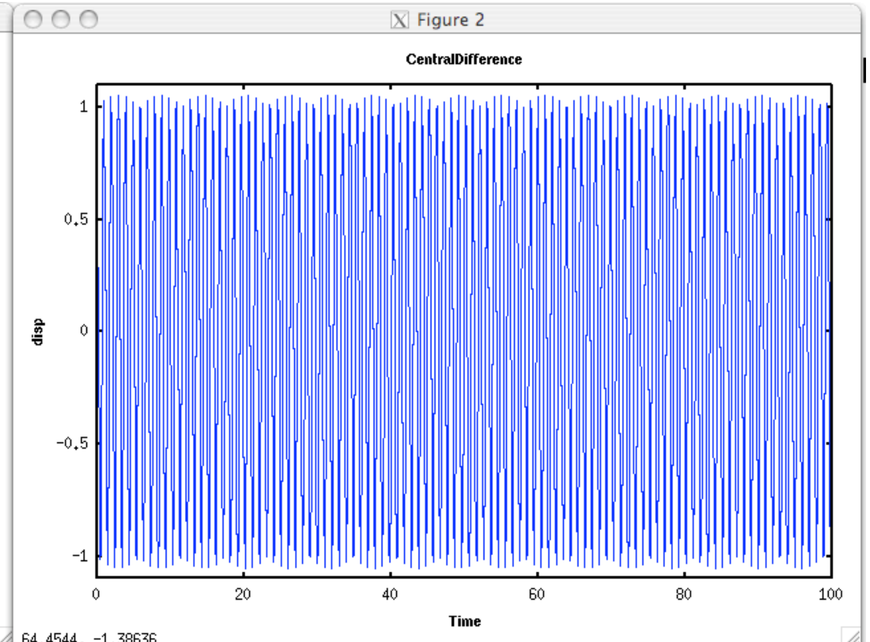
exact soln: $u(t) = u(0) \cos \omega_n t$.

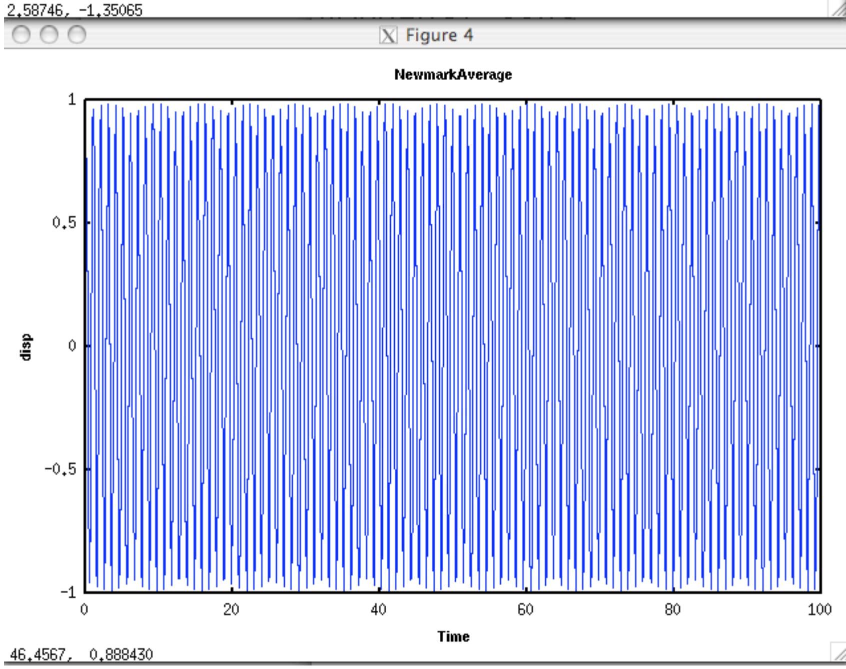
dT/Tn = 0.32 (ex1.tcl)

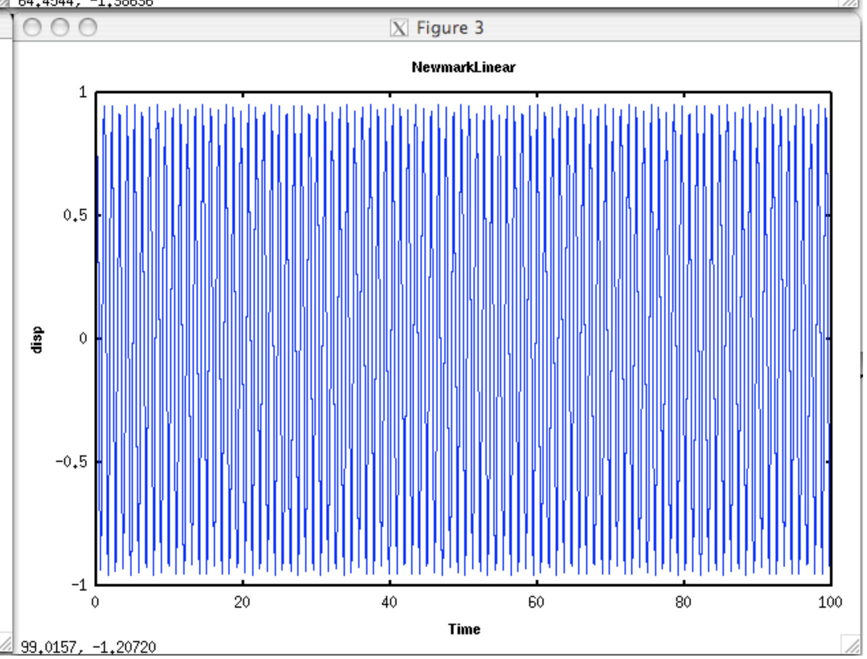


dT/Tn = 0.1 (ex1.tcl)

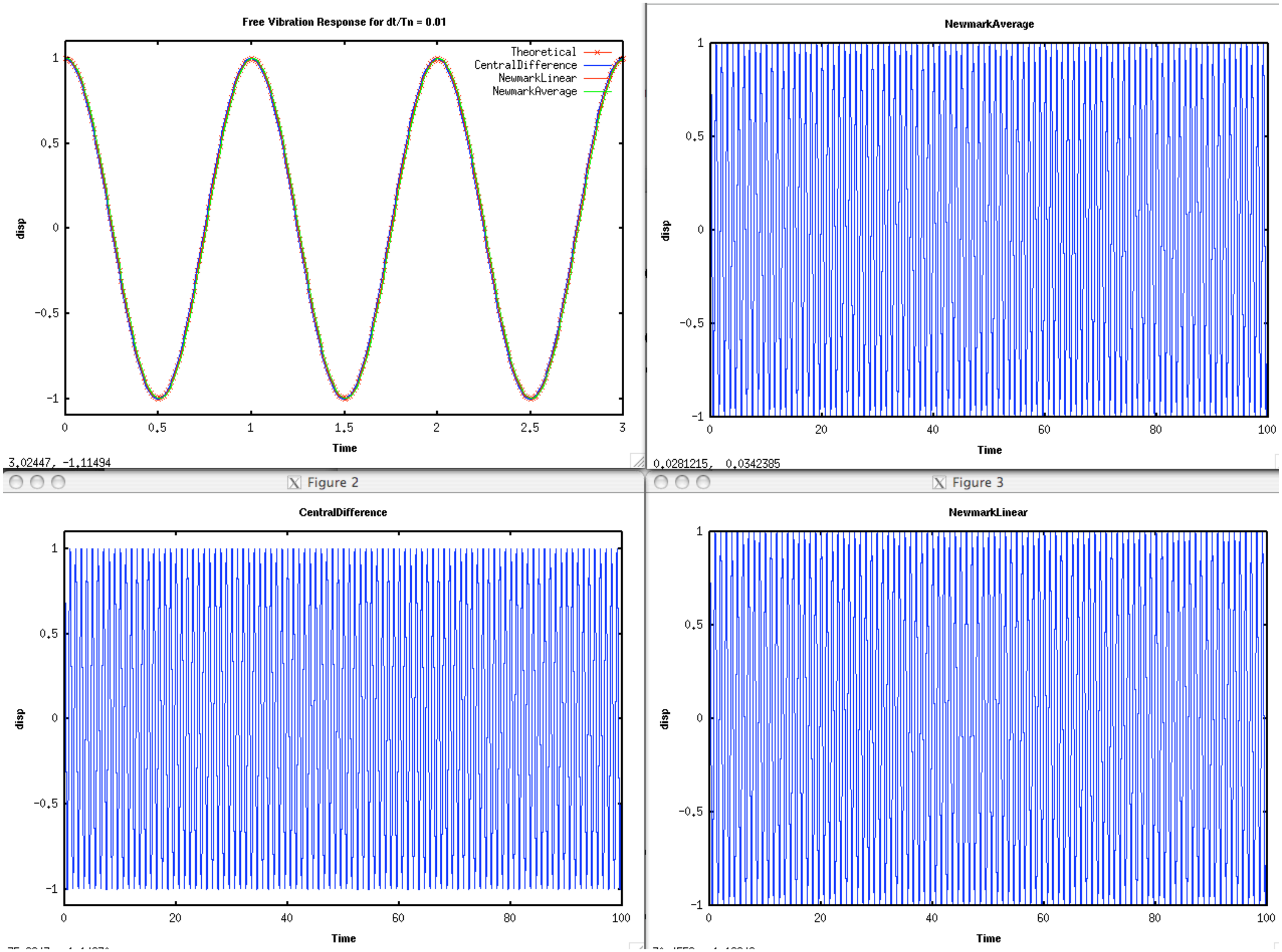








dT/Tn = 0.01 (ex1.tcl)



THINGS TO THINK ABOUT

- Computer models of large real structures contain a large number of periods. Some of these periods are smaller than the typical time step used (that from say the earthquake record). It is typically advised to select an algorithm that is unconditionally stable.
- There are situations when you might want to use a conditionally stable algorithm, e.g. convergence problems, accuracy, model size. In these cases you need to select the appropriate time step to ensure that higher frequencies do not cause instability.

Stability & Nonlinear Systems

- For nonlinear systems stability is the most important concern.
- Algorithms that are stable for linear dynamical systems ARE NOT NECESSARY STABLE in nonlinear case.
- A sufficient condition in non-linear systems for stability is the conservation of total energy within a step, expressed:

$$U_{n+1} - U_n + K_{n+1} - K_n <= W_{ext}$$

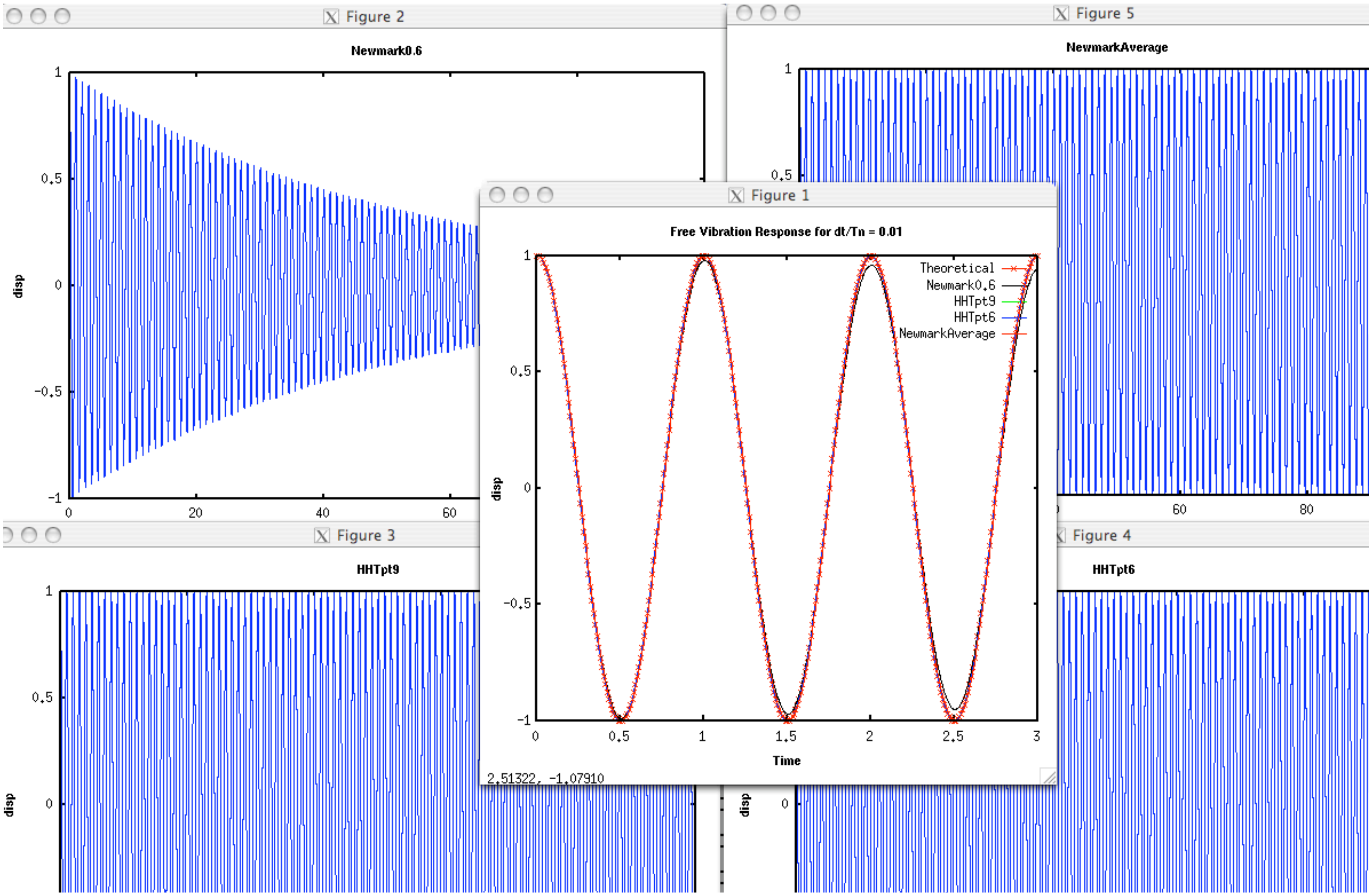
where U = strain energy and K = kinetic energy

- There are 3 groups of algorithms which ATTEMPT to satisfy this criterion:
  - Numerical Dissipation
  - Enforced Conservation of Energy
  - Algorithmic Conservation of Energy

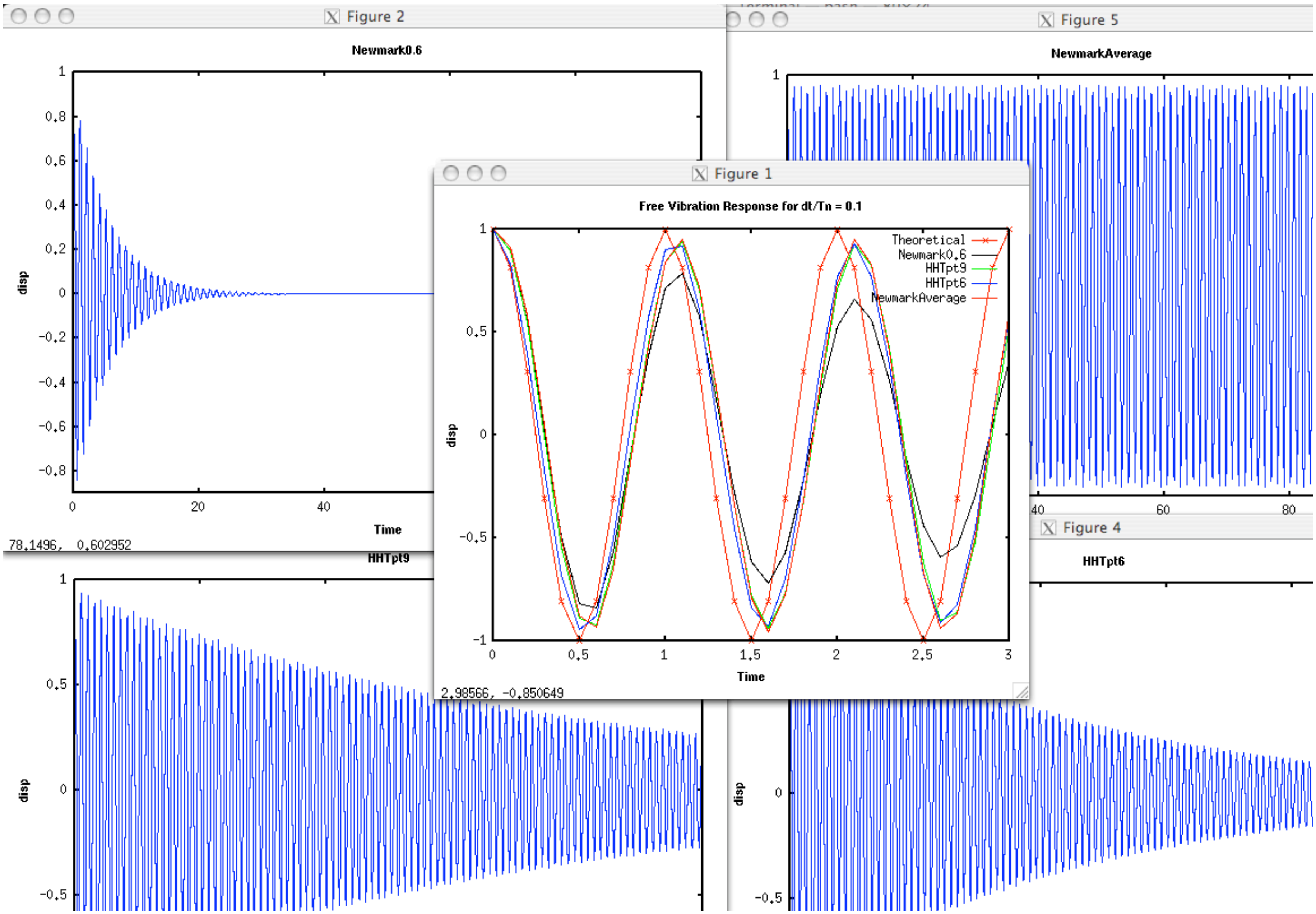
Dissipation Algorithms

- They were developed for large linear systems where typically only the low modes of response are of interest and the engineer wants to remove the high frequency noise.
- These controlled dissipation of high frequency modes is used in an ATTEMPT to conserve energy.
- For nonlinear systems they do not guarantee the dissipation of enough energy to always satisfy the conservation of energy.
- EXAMPLES: Newmark, HHT, …

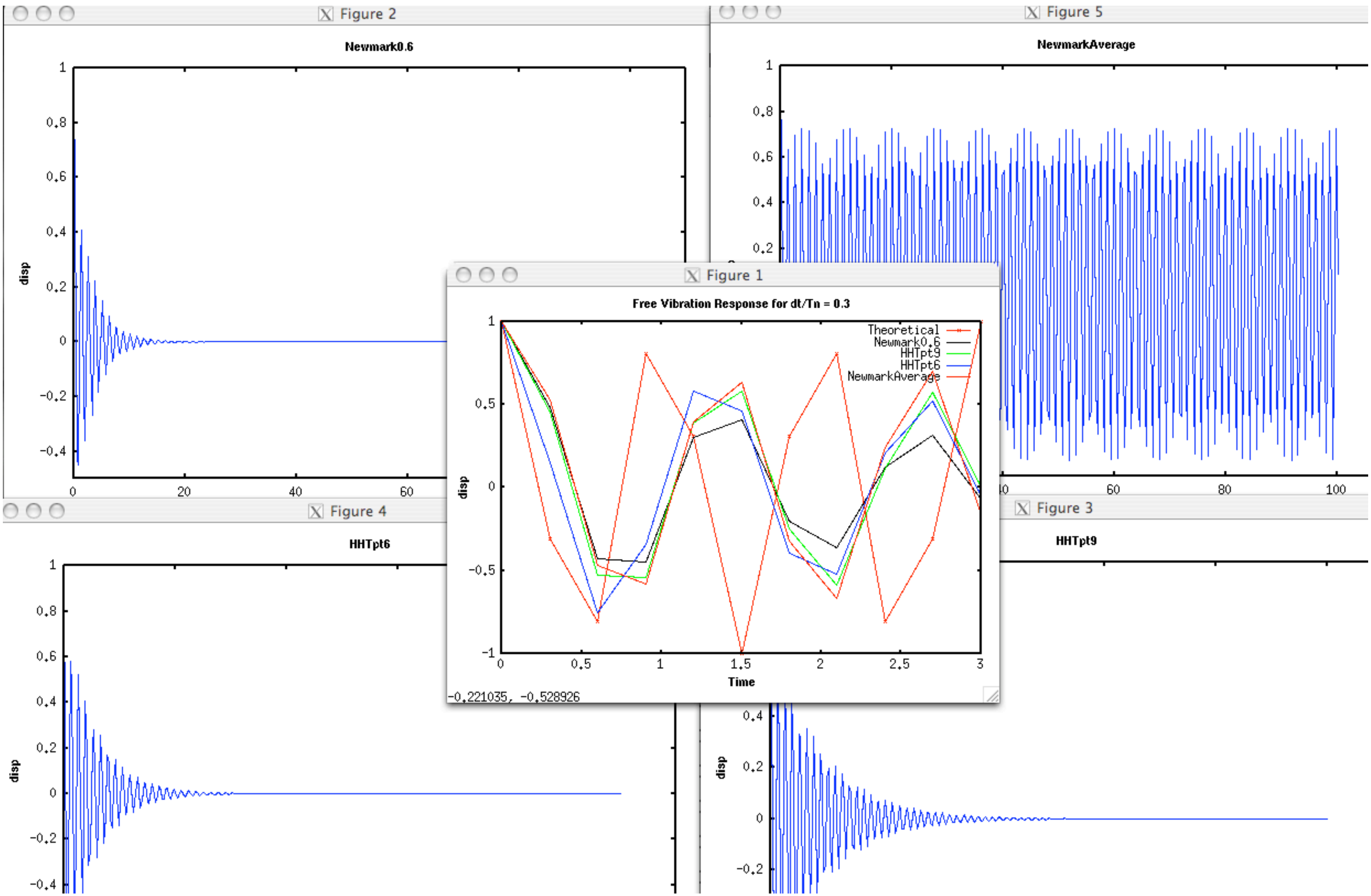
Numerical Dissipation: dT/Tn = 0.01 (ex2.tcl)



Numerical Dissipation: dT/Tn = 0.1 (ex2.tcl)



Numerical Dissipation: dT/Tn = 0.3 (ex2.tcl)



Example

(see “Dynamics of Structures” A.K. Chopra, section 5.5)

Harmonic Vibration (ex3.tcl)

$u(0) = 0$

$u'(0) = 0$

$p(t) = \sin \omega t$

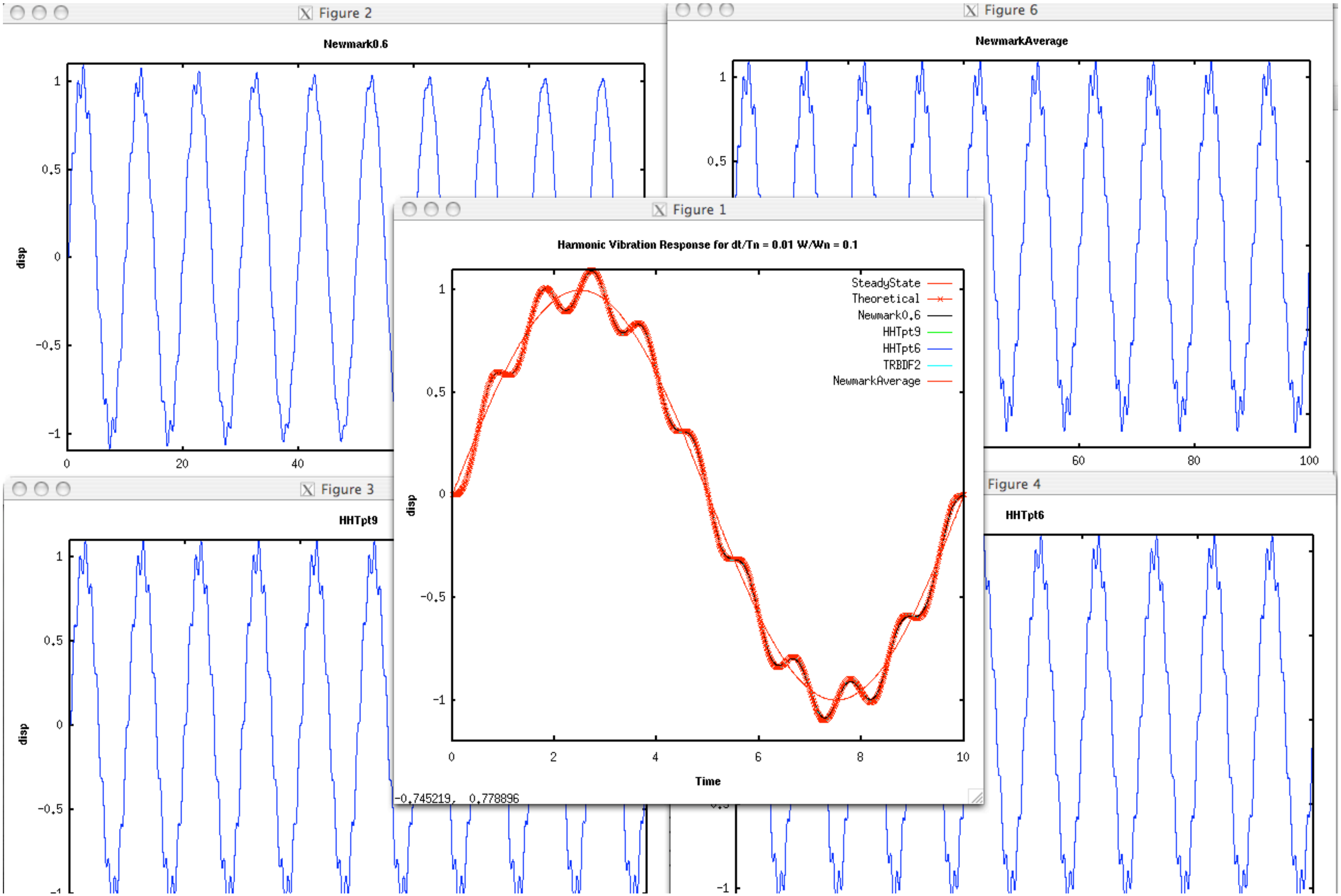
M again is chosen to give desired period

p is chosen such that $p/k = 1$.

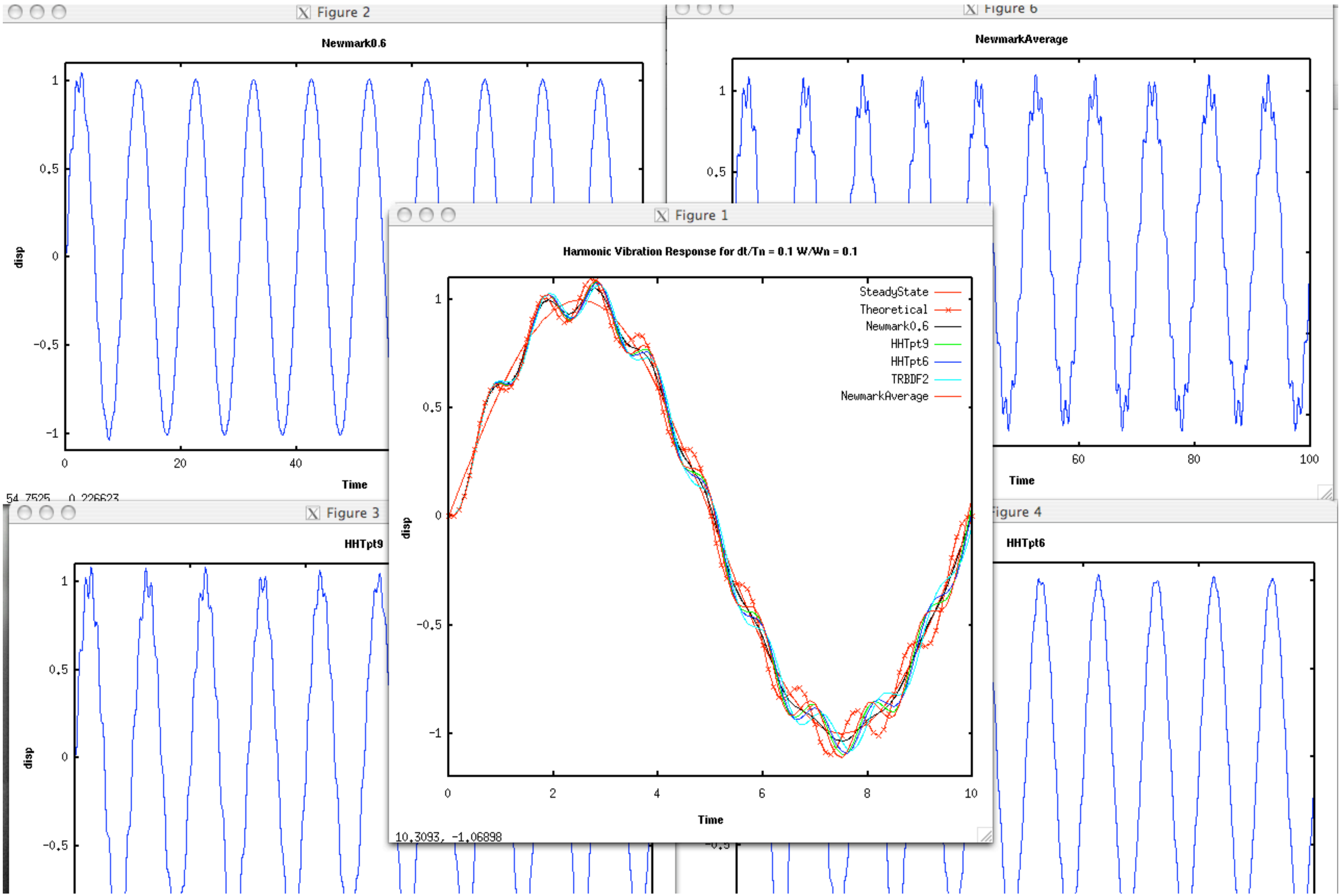
<u>exact soln</u>:

$$u(t) = \frac{p_0}{k} \frac{1}{1-(\omega/\omega_n)^2} \left(\sin \omega t - \frac{\omega}{\omega_n} \sin \omega_n t \right)$$

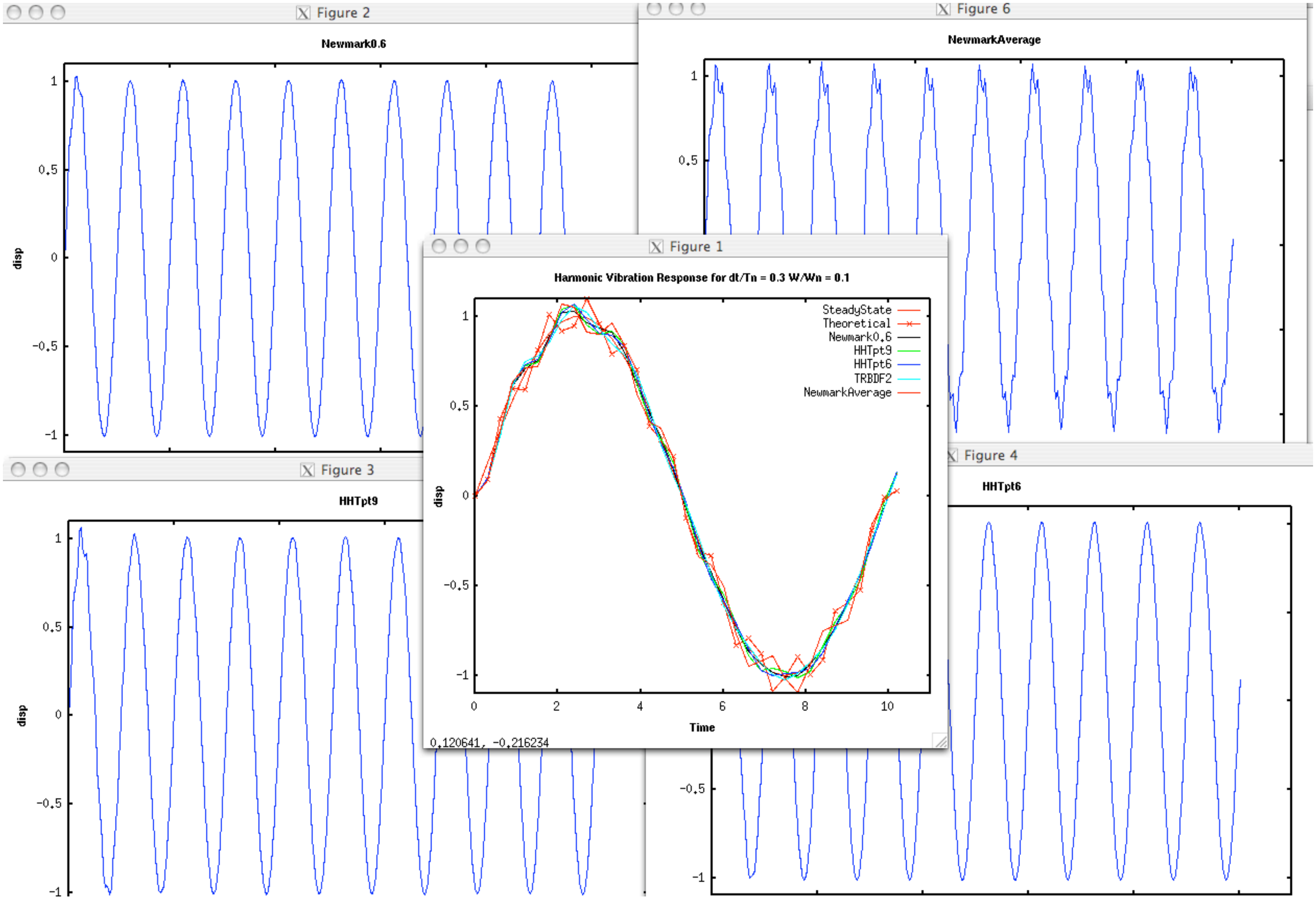
T/Tn = 0.1; dT/Tn = 0.01 (ex3.tcl)



T/Tn = 0.1; dT/Tn = 0.1 (ex3.tcl)



T/Tn = 0.1; dT/Tn = 0.3 (ex3.tcl)



Remember

- When using dissipation to damp out higher frequencies, the choice of dT is as important as choice of integrator parameters.
- Why damp out higher frequencies?

1. Not interested in spurious modes
2. Contact
3. (I know I am repeating but again) In nonlinear problems try to remove energy and hopefully allow conservation of energy (not guaranteed)

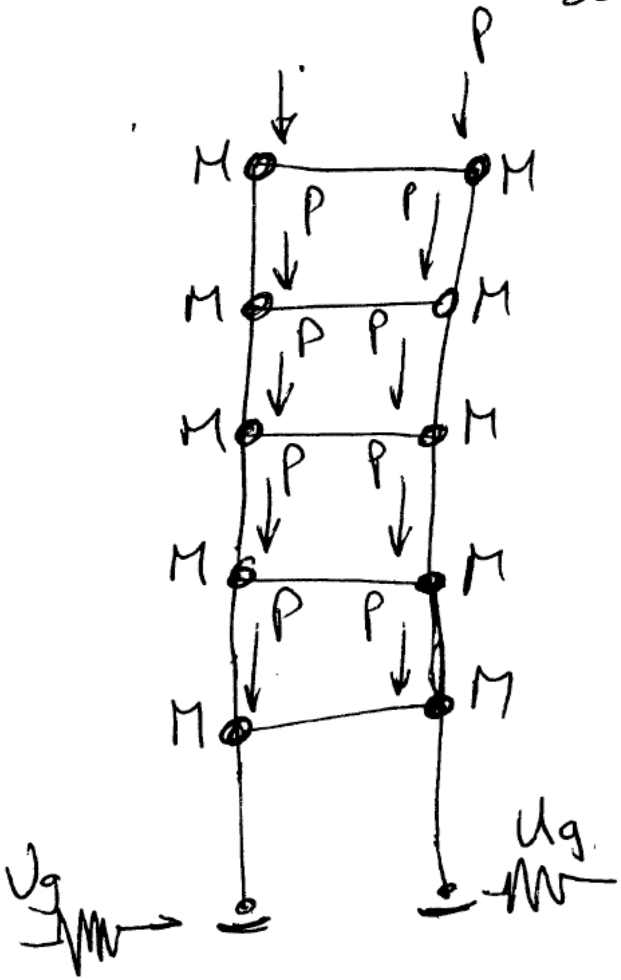
Algorithmic Energy Conserving Algorithms

- The only one available in OpenSees at the moment is TRBD2.

EXAMPLE

(Andreas Schellenberg, Rutherford and Chekenne)

– Building on friction pendulum bearing subjected to earthquake.



RESULTS

Periods Start: 0.50 sec to 0.0015 sec
Periods At Impact: 380590.94 sec to 0.0045 sec
Periods if successful Analysis: 1.29 sec to 0.0015

Eigenvalues at end of transient:

n	lambda	omega	period
0	2.725479e-10	1.6509024804633374e-5	380590.9423199949
1	4.841790e-10	2.2004067805749007e-5	285546.5345156761
2	1.900059e-09	4.358966620656781e-5	144143.9188225047
3	2.437622e+03	49.37227967189686	0.12726139746704931
4	5.094668e+03	71.37694305586363	0.08802822085364471
5	9.174452e+03	95.78335972391028	0.06559787968693609
6	1.297644e+04	113.91417822202818	0.05515718416484677
7	2.052232e+04	143.2561342491134	0.04385979937343223
8	1.388982e+05	372.69048820703756	0.016858990250615526
9	1.454944e+05	381.43728186950995	0.016472394298701693
10	5.183750e+05	719.9826386795726	0.008726856690187374
11	5.245590e+05	724.2644544639754	0.00867526394323706
12	1.036750e+06	1018.209212293819З	0.006170819544074681
13	1.036750e+06	1018.2092122938193	0.006170819544074681
14	1.037006e+06	1018.3349154379417	0.006170057818824233
15	1.038058e+06	1018.8513139806024	0.006166930562842862
16	1.040869e+06	1020.229876057352	0.00615859764023062
17	1.042444e+06	1021.0014691468372	0.006153943453607273
18	1.046875e+06	1023.1690964840562	0.006140906062126648
19	1.055707e+06	1027.476033783757	0.006115164831671342
20	1.555125e+06	1247.0465107605248	0.005038453059258967
21	1.560327e+06	1249.130497586221	0.00503004715625887
22	1.934602e+06	1390.8997088215958	0.004517353240732830
23	1.939489e+06	1392.6553773277867	0.004511658382589740

Eigenvalues at end of transient:

n	lambda	omega	period
1	2.365794e+01	4.863942845058935	1.291788474357259
2	2.444645e+03	49.443351423624186	0.12707846709957166
3	3.354993e+03	57.92230140455401	0.10847609909860358
4	7.038561e+03	83.89613221120506	0.07489243117146242
5	1.297724e+04	113.91768958331274	0.05515548401799733
6	2.045299e+04	143.01395036848677	0.0439340728019222
7	4.551268e+04	213.3370103849775	0.029451923488762022
8	1.422025e+05	377.0974675067442	0.016661966331203787
9	3.581179e+05	598.4295280147865	0.010499457351349708
10	5.214956e+05	722.1465225284962	0.008700707004971624
11	8.868152e+05	941.7086598306294	0.006672111636214162
12	1.036758e+06	1018.2131407519744	0.006170795735890136
13	1.037022e+06	1018.3427713692478	0.006170010220361572
14	1.038072e+06	1018.8581844398169	0.0061666888977423460 5
15	1.039638e+06	1019.6264021689512	0.006162242654578169 5
16	1.040878e+06	1020.2342868184738	0.006158571014872712
17	1.046880e+06	1023.1715398700259	0.006140891397328881 5
18	1.055708e+06	1027.4765204129972	0.006115161935431908
19	1.465409e+06	1210.5407882430068	0.005190395373871768
20	1.557795e+06	1248.1165810932887	0.005034133351289849
21	1.909504e+06	1381.8480379549699	0.00454694375546404
22	1.937262e+06	1391.855595958144	0.004514250850034686
23	1.823510e+07	4270.257603470779	0.001471383202285675

For a dT=0.001
Newmark Average Acceleration and HHT 0.9 failed
HHT 0.6, TRBDF2, and Newmark 0.6 0.3025 worked

Max recorded roof displacements: 6.03, 5.88, 5.87 respectively

Conclusion

- I hope you walk away from this webinar with the understanding and realization that **You actually need to spend some time thinking about the choice of integration strategy you use in the analysis**.
- For any graduate student/professor looking for something to do, **we could use some more integrators that look at conserving energy.**

Any Questions?