Modeling Steel Moment Resisting Frames with OpenSees

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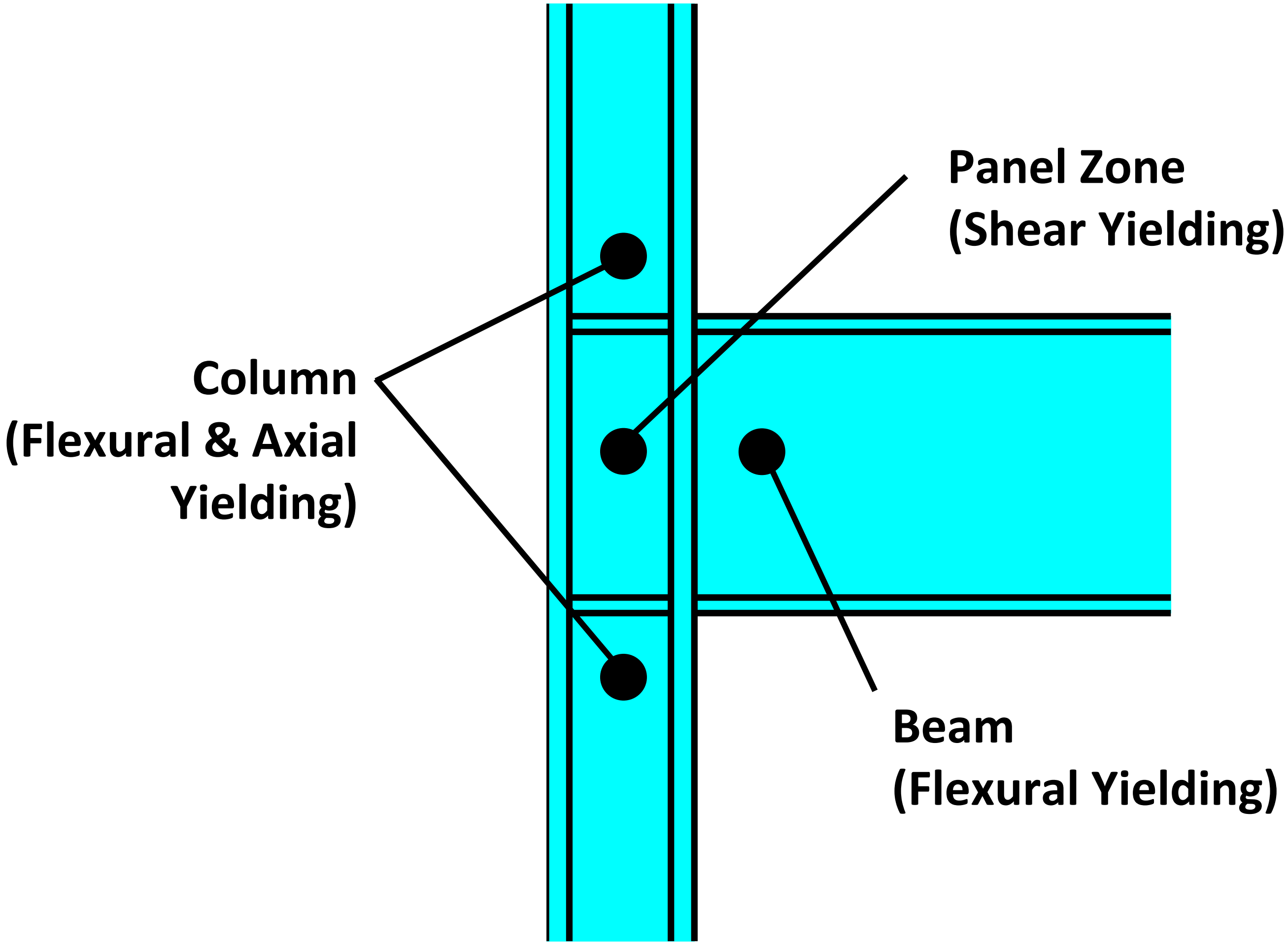
Agenda

- ✧ Nonlinear Modeling of Steel MRFs
  - ✧ Steel Components for Nonlinear Modeling
- ✧ Distributed and Concentrated Plasticity
  - ✧ Nonlinear Force-Based Elements
  - ✧ Zero Length Elements (Concentrated Plasticity)
- ✧ Background on Available Steel Materials in OpenSees
- ✧ Examples and Applications
- ✧ Summary Remarks

Steel Components for Nonlinear Modeling in MRFs

(Images courtesy of Prof. M. Engelhardt)

Steel Components for Nonlinear Modeling in MRFs



Panel Zone
(Shear Yielding)

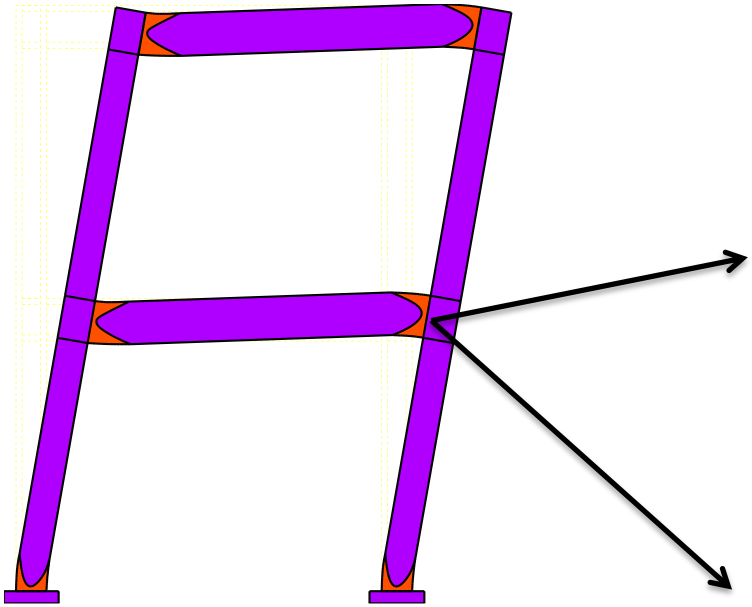
Column
(Flexural & Axial
Yielding)

Beam
(Flexural Yielding)

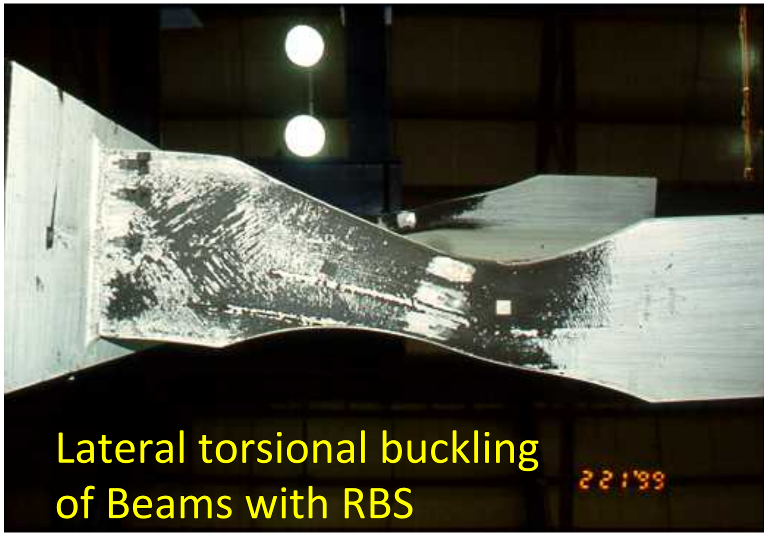
(Image courtesy of Prof. M. Engelhardt)

Steel Components for Nonlinear Modeling in MRFs

-Beam Plastic Hinging



Local buckling of Beams with RBS

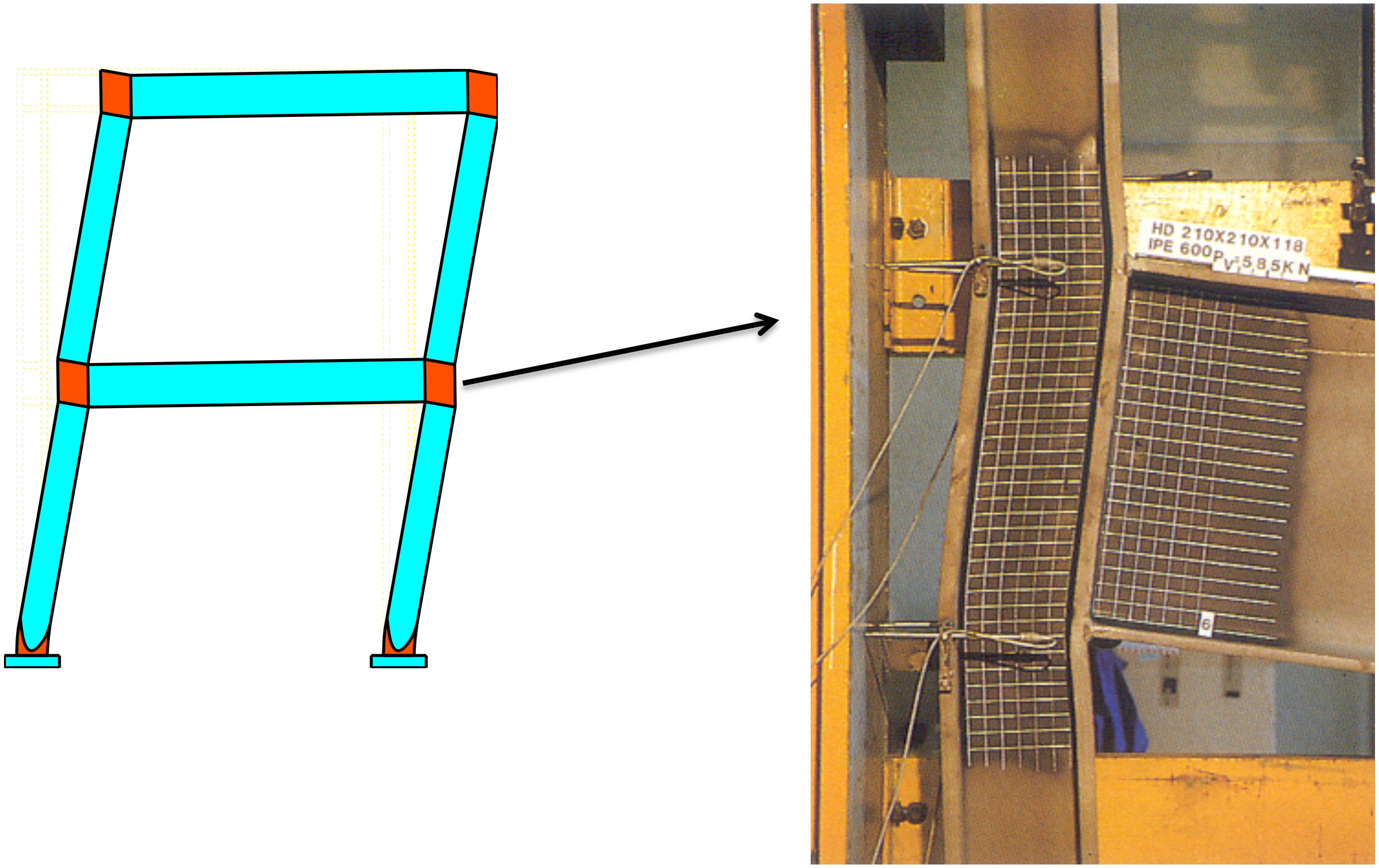


Lateral torsional buckling
of Beams with RBS

(Images courtesy of Prof. M. Engelhardt)

Steel Components for Nonlinear Modeling in MRFs

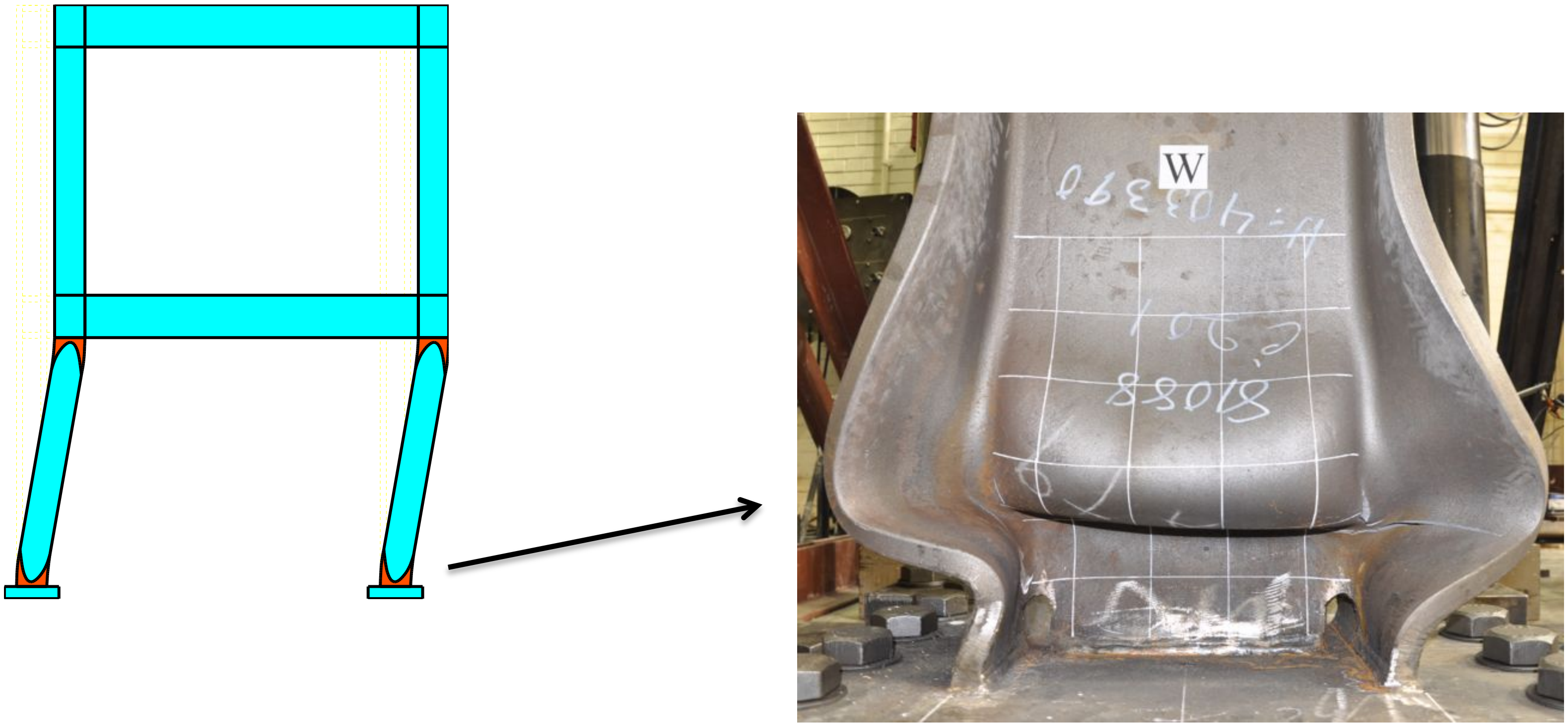
-Panel Zone Shear Yielding



(Image courtesy of Prof. M. Engelhardt)

Steel Components for Nonlinear Modeling in MRFs

-Column Plastic Hinging



(Image from Suzuki and Lignos 2014)

Simulation Approach

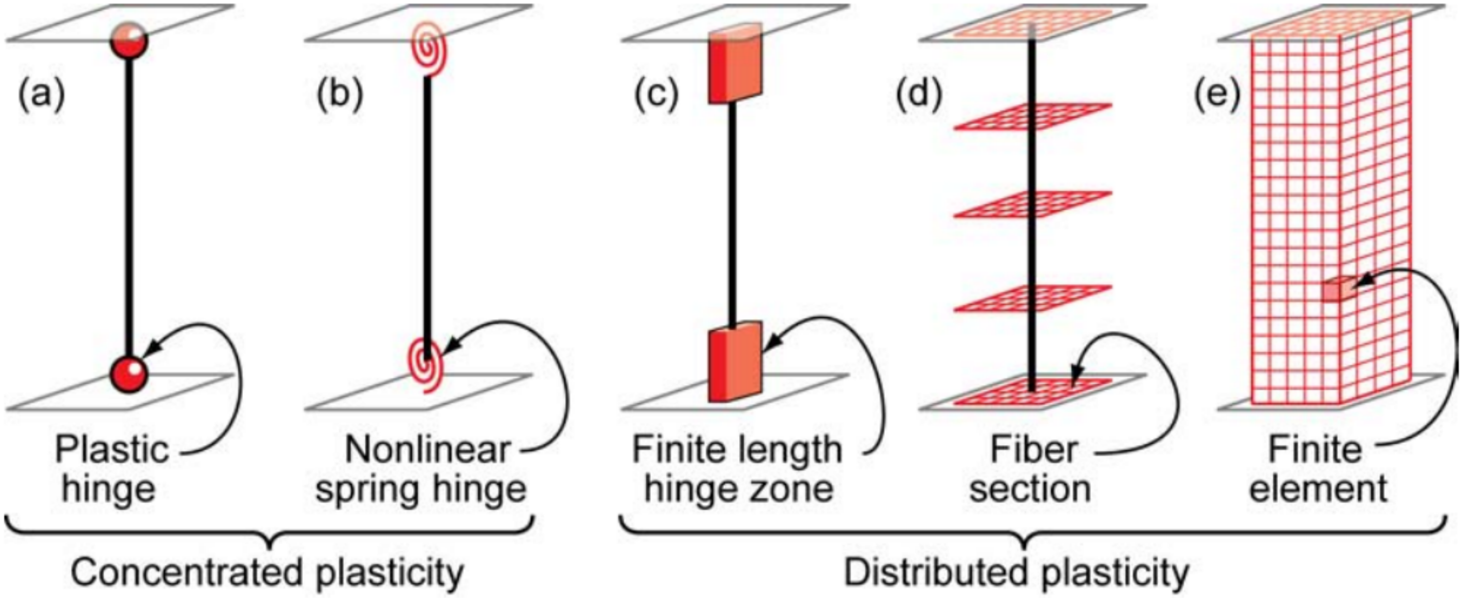


Image Source: NIST GSR 10-917-5

Concentrated Plasticity Models

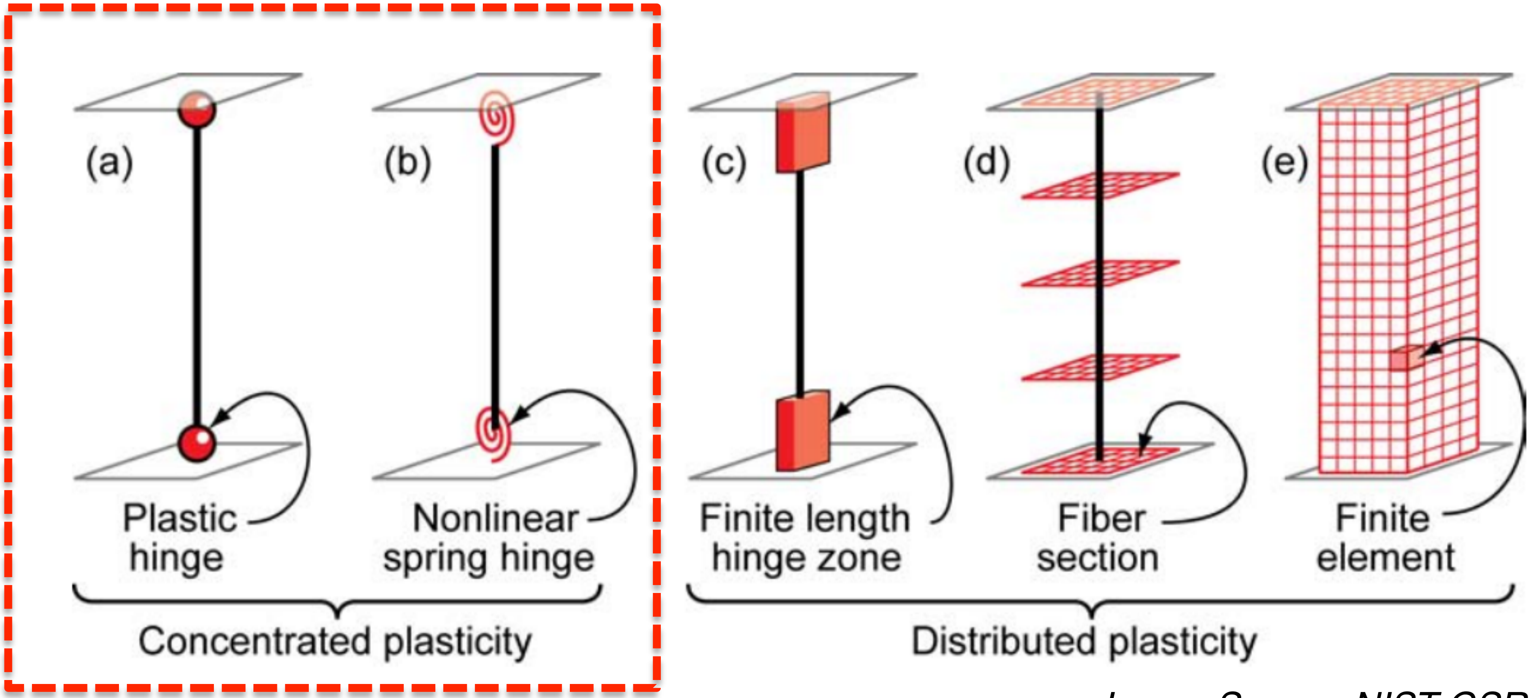


Image Source: NIST GSR 10-917-5

Advantages

✧ Fairly simple

✧ Effective for interface effects

✧ Computationally efficient

Disadvantages

✧ Require the moment – rotation relationship (as opposed to engineering stress-strain)

✧ They don’t capture P-M interaction (critical for columns)

Distributed Plasticity Models

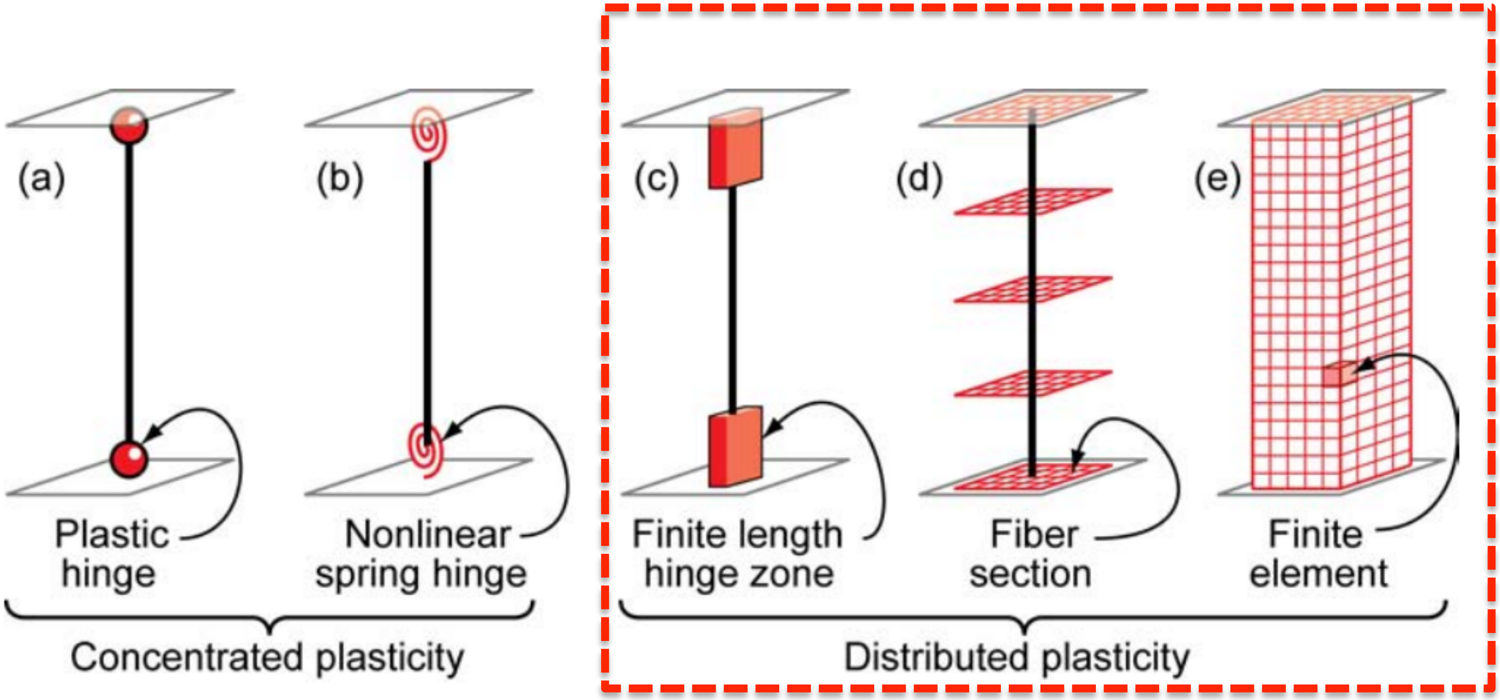


Image Source: NIST GSR 10-917-5

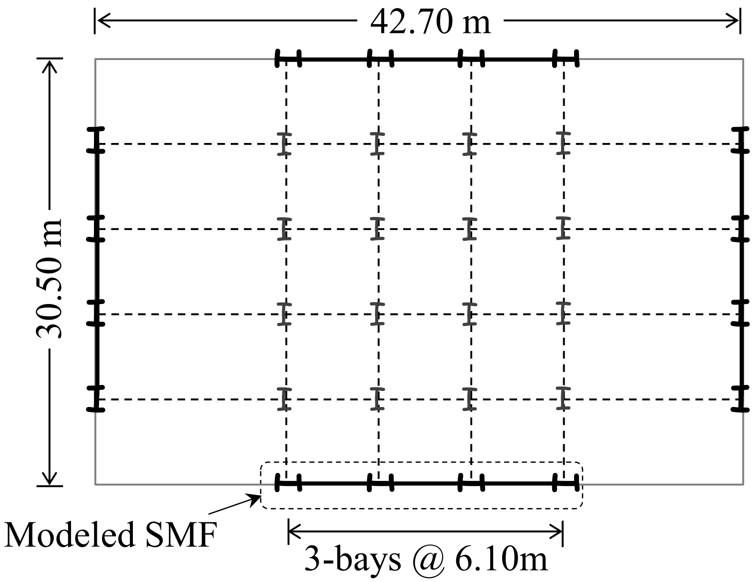
Advantages

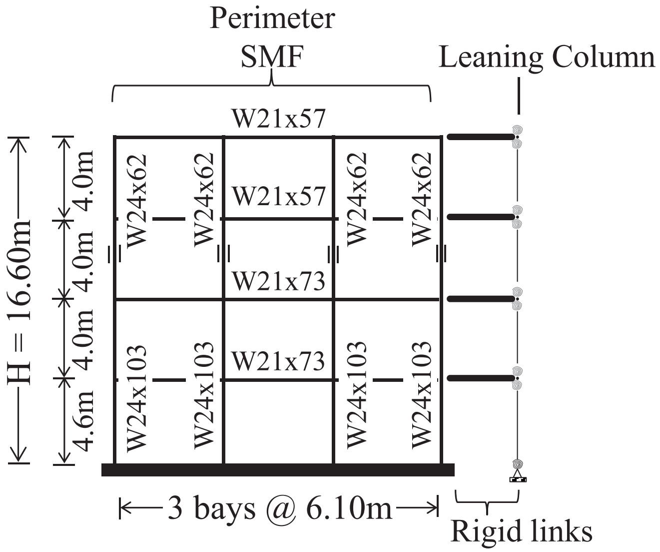
- ✧ Force- and displacement-based element permits spread of plasticity along the element
- ✧ P-M interaction can be captured

Disadvantages

- ✧ Localization
- ✧ Requires fiber section discretization (# fibers matters)
- ✧ Deteriorating phenomena (local buckling) difficult to capture with available engineering stress-strain models in OpenSees

Example for Today's Presentation

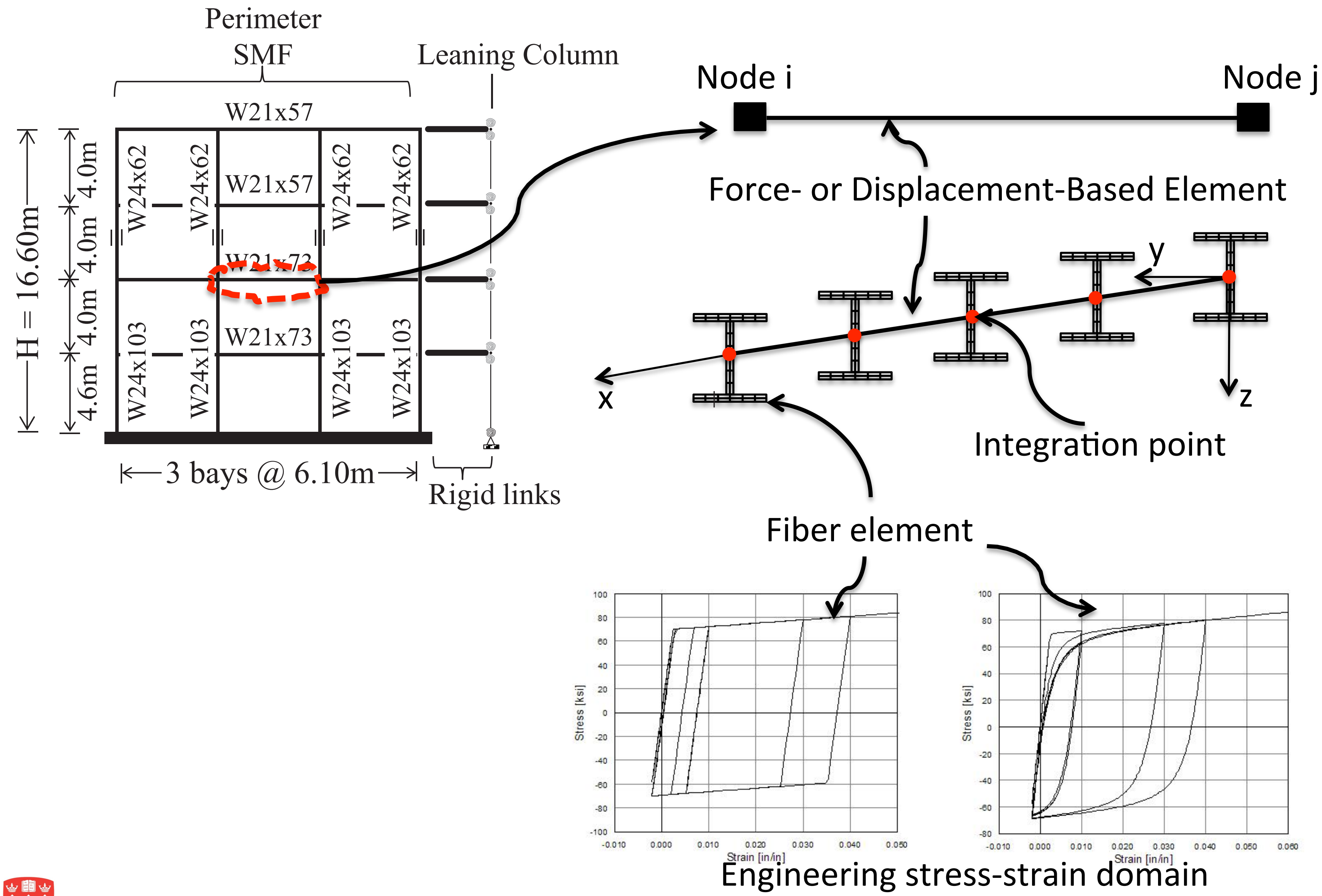




- ✧ 4-story building with perimeter steel special moment frames (SMFs)
- ✧ Design location Bulk Center Los Angeles, CA
- ✧ Design provisions: ASCE 7-10, AISC 2010
- ✧ Fully-Restrained Beam-to-Column connections with Reduced Beam Sections (RBS)→ See AISC-358-10 or FEMA-350
- ✧ First mode period: 1.51sec

(Source: Elkady and Lignos 2014)

Modeling with Distributed Plasticity



Node i

Node j

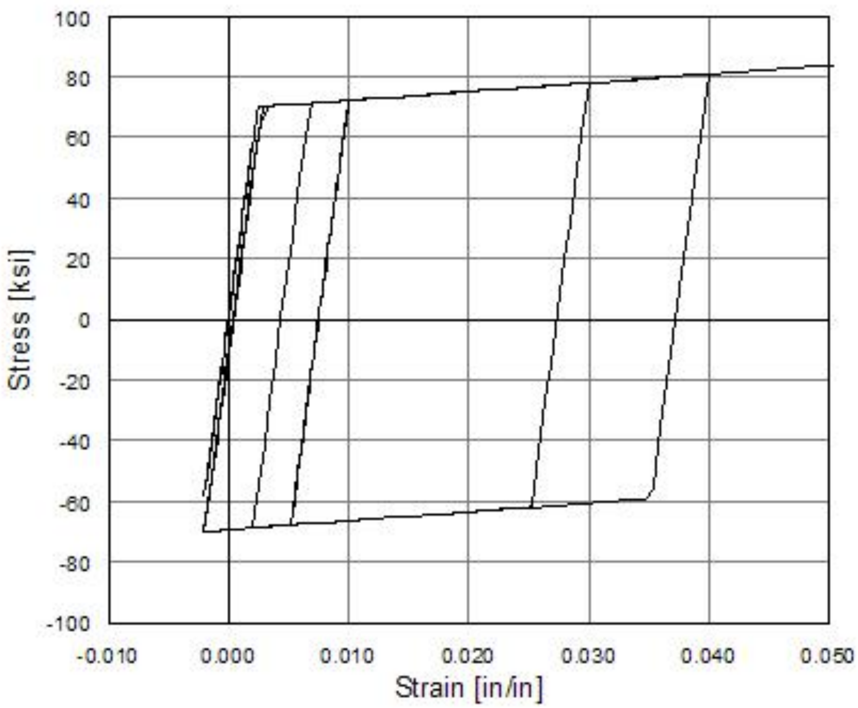
Force- or Displacement-Based Element

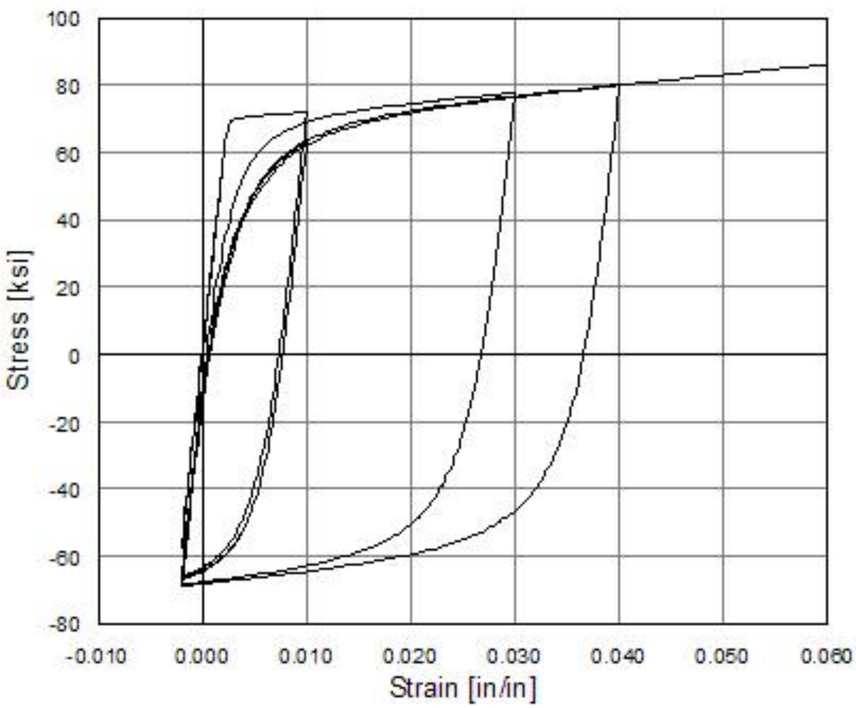
Integration point

Fiber element

Engineering stress-strain domain

Steel Material Models Available in OpenSees


Steel01-Simple Bilinear

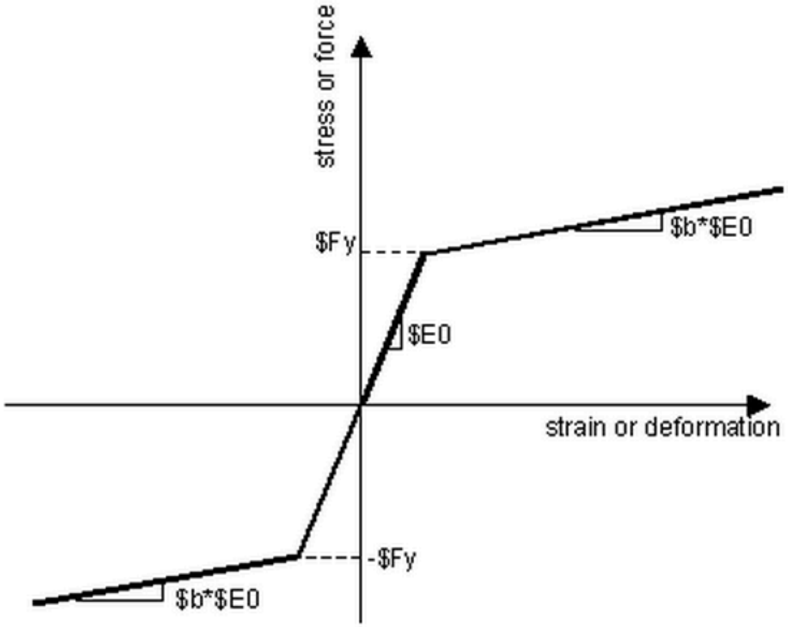

Steel02-Giuffre Menegotto-Pinto Model
with Isotropic Strain Hardening

Steel Material Models Available in OpenSees

Utilization of Steel01 for Modeling of Steel Components

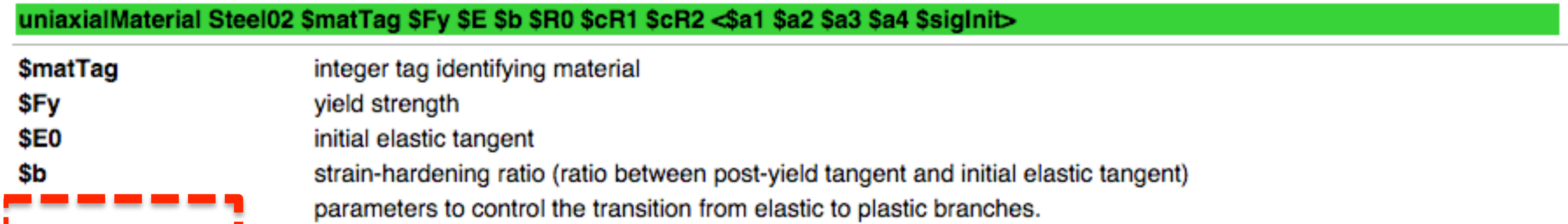
uniaxialMaterial Steel01 $matTag $Fy $E0 $b <$a1 $a2 $a3 $a4>

$matTag	integer tag identifying material
$Fy	yield strength
$E0	initial elastic tangent
$b	strain-hardening ratio (ratio between post-yield tangent and initial elastic tangent)
$a1	isotropic hardening parameter, increase of compression yield envelope as proportion of yield strength after a plastic strain of $a2*($Fy/E0). (optional)
$a2	isotropic hardening parameter (see explanation under $a1). (optional).
$a3	isotropic hardening parameter, increase of tension yield envelope as proportion of yield strength after a plastic strain of $a4*($Fy/E0). (optional)
$a4	isotropic hardening parameter (see explanation under $a3). (optional)



Steel Material Models Available in OpenSees

Utilization of Steel02 for Modeling of Steel Components



```
uniaxialMaterial Steel02 $matTag $Fy $E $b $R0 $cR1 $cR2 <$a1 $a2 $a3 $a4 $sigInit>
```

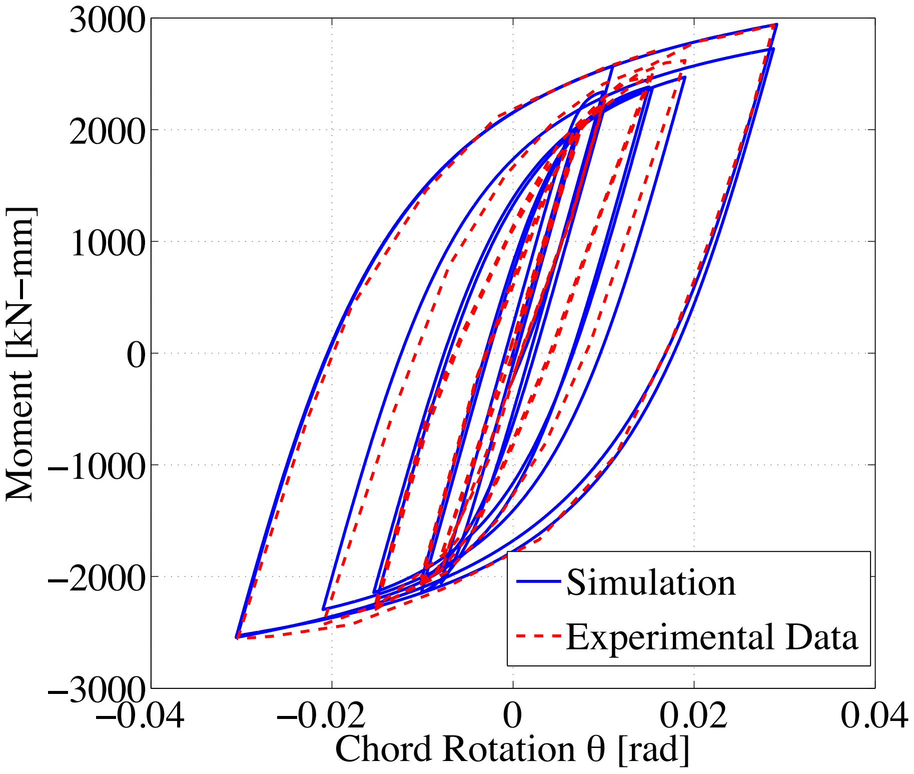
$matTag	integer tag identifying material
$Fy	yield strength
$E0	initial elastic tangent
$b	strain-hardening ratio (ratio between post-yield tangent and initial elastic tangent)
$R0 $CR1 $CR2	parameters to control the transition from elastic to plastic branches. Rec
$a1	isotr $a2'
$a2	isotr
$a3	isotr ($Fy
$a4	isotr
	Initia

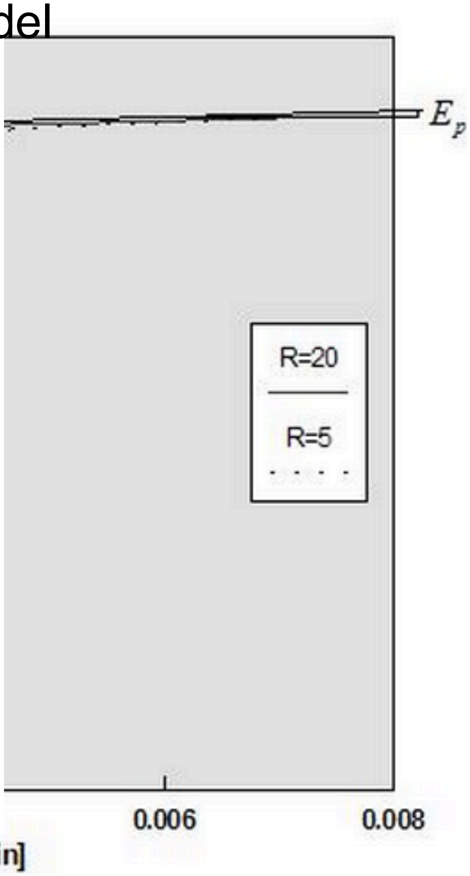
of

4*

From calibration

Example – Utilization of Steel02 Material Model





Number of Fibers for Cross Section Discretization

→Follow the recommendations by Kostic & Filippou (2012)

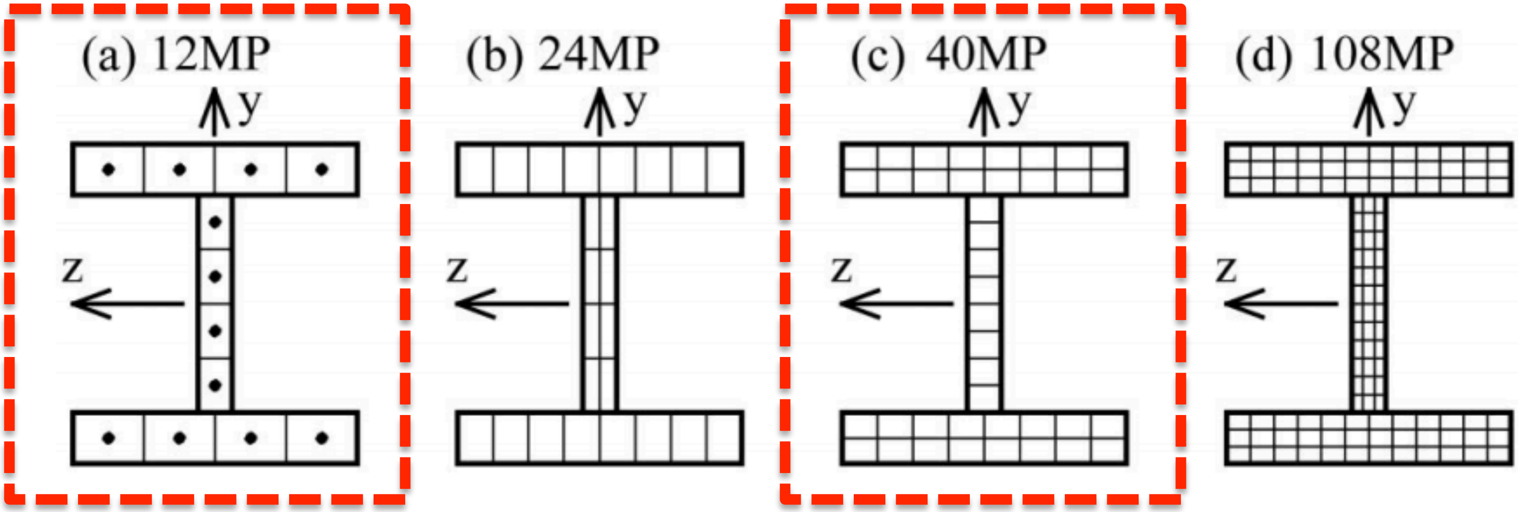
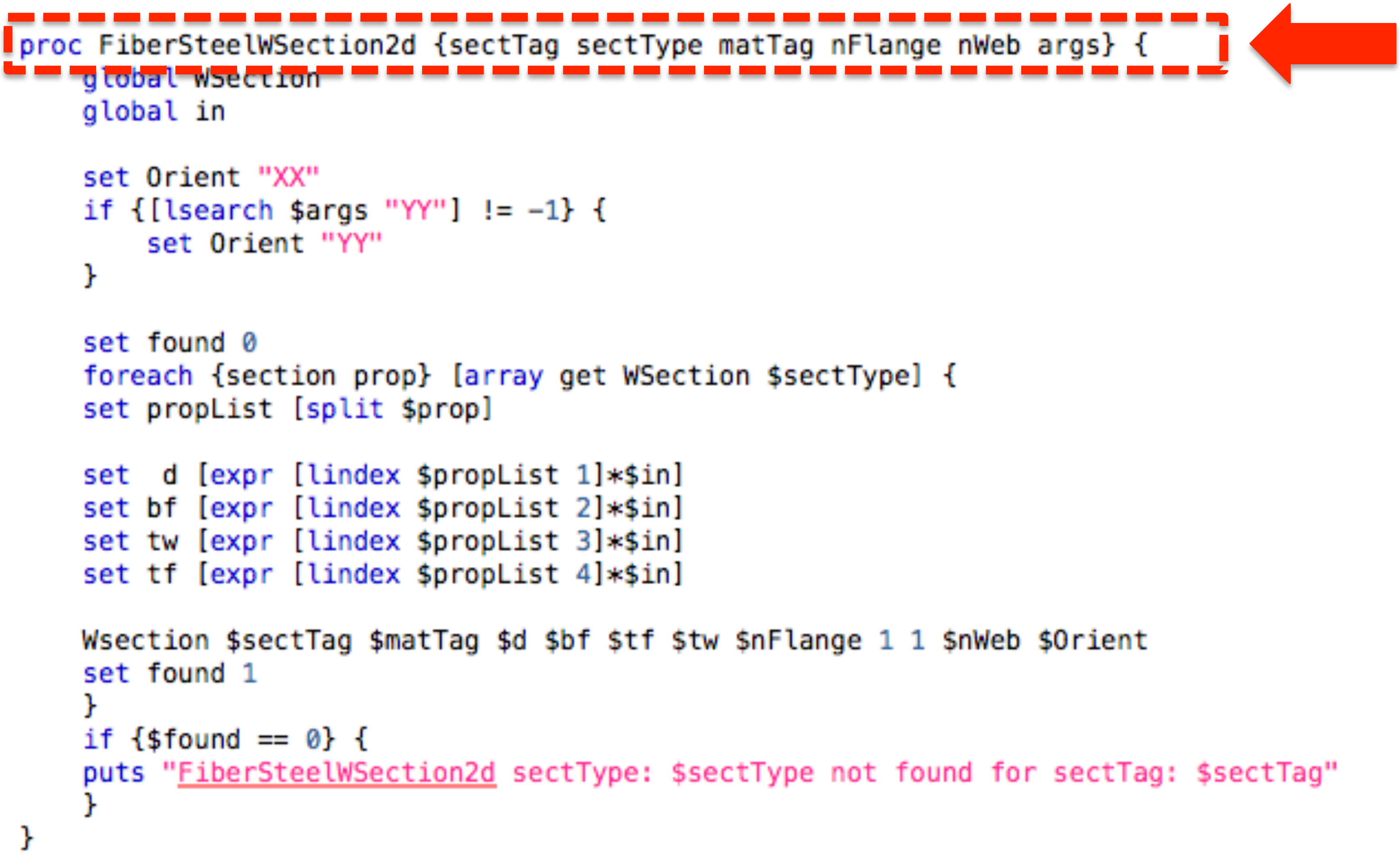


Fig. 1. Discretization of wide-flange cross section with midpoint integration rule (MP): (a) 12MP scheme; (b) 24MP scheme; (c) 40MP scheme, and (d) 108MP scheme

W-Shapes: 12 MP gives remarkable accuracy in terms of local response estimates

Biaxial bending and failures associated with weak axis bending: (40MP) 2x8 fibers (Flange) 1x8 fibers (Web)

Steel2.tcl – FiberSteelWsection2d

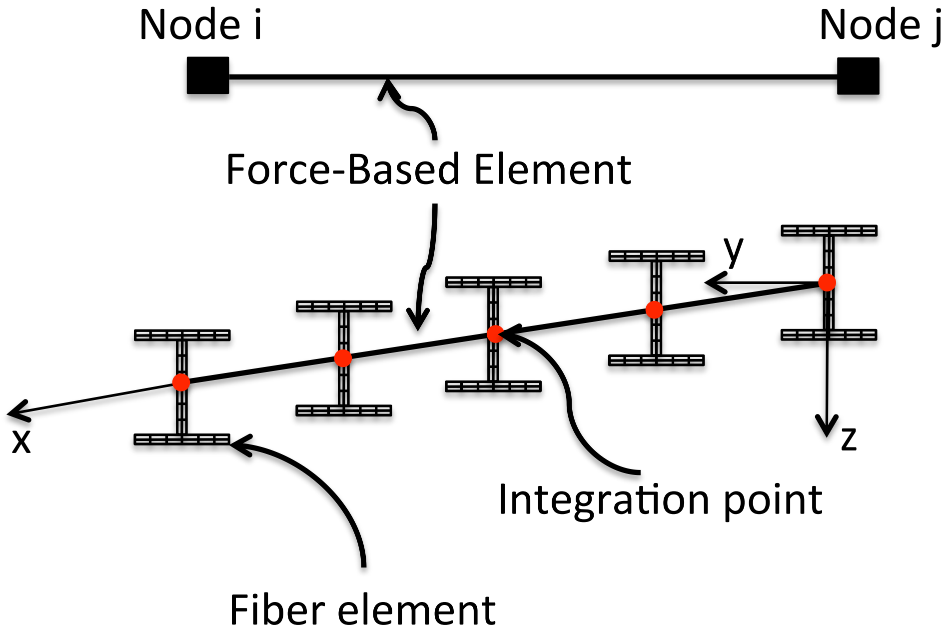


```
proc FiberSteelWSection2d {sectTag sectType matTag nFlange nWeb args} {
	global WSection
	global in

	set Orient "XX"
	if {[lsearch $args "YY"] != -1} {
		set Orient "YY"
	}

	set found 0
	foreach {section prop} [array get WSection $sectType] {
	set propList [split $prop]

	set  d [expr [lindex $propList 1]*$in]
	set bf [expr [lindex $propList 2]*$in]
	set tw [expr [lindex $propList 3]*$in]
	set tf [expr [lindex $propList 4]*$in]

	Wsection $sectTag $matTag $d $bf $tf $tw $nFlange 1 1 $nWeb $Orient
	set found 1
	}
	if {$found == 0} {
	puts "FiberSteelWSection2d sectType: $sectType not found for sectTag: $sectTag"
	}
}
```

Nonlinear Beam-Column Elements in OpenSees

-Use of forceBeamColumn element

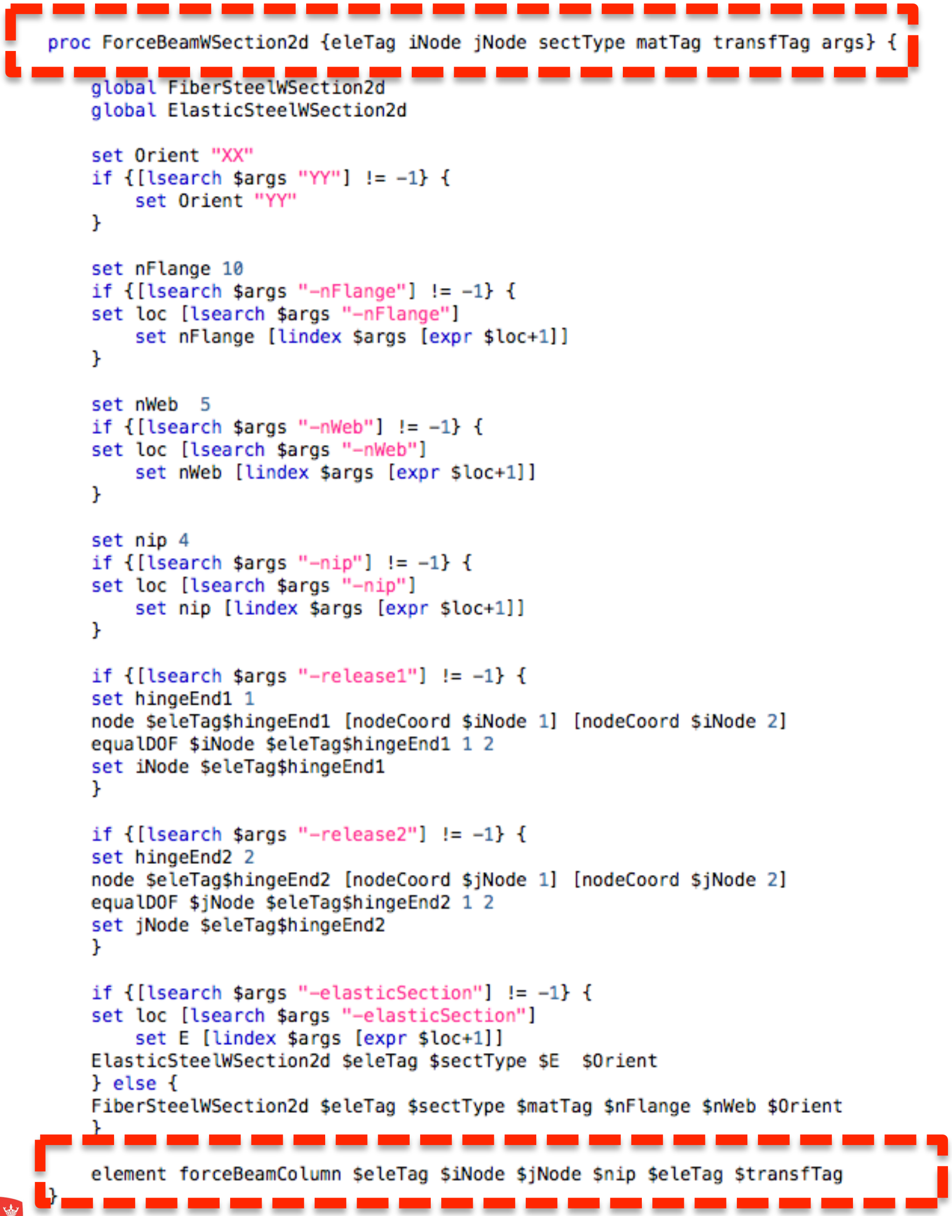
(Neuenhofer and Filippou 1997) → good discussion why you may want to consider using force based elements over displacement-based ones.



- Only one element is adequate
- 5 integration points along the member length are sufficient most of the times for modeling of steel MRFs

Recommendations by Kostic & Filippou (2012)

Steel2.tcl – Procedure: ForceBeamWSection2d



```
proc ForceBeamWSection2d {eleTag iNode jNode sectType matTag transfTag args} {
    global FiberSteelWSection2d
    global ElasticSteelWSection2d

    set Orient "XX"
    if {[lsearch $args "YY"] != -1} {
        set Orient "YY"
    }

    set nFlange 10
    if {[lsearch $args "-nFlange"] != -1} {
    set loc [lsearch $args "-nFlange"]
        set nFlange [lindex $args [expr $loc+1]]
    }

    set nWeb  5
    if {[lsearch $args "-nWeb"] != -1} {
    set loc [lsearch $args "-nWeb"]
        set nWeb [lindex $args [expr $loc+1]]
    }

    set nip 4
    if {[lsearch $args "-nip"] != -1} {
    set loc [lsearch $args "-nip"]
        set nip [lindex $args [expr $loc+1]]
    }

    if {[lsearch $args "-release1"] != -1} {
    set hingeEnd1 1
    node $eleTag$hingeEnd1 [nodeCoord $iNode 1] [nodeCoord $iNode 2]
    equalDOF $iNode $eleTag$hingeEnd1 1 2
    set iNode $eleTag$hingeEnd1
    }

    if {[lsearch $args "-release2"] != -1} {
    set hingeEnd2 2
    node $eleTag$hingeEnd2 [nodeCoord $jNode 1] [nodeCoord $jNode 2]
    equalDOF $jNode $eleTag$hingeEnd2 1 2
    set jNode $eleTag$hingeEnd2
    }

    if {[lsearch $args "-elasticSection"] != -1} {
    set loc [lsearch $args "-elasticSection"]
        set E [lindex $args [expr $loc+1]]
    ElasticSteelWSection2d $eleTag $sectType $E  $Orient
    } else {
    FiberSteelWSection2d $eleTag $sectType $matTag $nFlange $nWeb $Orient
    }

    element forceBeamColumn $eleTag $iNode $jNode $nip $eleTag $transfTag
}
```

→Uses forceBeamColumn element

MRF1.tcl

Same Model in 35 lines (Tcl Code by F. McKenna)

```
model Basic –ndm 2 –ndf 3
source Steel2d.tcl

# set some lists containing floor and column line locations and nodal masses
set floorLocs {0. 204. 384. 564.}; # floor locations in inches
set colLocs {0. 360. 720. 1080. 1440. 1800.}; #column line locations in inches
set massesX {0. 0.419 0.419 0.430}; # mass at nodes on each floor in x dirn
set massesY{0. 0.105 0.105 0.096} ; # “ “ “ “ “ “ in y dirn

# add nodes at each floor at each column line location & fix nodes if at floor 1
    foreach floor {1 2 3 4 5} floorLoc $floorLocs massX $massesX massY $massesY {
        foreach colLine {1 2 3 4} colLoc $colLocs {
            node $colLine$floor $colLoc $floorLoc -mass $massX $massY 0.
            if {$floor == 1} {fix $colLine$floor 1 1 1}
        }
    }
```

Selection of material model

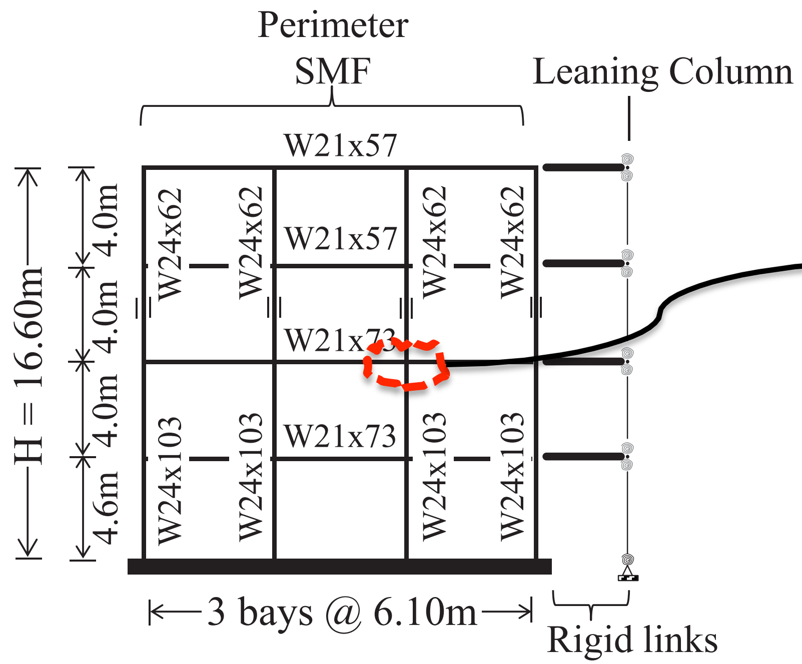
```
#uniaxialMaterial Steel02 $tag $Fy $E $b $R0 $cr1 $cr2 $a1 $a2 $a3 $a4
uniaxialMaterial Steel02 1 55.0 29000. 0.02 19.0 0.925 0.15 0.12 0.90 0.18 0.90; # material to be used for steel elements
```

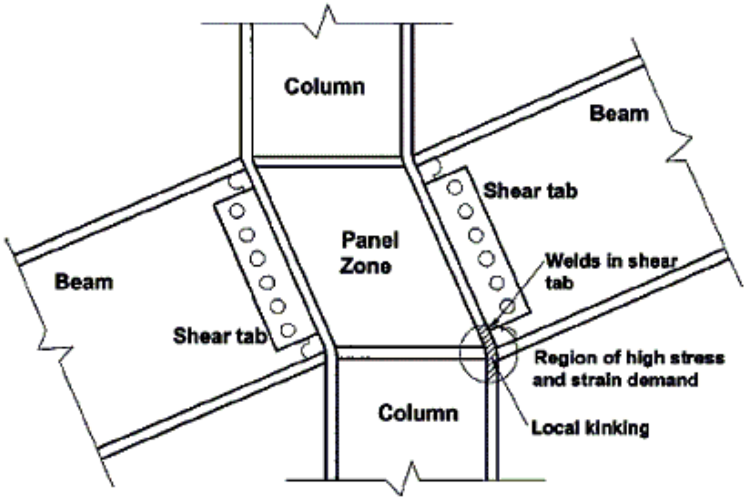
```
# set some list for col and beam sizes
set colSizes {W24x103 W24x103 W24X103 W24x64}; #col sizes stories 1, 2, 3 and 4
set beamSizes {W21x73 W21X73 W21X57 W21x57}; #beams sizes floor 1, 2, 3 and 4

# add columns at each column line between floors
geomTransf PDelta 1
    foreach colLine {1 2 3 4} {
        foreach floor1 {1 2 3 } floor2 { 2 3 4 5 } {
```

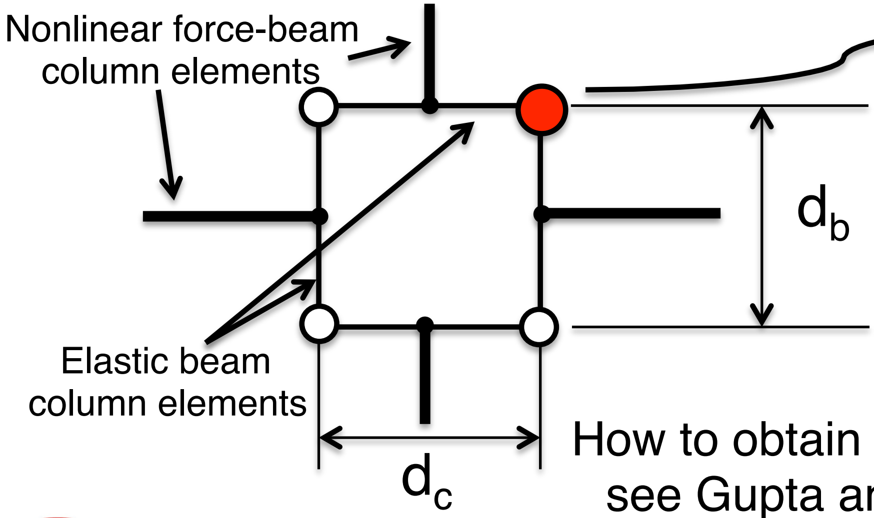
Selection of nonlinear force beamColumn element

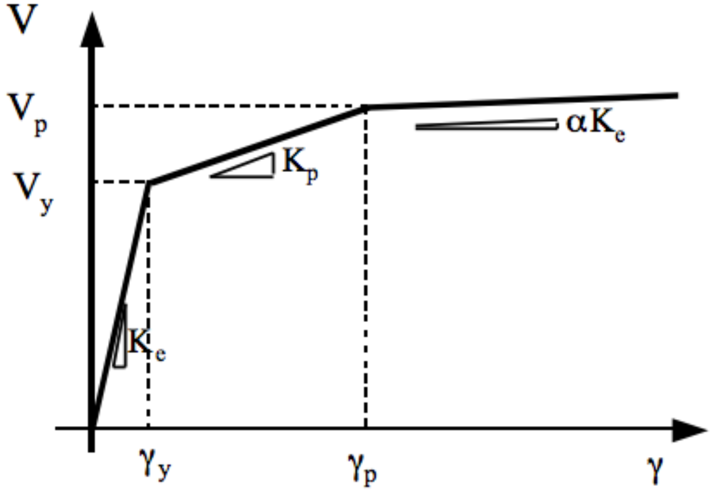
```
            set theSection [lindex $colSizes [expr $floor1 -1]]; # obtain section size for column
             ForceBeamWSection2d $colLine$floor1$colLine$floor2 $colLine$floor1 $colLine$floor2 $theSection 1 1 –nip 5
        }
    }
#add beams between column lines at each floor
geomTransf Linear 2
foreach colLine1 {1 2 3} colLine2 {2 3 4} {
foreach floor {2 3 4 5} {
    set theSection [lindex $beamSizes [expr $floor -2]]; # obtain section size for floor
    ForceBeamWSection2d $colLine1$floor$colLine2$floor $colLine1$floor $colLine2$floor $theSection 1 2
}
}
```

Panel Zone Modeling





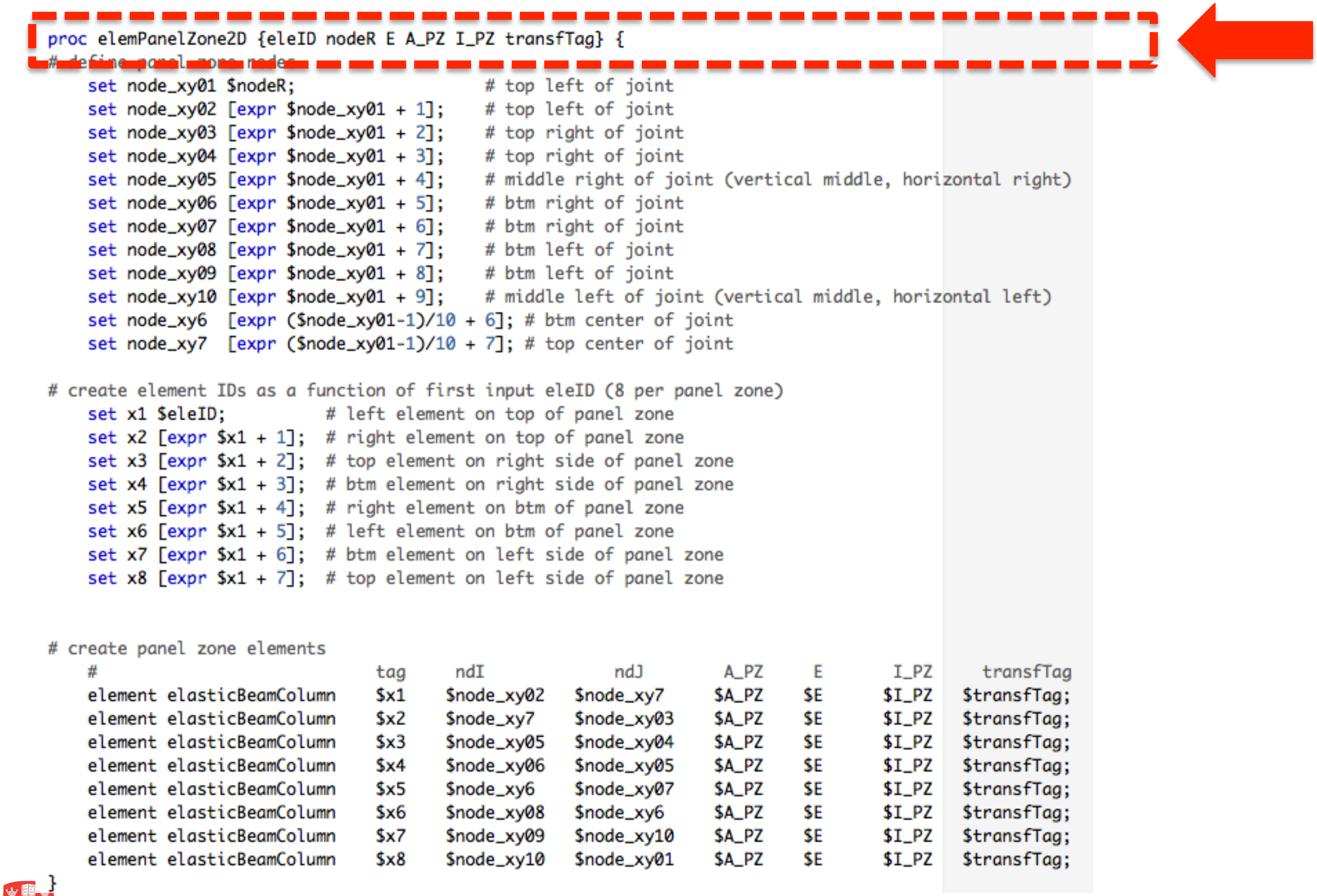




How to obtain the input parameters?
see Gupta and Krawinkler (1999)

Procedure for Modeling Panel Zones

-Available from OpenSees Examples Posted by Dr. L. Eads

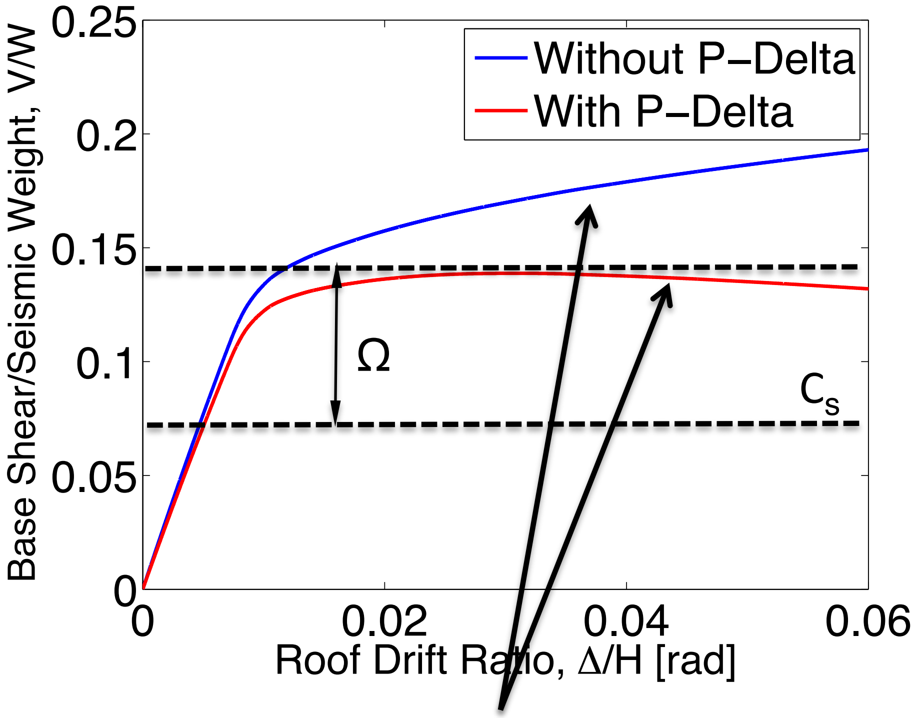


```
proc elemPanelZone2D {eleID nodeR E A_PZ I_PZ transfTag} {
# define panel zone nodes
    set node_xy01 $nodeR;                  # top left of joint
    set node_xy02 [expr $node_xy01 + 1];   # top left of joint
    set node_xy03 [expr $node_xy01 + 2];   # top right of joint
    set node_xy04 [expr $node_xy01 + 3];   # top right of joint
    set node_xy05 [expr $node_xy01 + 4];   # middle right of joint (vertical middle, horizontal right)
    set node_xy06 [expr $node_xy01 + 5];   # btm right of joint
    set node_xy07 [expr $node_xy01 + 6];   # btm right of joint
    set node_xy08 [expr $node_xy01 + 7];   # btm left of joint
    set node_xy09 [expr $node_xy01 + 8];   # btm left of joint
    set node_xy10 [expr $node_xy01 + 9];   # middle left of joint (vertical middle, horizontal left)
    set node_xy6  [expr ($node_xy01-1)/10 + 6]; # btm center of joint
    set node_xy7  [expr ($node_xy01-1)/10 + 7]; # top center of joint

# create element IDs as a function of first input eleID (8 per panel zone)
    set x1 $eleID;              # left element on top of panel zone
    set x2 [expr $x1 + 1];  # right element on top of panel zone
    set x3 [expr $x1 + 2];  # top element on right side of panel zone
    set x4 [expr $x1 + 3];  # btm element on right side of panel zone
    set x5 [expr $x1 + 4];  # right element on btm of panel zone
    set x6 [expr $x1 + 5];  # left element on btm of panel zone
    set x7 [expr $x1 + 6];  # btm element on left side of panel zone
    set x8 [expr $x1 + 7];  # top element on left side of panel zone


# create panel zone elements
    #                          tag     ndI          ndJ          A_PZ    E      I_PZ    transfTag
    element elasticBeamColumn  $x1     $node_xy02   $node_xy7    $A_PZ   $E     $I_PZ   $transfTag;
    element elasticBeamColumn  $x2     $node_xy7    $node_xy03   $A_PZ   $E     $I_PZ   $transfTag;
    element elasticBeamColumn  $x3     $node_xy05   $node_xy04   $A_PZ   $E     $I_PZ   $transfTag;
    element elasticBeamColumn  $x4     $node_xy06   $node_xy05   $A_PZ   $E     $I_PZ   $transfTag;
    element elasticBeamColumn  $x5     $node_xy6    $node_xy07   $A_PZ   $E     $I_PZ   $transfTag;
    element elasticBeamColumn  $x6     $node_xy08   $node_xy6    $A_PZ   $E     $I_PZ   $transfTag;
    element elasticBeamColumn  $x7     $node_xy09   $node_xy10   $A_PZ   $E     $I_PZ   $transfTag;
    element elasticBeamColumn  $x8     $node_xy10   $node_xy01   $A_PZ   $E     $I_PZ   $transfTag;
}
```

4-Story SMF – Distributed Plasticity Approach

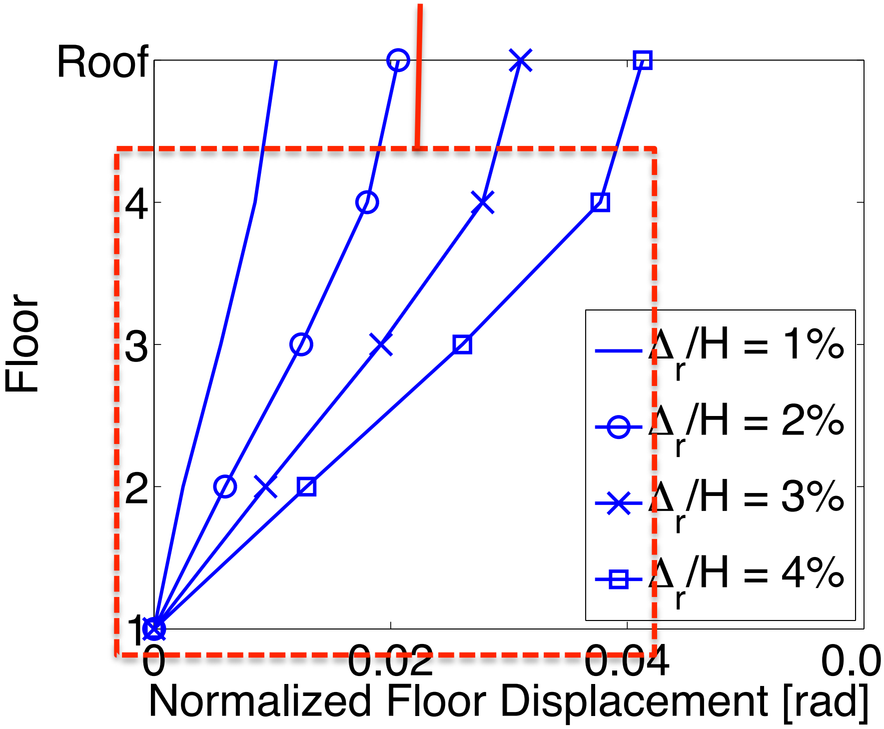
Basic Results – Nonlinear Static Procedure

First Mode Lateral Load Pattern



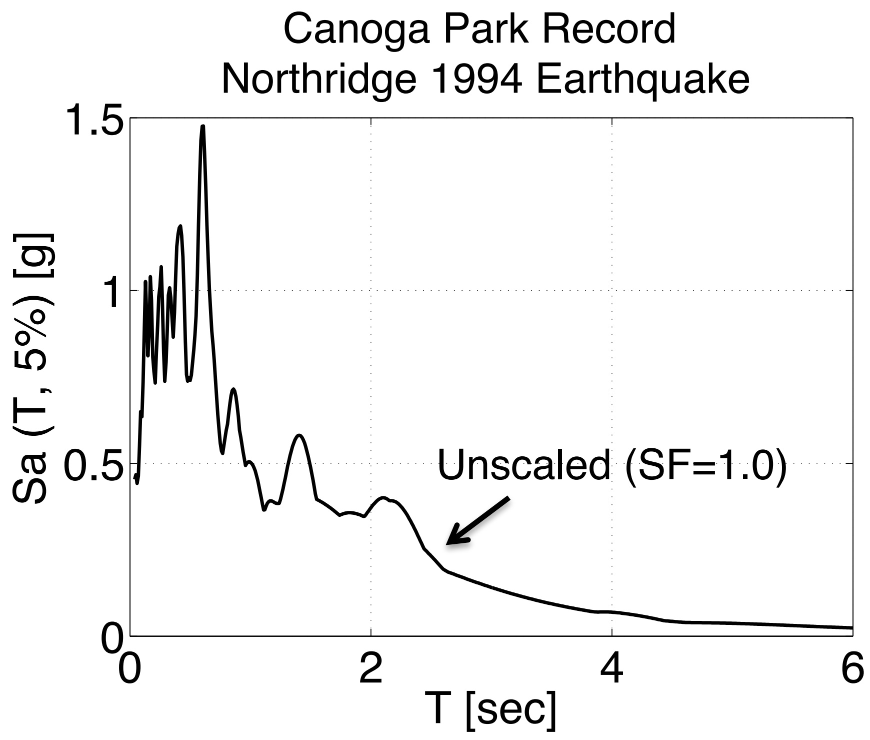
Does not capture strength deterioration of steel components

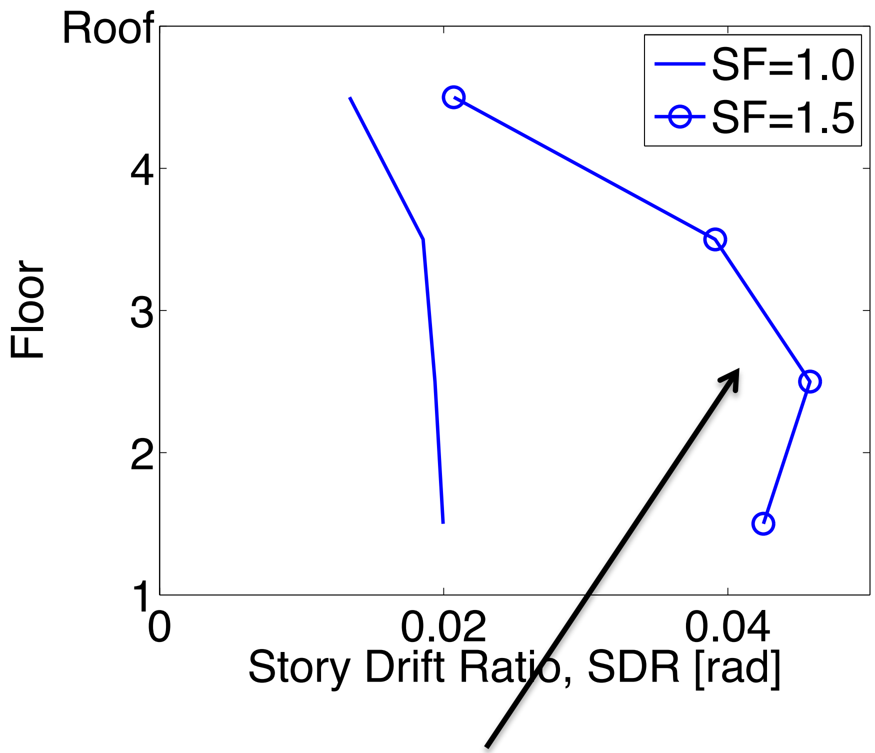
Formation of a 3-story mechanism



4-Story SMF – Distributed Plasticity Approach

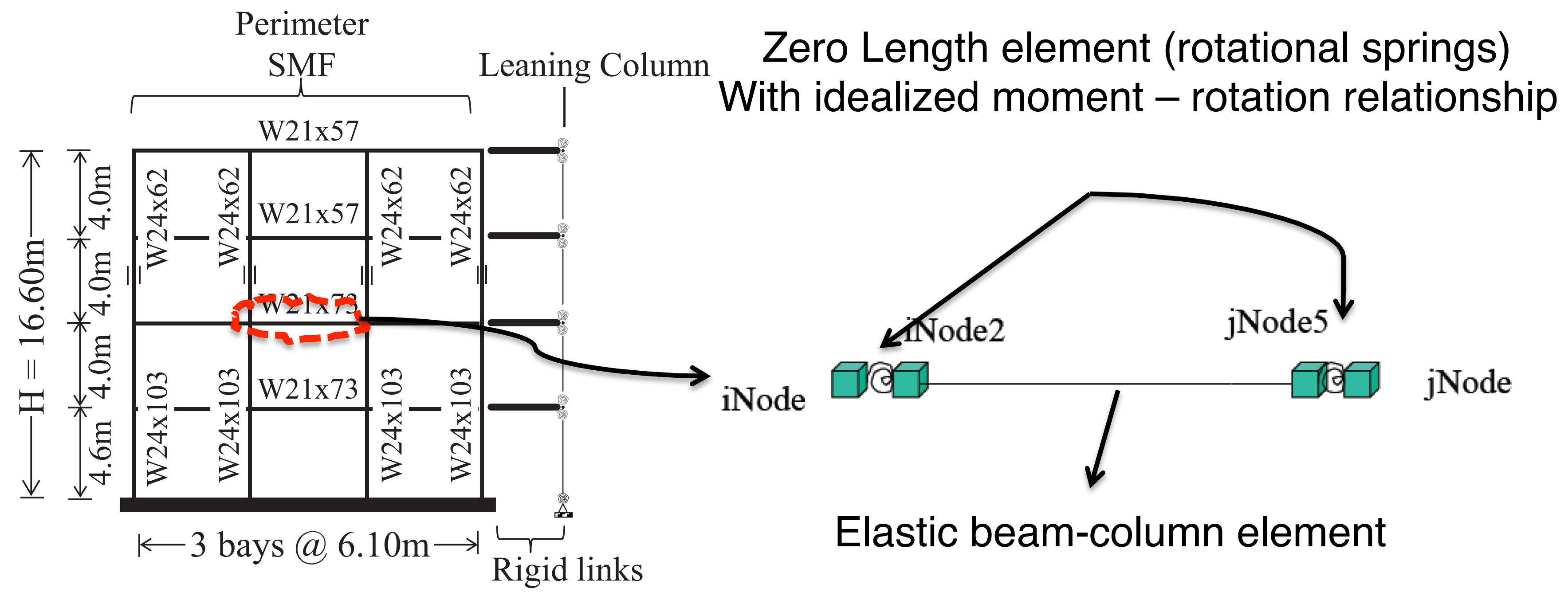
Basic Results – Nonlinear Response History Analysis





Does not include the effect of cyclic deterioration on flexural strength and stiffness of steel components

4-Story SMF – Concentrated Plasticity Approach

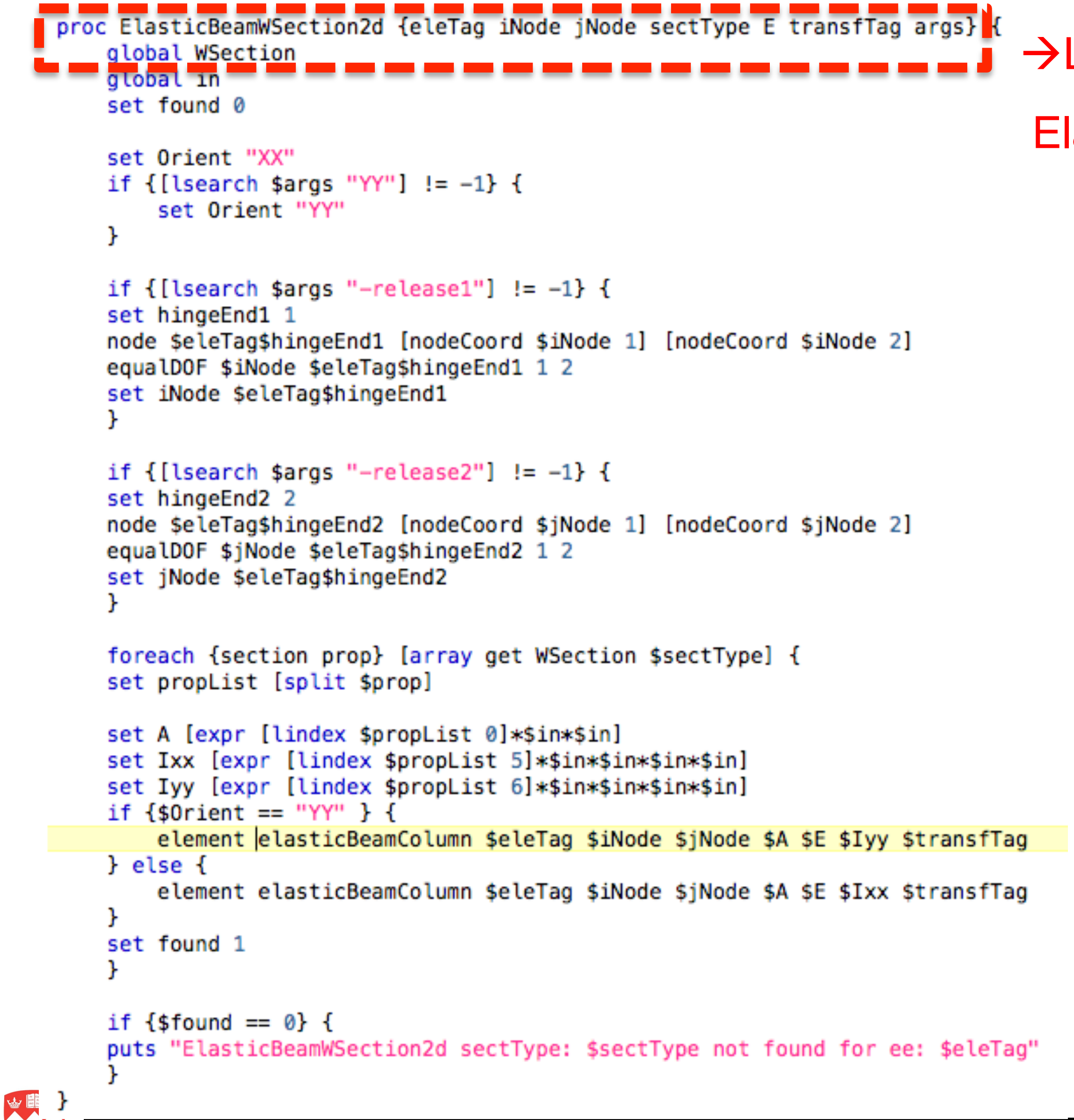


Zero Length element (rotational springs)
With idealized moment – rotation relationship

Elastic beam-column element

4-Story SMF – Concentrated Plasticity Approach

From Steel2d.tcl: Procedure to Create Elastic Beam-Column Element



```
proc ElasticBeamWSection2d {eleTag iNode jNode sectType E transfTag args} {
    global WSection
    global in
    set found 0

    set Orient "XX"
    if {[lsearch $args "YY"] != -1} {
        set Orient "YY"
    }

    if {[lsearch $args "-release1"] != -1} {
    set hingeEnd1 1
    node $eleTag$hingeEnd1 [nodeCoord $iNode 1] [nodeCoord $iNode 2]
    equalDOF $iNode $eleTag$hingeEnd1 1 2
    set iNode $eleTag$hingeEnd1
    }

    if {[lsearch $args "-release2"] != -1} {
    set hingeEnd2 2
    node $eleTag$hingeEnd2 [nodeCoord $jNode 1] [nodeCoord $jNode 2]
    equalDOF $jNode $eleTag$hingeEnd2 1 2
    set jNode $eleTag$hingeEnd2
    }

    foreach {section prop} [array get WSection $sectType] {
    set propList [split $prop]

    set A [expr [lindex $propList 0]*$in*$in]
    set Ixx [expr [lindex $propList 5]*$in*$in*$in*$in]
    set Iyy [expr [lindex $propList 6]*$in*$in*$in*$in]
    if {$Orient == "YY" } {
        element elasticBeamColumn $eleTag $iNode $jNode $A $E $Iyy $transfTag
    } else {
        element elasticBeamColumn $eleTag $iNode $jNode $A $E $Ixx $transfTag
    }
    set found 1
    }

    if {$found == 0} {
    puts "ElasticBeamWSection2d sectType: $sectType not found for ee: $eleTag"
    }
}
```

→Look for Procedure

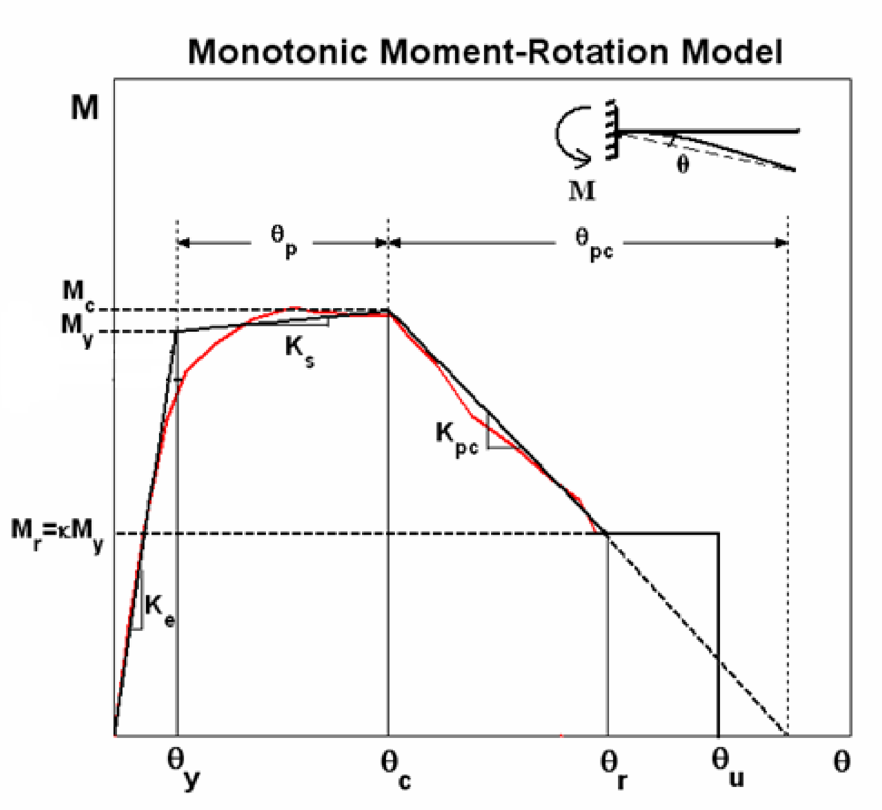
ElasticBeamSection2d

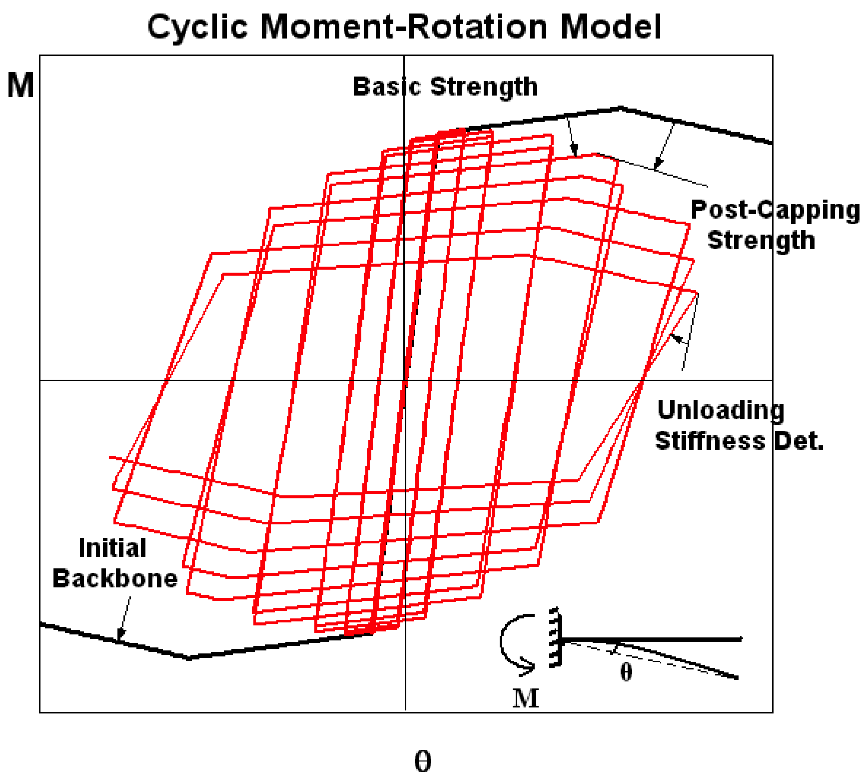
Available Steel Material Models for Modeling the Moment – Rotation Relationship of a Steel Component

- Steel01 (Basic Bilinear)
- Steel02 (Giuffre Menegotto-Pinto Model with Isotropic Strain Hardening)
- Modified Ibarra-Medina-Krawinkler (IMK) Deterioration Model with Bilinear Hysteretic Response (or Bilin in OpenSees) → Considers Strength and Stiffness Deterioration of Steel Components

The Modified IMK Deterioration Model

-Input Model Parameters



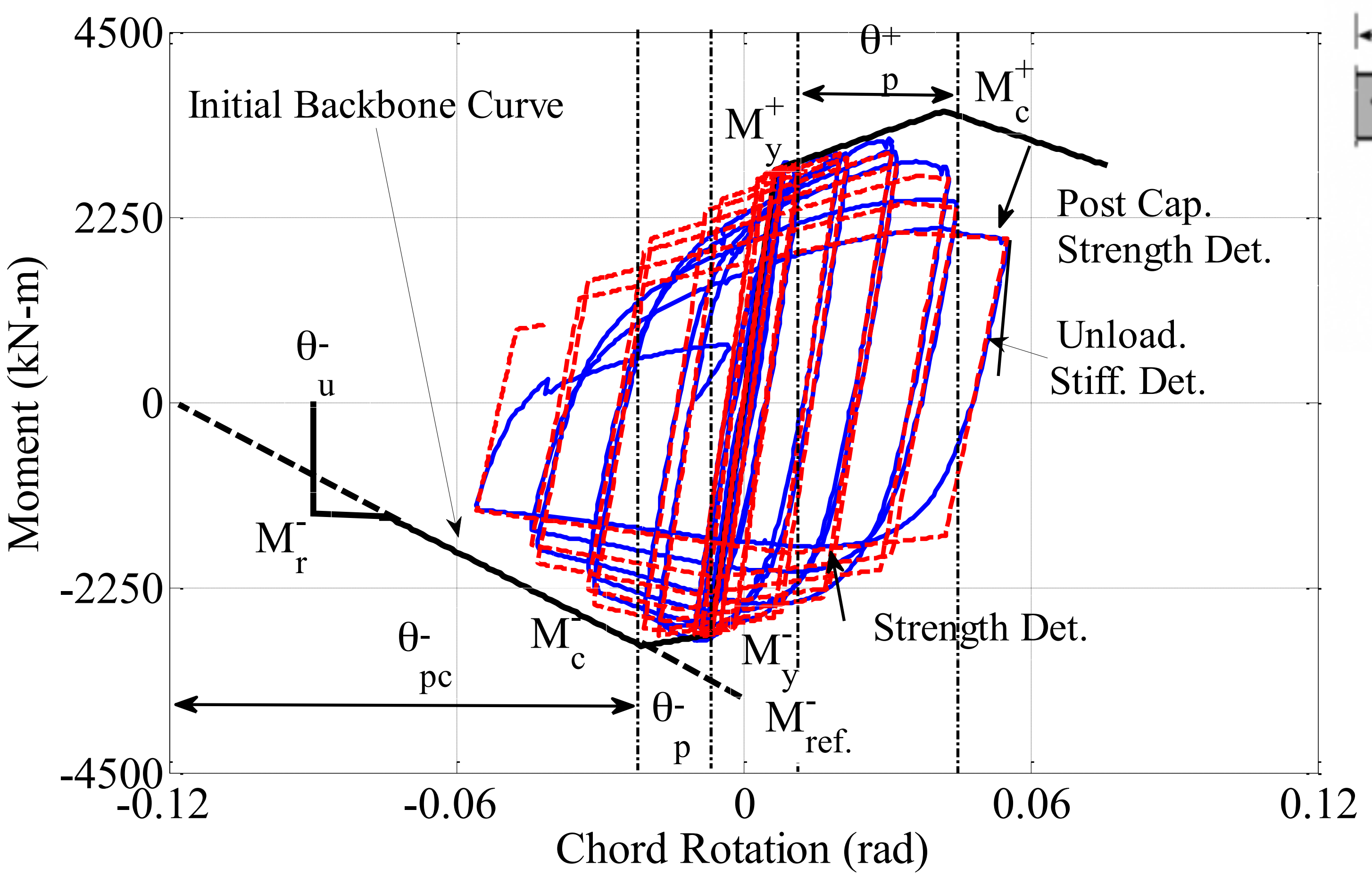


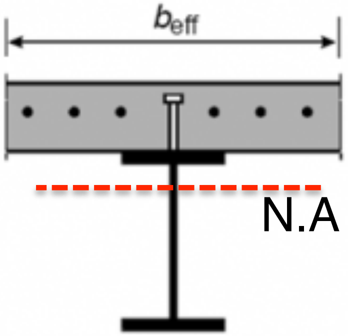
(Image Source: Lignos and Krawinkler 2012)

The deduced moment rotation relationship of the component under consideration is needed for input parameter identification

The Modified IMK Deterioration Model

-Considering Slab Effects (see Elkady and Lignos 2014)

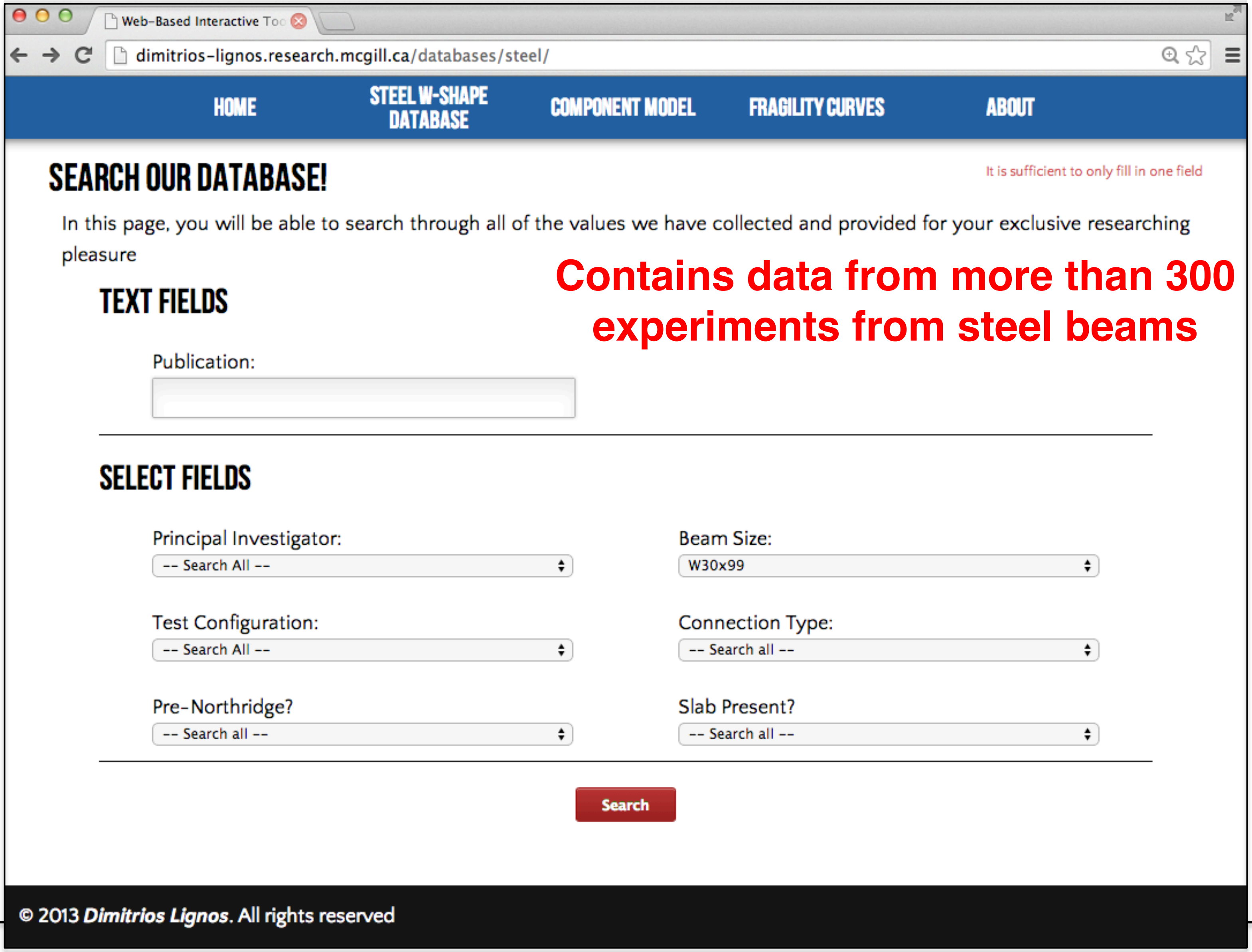




The deduced moment rotation relationship of the component under consideration is needed for input parameter identification

Utilizing the Modified IMK Model in OpenSees

Visit: dimitrios.lignos.research.mcgill.ca/databases/steel/



SEARCH OUR DATABASE!

It is sufficient to only fill in one field

In this page, you will be able to search through all of the values we have collected and provided for your exclusive researching pleasure

TEXT FIELDS

Publication:

SELECT FIELDS

Principal Investigator: -- Search All --

Beam Size: W30x99

Test Configuration: -- Search All --

Connection Type: -- Search all --

Pre-Northridge? -- Search all --

Slab Present? -- Search all --

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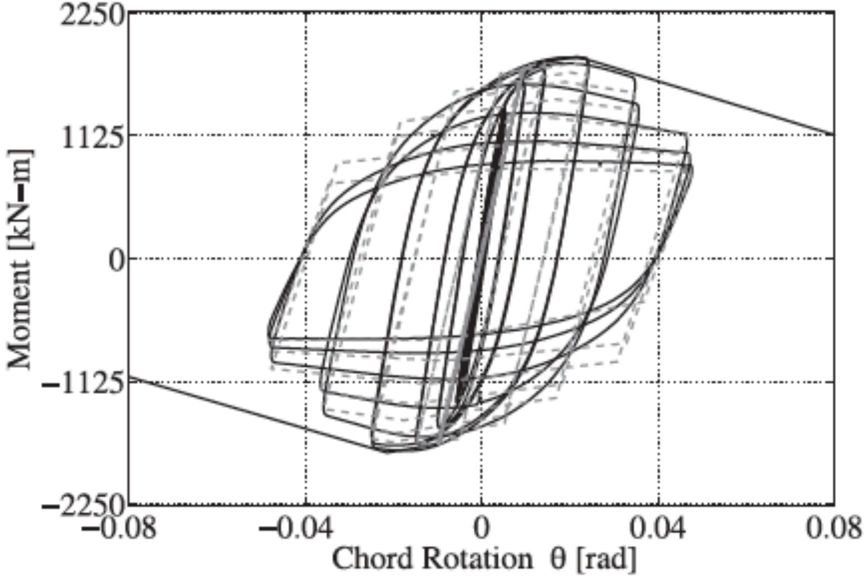
Contains data from more than 300 experiments from steel beams

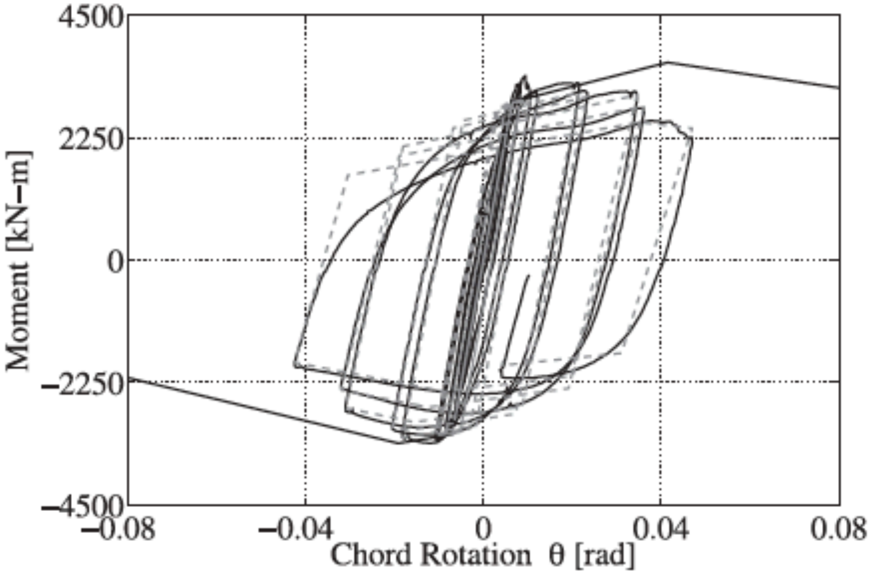
Utilizing the Modified IMK Model in OpenSees

Sample Model Calibrations

Steel Beam (bare) with RBS



Steel Beam (Composite) with RBS



(Sources: Lignos and Krawinkler 2011, 2013)

Utilizing the Modified IMK Model in OpenSees

Visit: dimitrios.lignos.research.mcgill.ca/databases/component/

Pre-capping plastic rotation, θ_p, for beams with non-RBS connections:

$$\theta_p = 0.087 \cdot \left(\frac{h}{t_w}\right)^{-0.365} \cdot \left(\frac{b_f}{2 \cdot t_f}\right)^{-0.14} \cdot \left(\frac{L}{d}\right)^{0.34} \cdot \left(\frac{d}{c_{unit}^{1} \cdot 21"}\right)^{-0.721} \cdot \left(\frac{c_{unit}^{2} \cdot F_y}{50}\right)^{-0.23}$$

Pre-capping plastic rotation, θ_p, for beams with RBS connections:

$$\theta_p = 0.19 \cdot \left(\frac{h}{t_w}\right)^{-0.314} \cdot \left(\frac{b_f}{2 \cdot t_f}\right)^{-0.10} \cdot \left(\frac{L_b}{r_y}\right)^{-0.1185} \cdot \left(\frac{L}{d}\right)^{0.113} \cdot \left(\frac{d}{c_{unit}^{1} \cdot 21"}\right)^{-0.76} \cdot \left(\frac{c_{unit}^{2} \cdot F_y}{50}\right)^{-0.07}$$

Post-capping rotation, θ_{pc}, for beams with non-RBS connections:

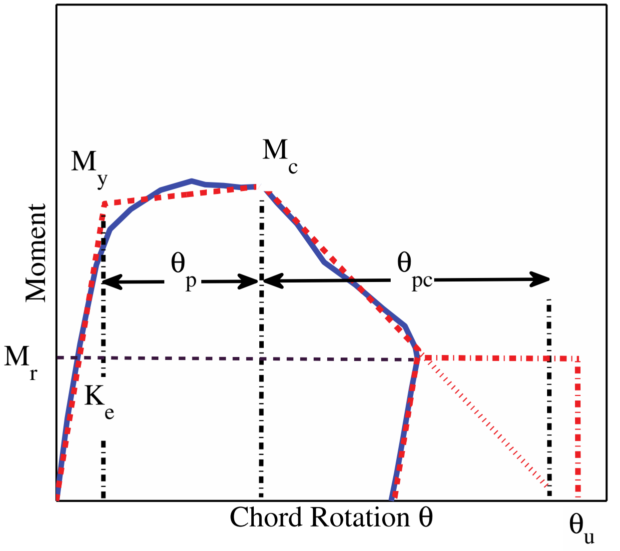
$$\theta_{pc} = 5.70 \cdot \left(\frac{h}{t_w}\right)^{-0.565} \cdot \left(\frac{b_f}{2 \cdot t_f}\right)^{-0.80} \cdot \left(\frac{d}{c_{unit}^{1} \cdot 21"}\right)^{-0.28} \cdot \left(\frac{c_{unit}^{2} \cdot F_y}{50}\right)^{-0.43}$$

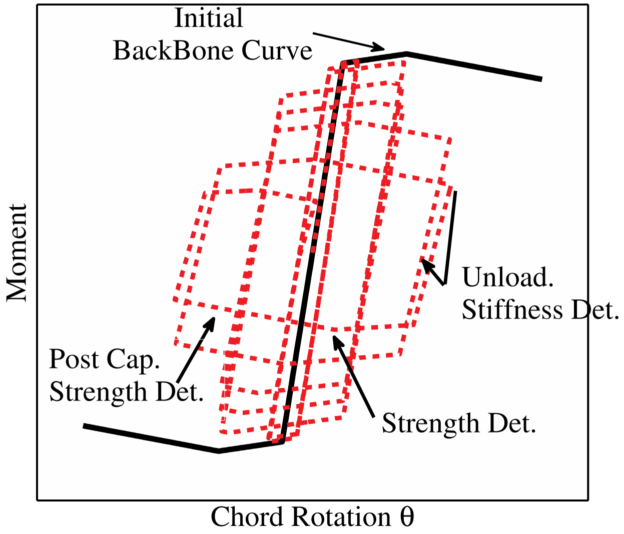
Post-capping rotation, θ_{pc}, for beams with RBS connections:

$$\theta_{pc} = 9.62 \cdot \left(\frac{h}{t_w}\right)^{-0.513} \cdot \left(\frac{b_f}{2 \cdot t_f}\right)^{-0.863} \cdot \left(\frac{L_b}{r_y}\right)^{-0.108} \cdot \left(\frac{c_{unit}^{2} \cdot F_y}{50}\right)^{-0.36}$$

Reference cumulative plastic rotation, Λ, for beams with non-RBS connections:

$$\Lambda = \frac{E_t}{M_y} = 500 \left(\frac{h}{t_w}\right)^{-1.34} \cdot \left(\frac{b_f}{2 \cdot t_f}\right)^{-0.595} \cdot \left(\frac{c_{unit}^{2} \cdot F_y}{50}\right)^{-0.36}$$

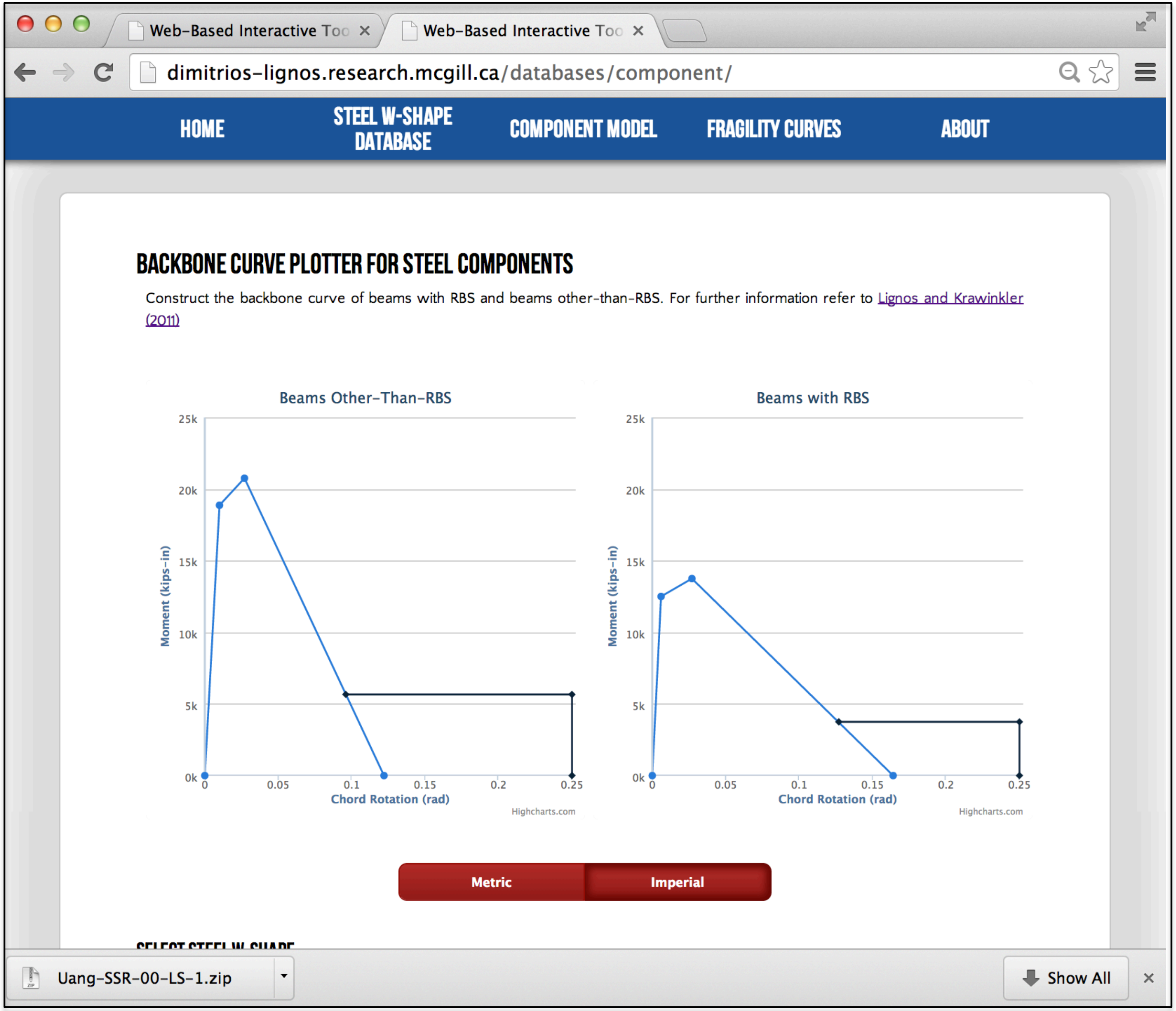




(Sources: Lignos and Krawinkler 2011, 2013)

Utilizing the Modified IMK Model in OpenSees

Visit: dimitrios.lignos.research.mcgill.ca/databases/component/

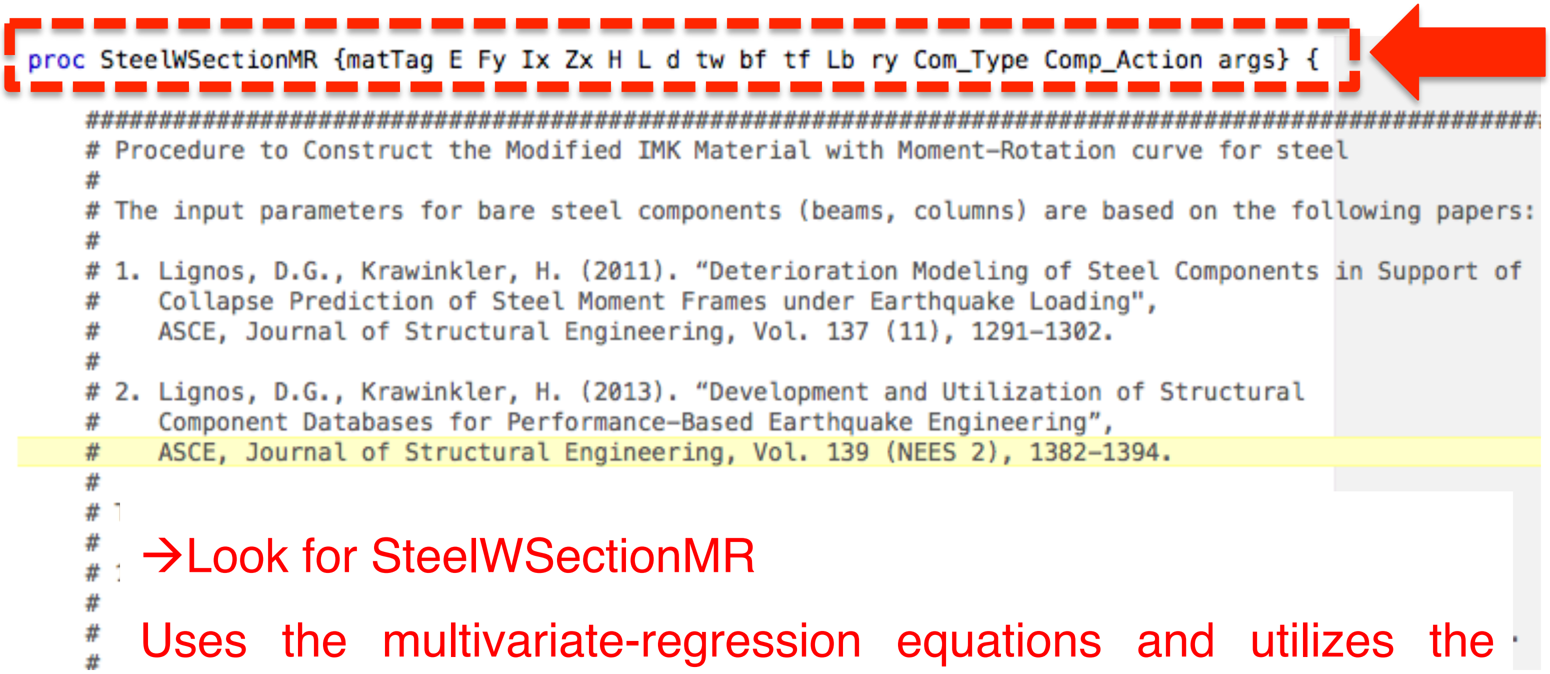


BACKBONE CURVE PLOTTER FOR STEEL COMPONENTS

Construct the backbone curve of beams with RBS and beams other-than-RBS. For further information refer to Lignos and Krawinkler (2011)

4-Story – Concentrated Plasticity Approach

Steel2d.tcl: Rotational Spring with SteelWsectionMR



```
proc SteelWSectionMR {matTag E Fy Ix Zx H L d tw bf tf Lb ry Com_Type Comp_Action args} {
    ##############################################################################################
    # Procedure to Construct the Modified IMK Material with Moment-Rotation curve for steel
    #
    # The input parameters for bare steel components (beams, columns) are based on the following papers:
    #
    # 1. Lignos, D.G., Krawinkler, H. (2011). "Deterioration Modeling of Steel Components in Support of
    #    Collapse Prediction of Steel Moment Frames under Earthquake Loading",
    #    ASCE, Journal of Structural Engineering, Vol. 137 (11), 1291-1302.
    #
    # 2. Lignos, D.G., Krawinkler, H. (2013). "Development and Utilization of Structural
    #    Component Databases for Performance-Based Earthquake Engineering",
    #    ASCE, Journal of Structural Engineering, Vol. 139 (NEES 2), 1382-1394.
    #
    #
    #
    #
    #
    #
    #
```

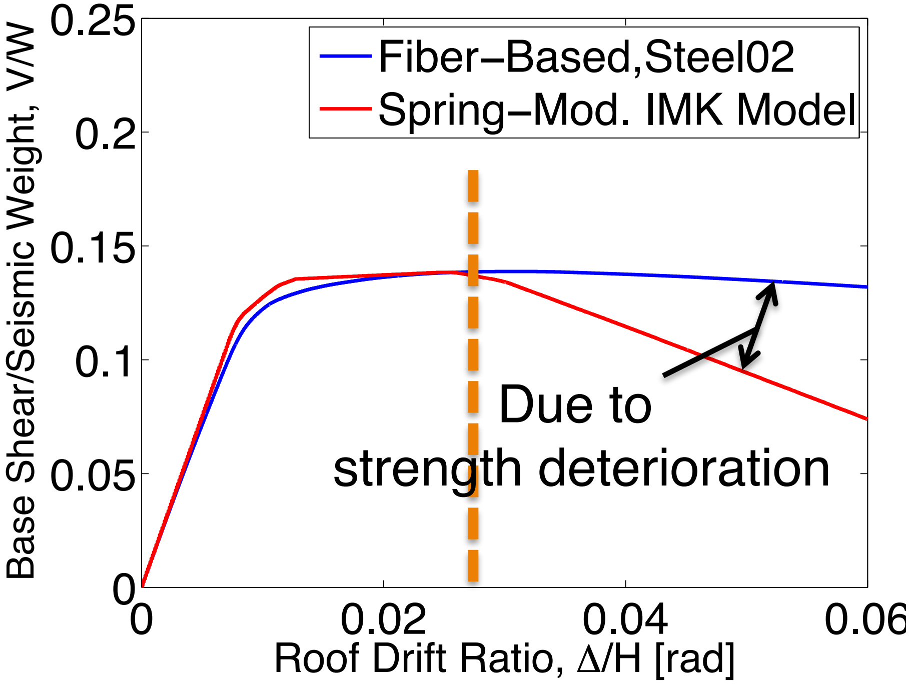
→Look for SteelWSectionMR

Uses the multivariate-regression equations and utilizes the modified IMK model in OpenSees

4-Story SMF – Concentrated Plasticity- Deterioration

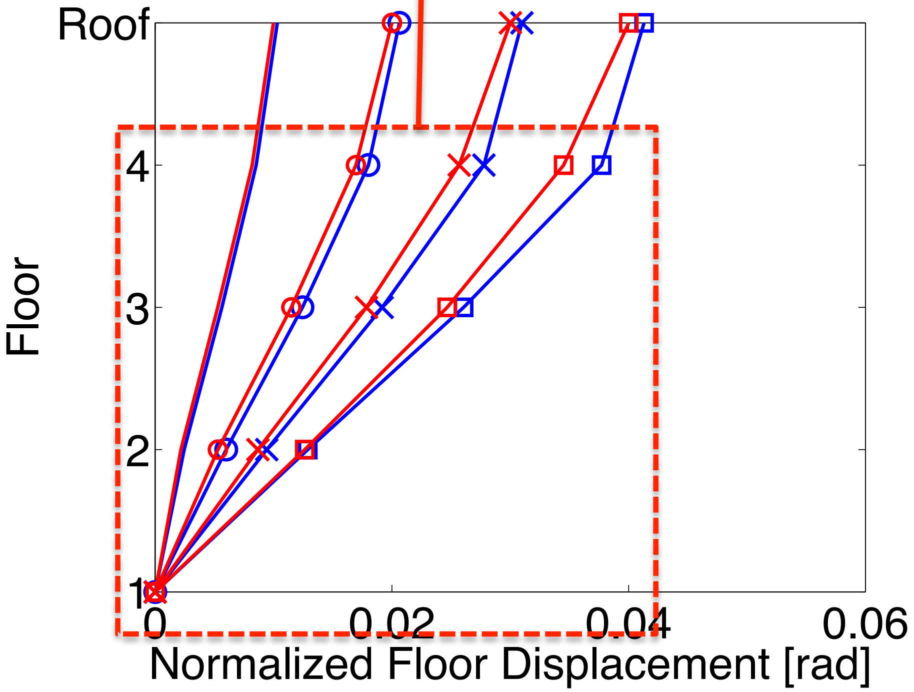
Nonlinear Static Analysis – First Mode Lateral Load Pattern

Comparisons with Distributed Plasticity Model



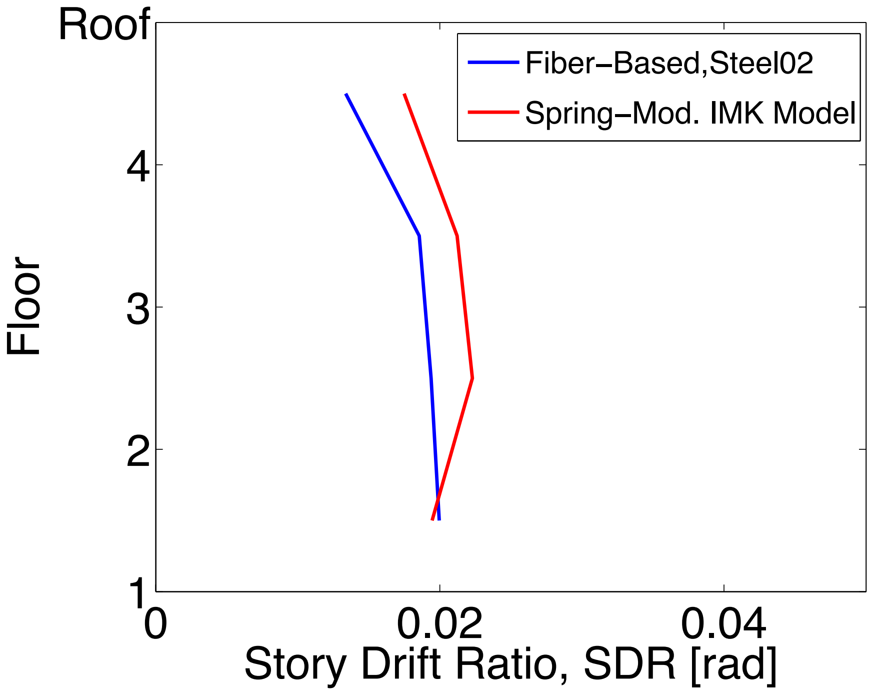
Due to strength deterioration

Formation of a 3-story mechanism (Did not change in this case)

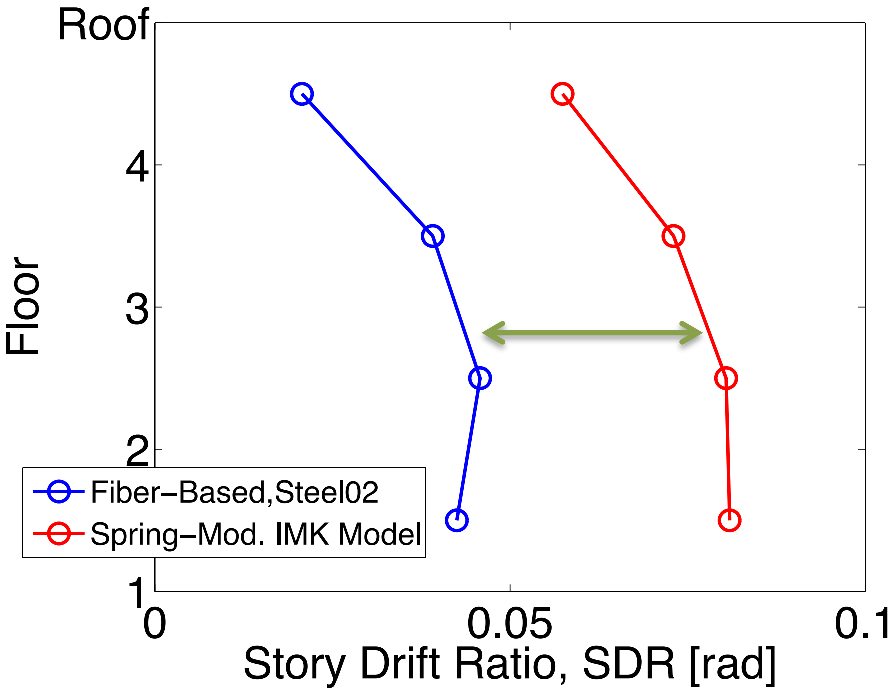


4-Story SMF – Concentrated Plasticity- Deterioration

Nonlinear Response History Analysis with Canoga Park Record Comparisons with Distributed Plasticity Model



Canoga Park SF=1.0



Canoga Park SF=1.5

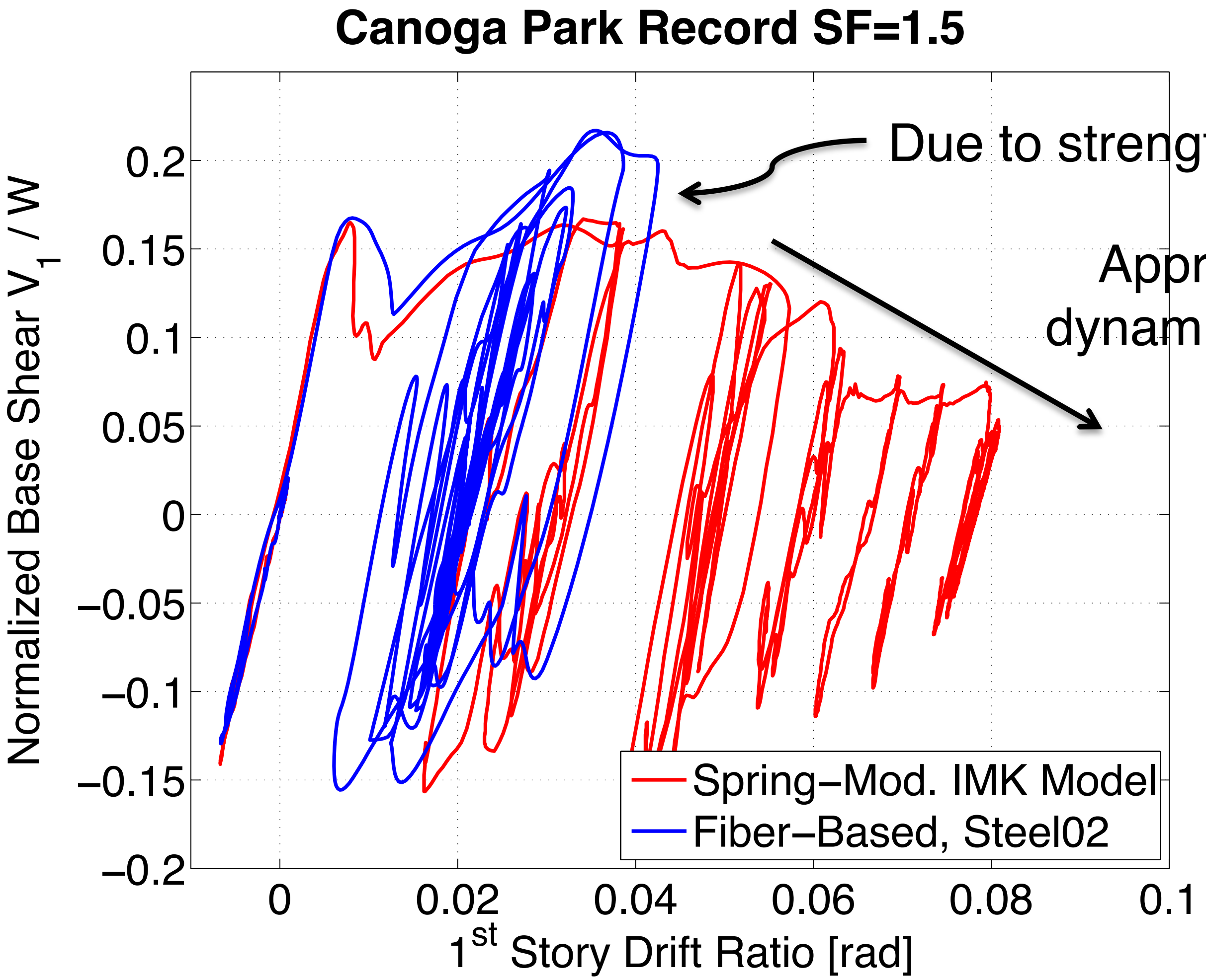
The effect of cyclic deterioration on the structural response is minimal for SF = 1.0 (close to design level earthquake)

The effect of cyclic deterioration on the structural response is significant

4-Story SMF – Concentrated Plasticity- Deterioration

Nonlinear Response History Analysis with Canoga Park Record

Comparisons with Distributed Plasticity Model



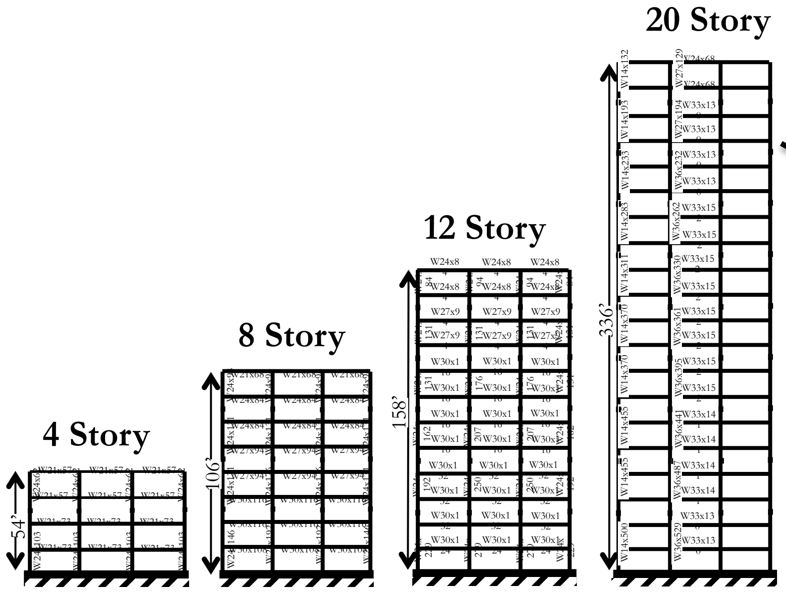
Due to strength deterioration

Approaching dynamic collapse

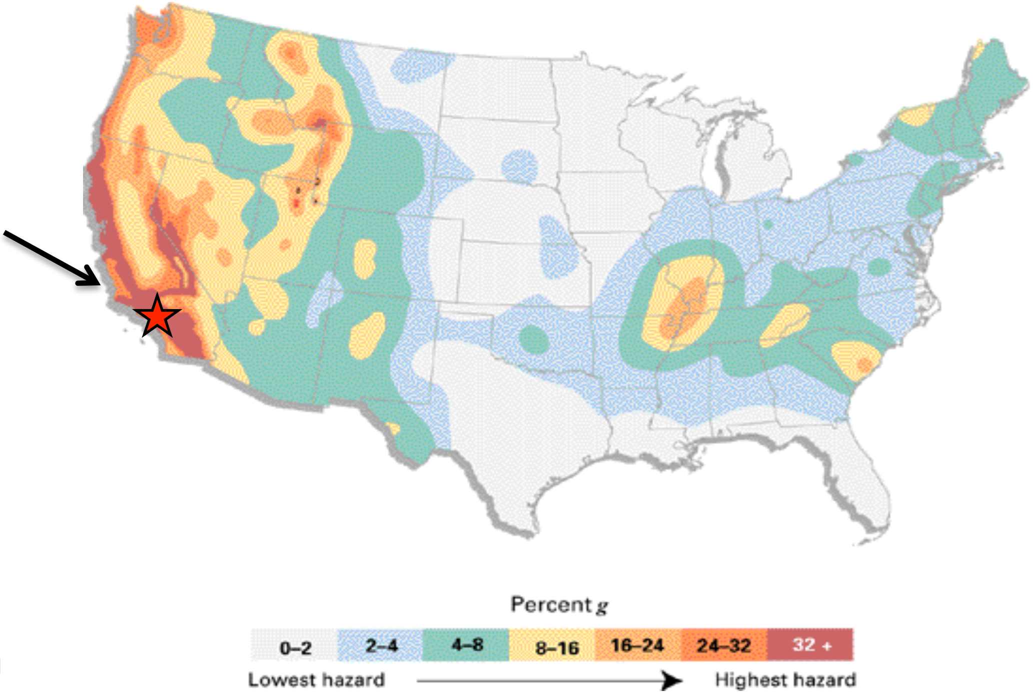
Collapse Assessment of Steel SMFs

Using Concentrated Plasticity Approach

Case studies: Archetype office steel buildings with perimeter steel special moment frames designed in Urban California (ASCE 7-10, AISC-2010)



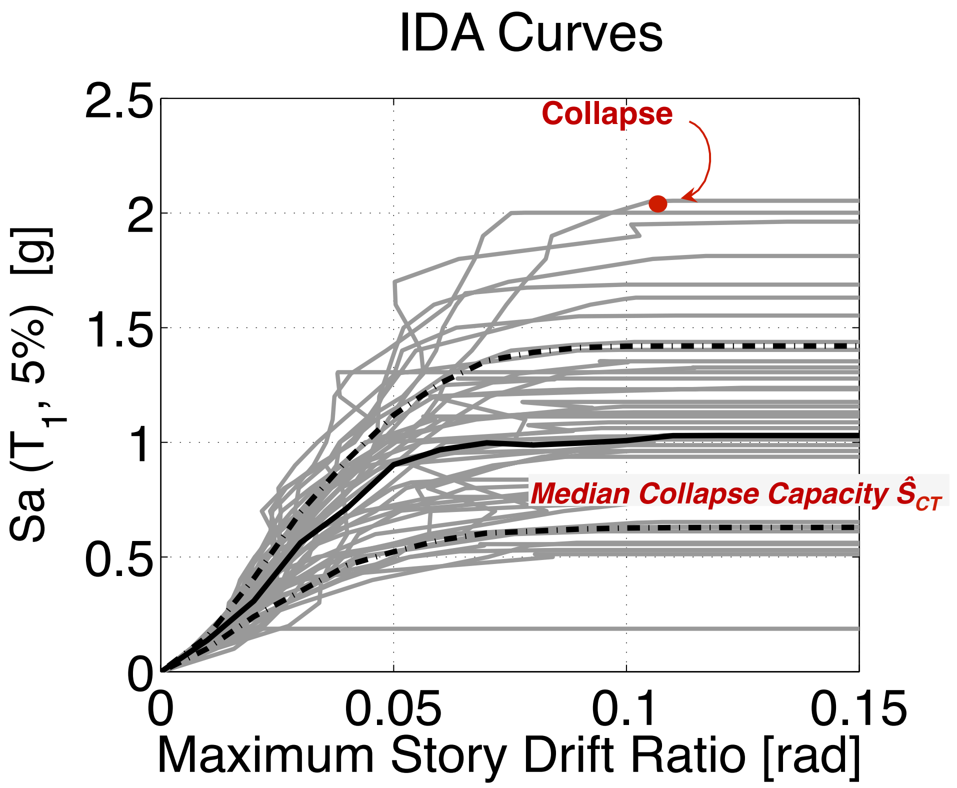
(Sources: Elkady and Lignos 2014)

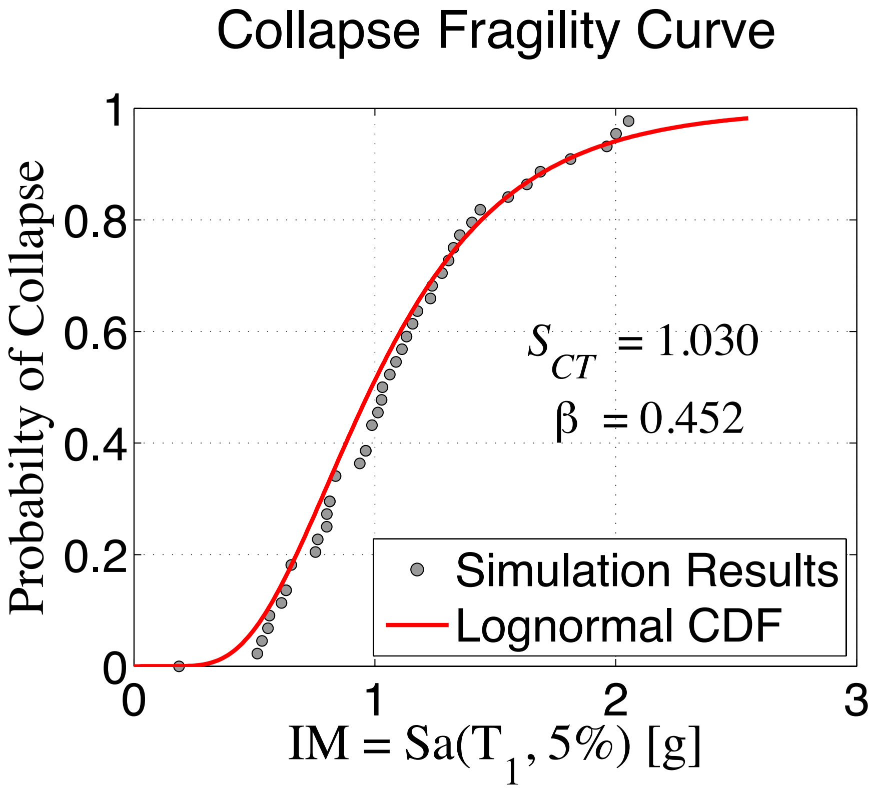


Source: National Seismic Hazard Map (USGS 2008)

Example: Collapse Risk of 4-Story Steel SMF

Incremental Dynamic Analysis – Utilization of 44 Ground Motions



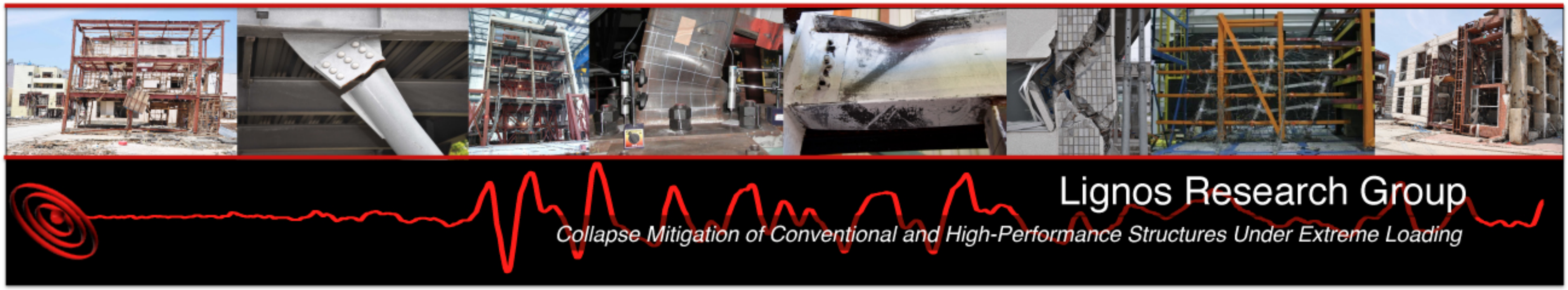


Source: Elkady and Lignos (2014)

Concluding Remarks

- ✧ **Modeling Steel Moment Resisting Frames in OpenSees**
  - ➢ *You can use a number of readily available tools (procedures in tcl, examples, web-based tools, etc)*
- ✧ **Steel Components to Consider**
  - ✓ Steel Beams, Columns and Panel Zones
- ✧ **Distributed Versus Concentrated Plasticity Approach**
  - ✓ For low rise code-compliant steel buildings with perimeter MRFs the differences should not be large for "design level earthquakes" – Distributed plasticity models capture cyclic hardening, P-M interaction
- ✧ **If Performance Evaluation at Large Deformations is the Objective: <u>Component Deterioration</u> Must be Considered**
  - ✓ Concentrated Plasticity with Degrading Phenomenological Models will do reasonably well <u>for low- to mid-rise</u> steel MRFs.

Thank you for your kind attention!



For more information visit: dimitrios-lignos.research.mcgill.ca

References:

1. Elkady, A., Lignos, D.G. (2014). "Modeling of the Composite Action in Fully Restrained Beam-to-Column Connections: Implications in the Seismic Design and Collapse Capacity of Steel Special Moment Frames", Earthquake Engineering and Structural Dynamics, EESD, doi: 10.1002/eqe.2430 (available in early view).
2. Lignos, D.G., Krawinkler, H. (2013). "Development and Utilization of Structural Component Databases for Performance-Based Earthquake Engineering", ASCE, Journal of Structural Engineering, Vol. 139 (NEES 2), pp. 1382-1394, doi:10/1061/(ASCE)ST.1943-541X.0000646.
3. Lignos, D.G., Krawinkler, H. (2011). "Deterioration Modeling of Steel Components in Support to Collapse Prediction of Steel Moment Frames", ASCE Journal of Structural Engineering, Vol. 137 (11), pp. 1291-1302, doi: 10.1061/(ASCE)ST.1943-541X.0000376.
4. Lignos, D.G., Krawinkler, H. (2012). "Sidesway Collapse of Deteriorating Structural Systems under Seismic Excitations," Report No. TB 177, The John A. Blume Earthquake Engineering Center, Stanford, CA.
5. Ibarra, L., Medina, R., Krawinkler, H. (2005). "Hysteretic Models that Incorate Strength and Stiffness Deterioration", Earthquake Engineering and Structural Dynamics, EESD, Vol. 34(12), pp. 1489-1511.