Nonlinear Frame Finite Elements in OpenSees

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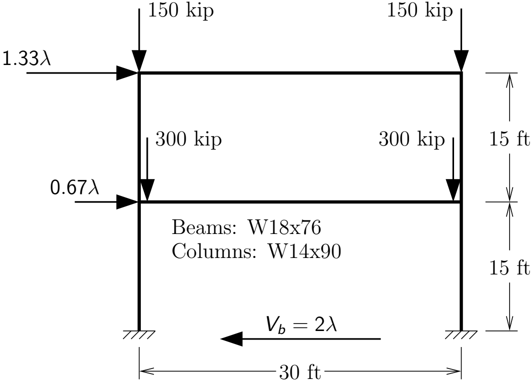
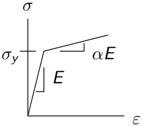
Types of Nonlinearity

- Two sources of nonlinear frame element response:
  - **Material** – yielding, strain hardening, crushing of concrete, etc.
  - **Geometry** – loss of stability due to loads acting through large displacements
- An analysis can account for each source of nonlinearity separately, giving four possible approaches

	Geometry Linear (GL)	Geometry Nonlinear (GN)
Material Linear (ML)	ML, GL	ML, GN
Material Nonlinear (MN)	MN, GL	MN, GN

Steel Frame Pushover Analysis

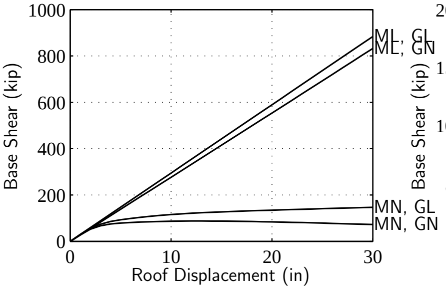


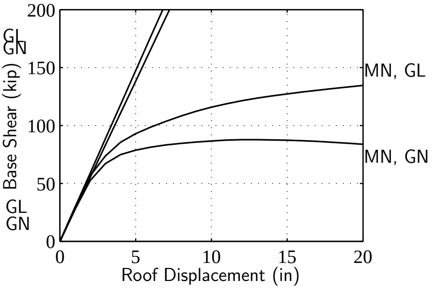


σ_y=36 ksi
E=30,000 ksi
α=0.02

- Simple steel frame model analyzed under four approahces
- Relatively large column axial loads will intensify both material and geometric nonlinear response for demonstration purposes

Steel Frame Pushover Analysis



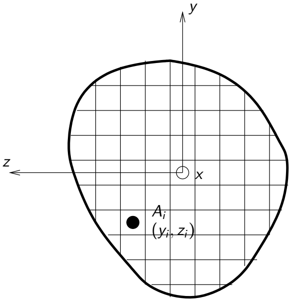


We observe the following:

- Material nonlinearity kicks in well before geometric nonlinearity
- Geometric nonlinearity allows for prediction of loss of stability for increasing displacement

Section Force-Deformation Response

- At each cross-section along a frame element, we must determine the section forces for any given section deformations
- Material nonlinearity eminates from the stress-strain response in each frame element
  - Heuristic approach through stress-resultant section models, e.g., moment-curvature; or
  - Integrate stress-strain response via “fiber section” approach



Commands for Section Definition

`uniaxialMaterial modelName $tag ...`

- Define uniaxial stress-strain models for use in Bernoulli beam elements
- Elastic, Steel01, Steel02, Concrete01, Concrete02, etc.

`nDMaterial modelName $tag ...`

- Define multiaxial stress-strain models for use in Timoshenko beam elements
- ElasticIsotropic, J2Plasticity, ConcreteMCFT, etc.

Commands for Section Definition

- General definition of Bernoulli cross-section using patches and layers of fibers whose stress-strain response is defined by uniaxialMaterial objects

```
section Fiber $tag {
 patch $type $matTag ...
 layer $type $matTag ...
 fiber $matTag ...
 ...
}
```

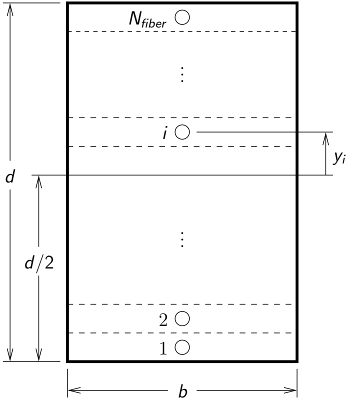
  Use `NDFiber` with nDMaterial objects instead of `Fiber` with uniaxialMaterial objects for Timoshenko beams

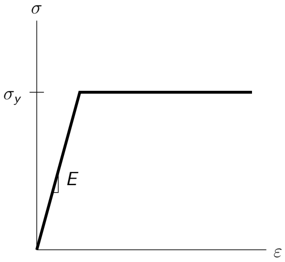
- Specific cross-sections obtained with “canned” models

```
section WFSection2d $tag $matTag ...
section RCSection2d $tag $matTag ...
```

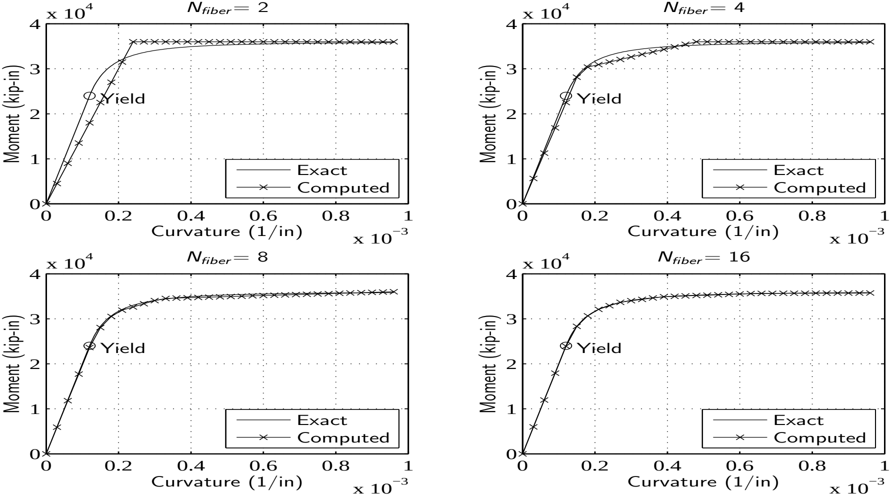
Rectangular Steel Section

- Rectangular section with EPP uniaxial stress-strain response
- Compute moment-curvature response for increasing number of fibers
- Exact solution for $M_y = f_y bd^2/6$ and $M_p = bd^2/4$



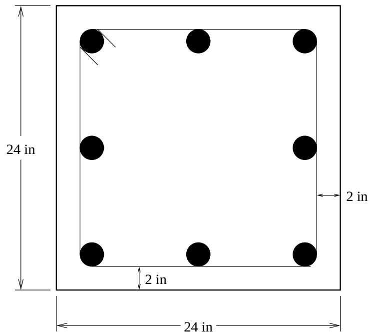


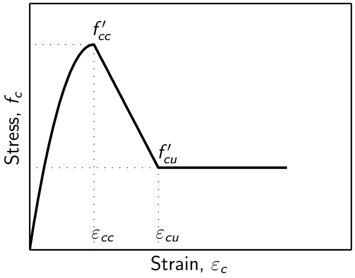
Rectangular Steel Section



Reinforced Concrete Section

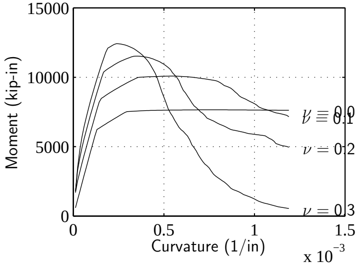
- EPP steel and Concrete01 concrete
- Using “canned” RCSection2d command
- Confined and unconfined concrete



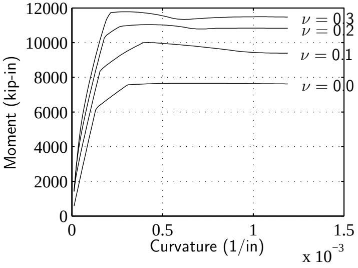


Reinforced Concrete Section

- Moment-curvature response for increasing levels of axial load
- With and without confining effects of transverse reinforcement
- Modify the Concrete01 input parameters for confined concrete



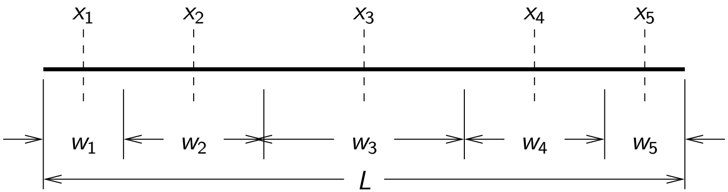
(c) Without Confining Effects



(d) With Confining Effects

Numerical Integration of Element Response

- For most material nonlinear element formulations, cross-section response is integrated numerically along the frame element length in order to determine element force-deformation response
- Sections located at discrete points along the element length, each with a prescribed weight
- Highly accurate Gauss-based quadrature commonly used



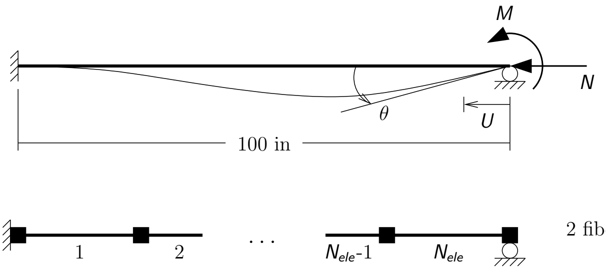
Displacement-Based Frame Element

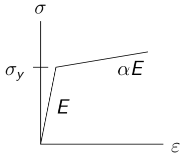
```
element dispBeamColumn $tag $ndI $ndJ $transfTag Legendre $secTag 2
```

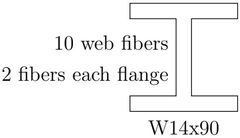
- Strict compatibility
  - Linear axial and cubic Hermitian transverse displacement fields
  - Constant axial deformation and linear curvature along element length
- Weak equilibrium
  - Equilibrium satisfied only at the nodes, not at every section along the element
  - Two-point Gauss-Legendre integration along element length
- Improve numerical solution by using more elements per member (mesh- or *h*-refinement)

Propped Cantilever

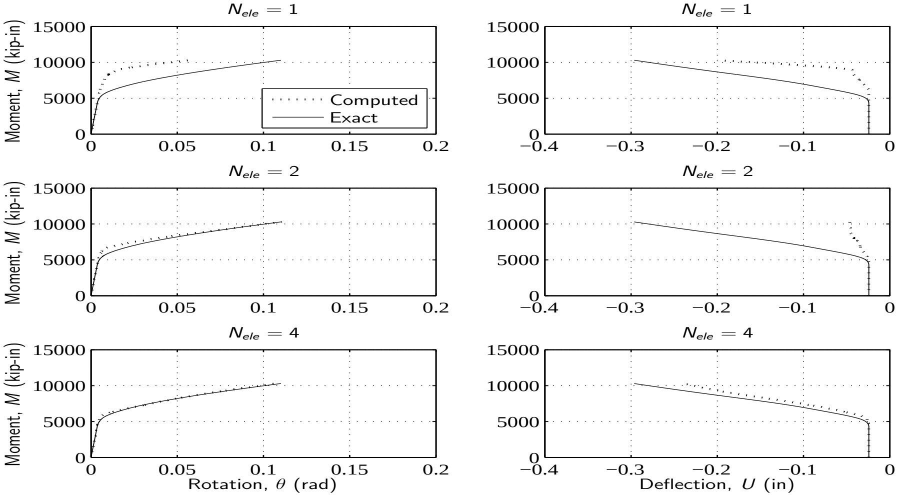
- Constant axial load and increasing moment applied at propped end
- Fiber-discretized section response with strain-hardening stress-strain
- Two Gauss-points per element
- Investigate refinement for increasing number of elements



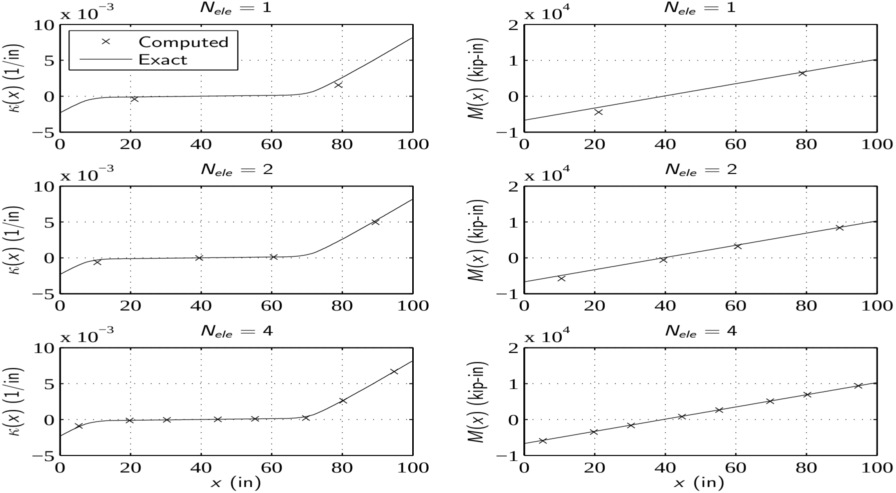




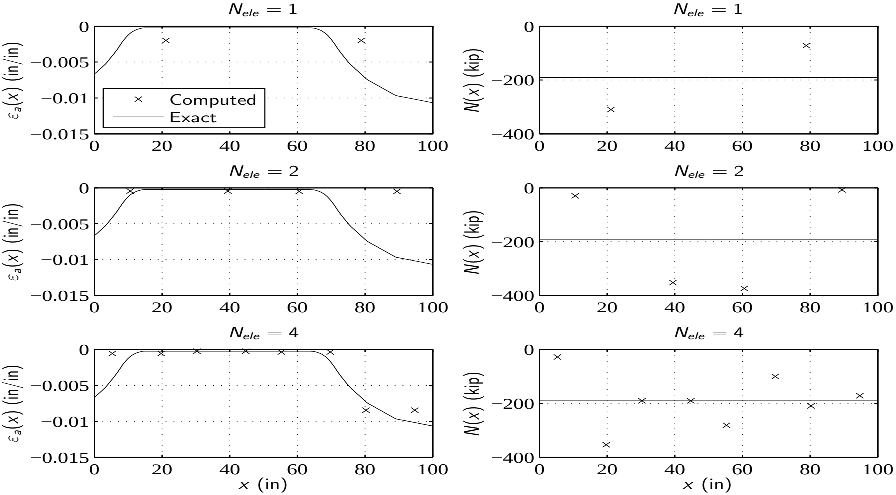
Global Response



Local Flexural Response

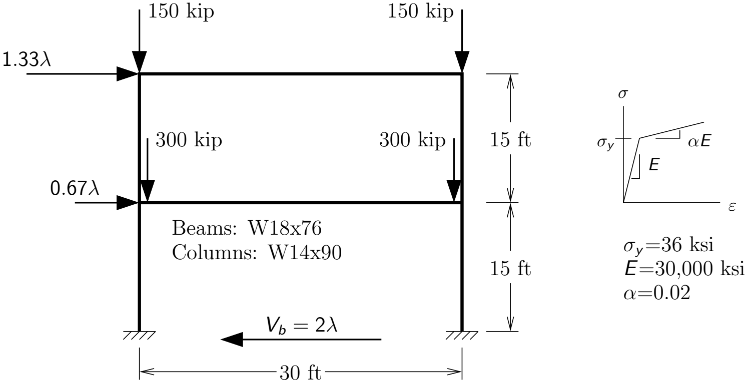


Local Axial Response



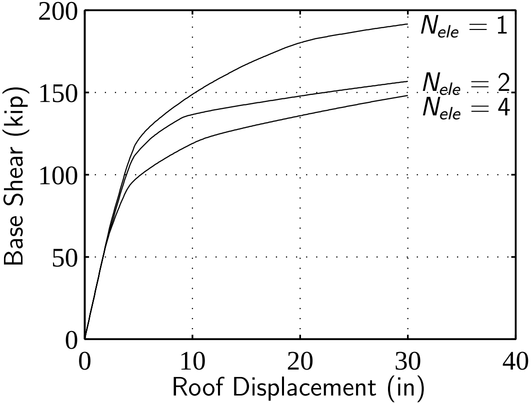
Steel Frame Pushover Analysis

- Investigate refinement of load-displacement response for increasing number of displacement-based elements per member



Steel Frame Pushover Analysis

- Coarse mesh over-predicts strength – *unconservative*
- Improved solution with refined mesh



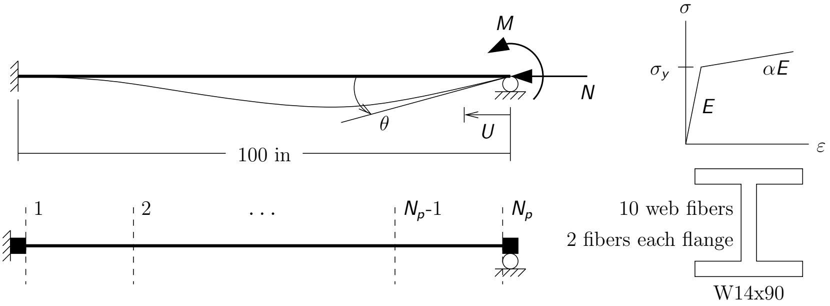
Force-Based Frame Element

```
element forceBeamColumn $tag $ndI $ndJ $transfTag Lobatto $secTag $Np
```

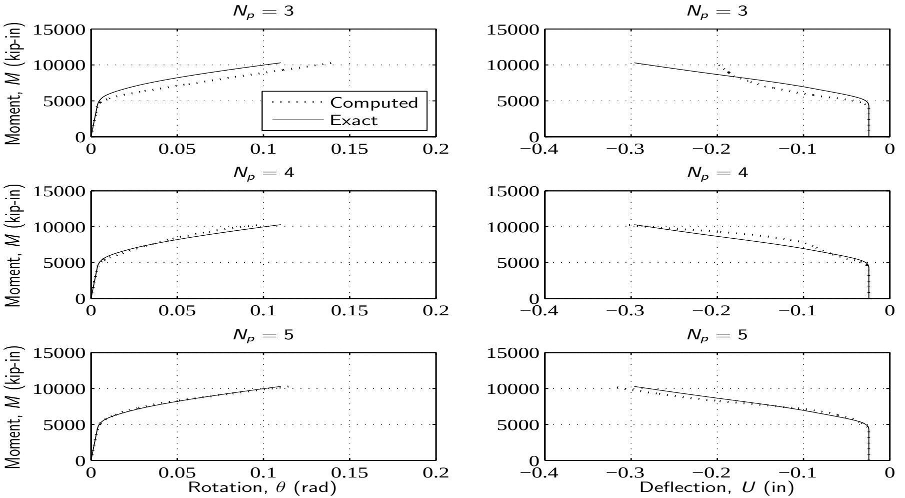
- Average compatibility
    - Nodal displacements are balanced by weighted integral of section deformations
    - Complex state determination
    - Use Gauss-Lobatto integration so that extreme flexural response captured at element ends
- Strong equilibrium
    - Equilibrum of nodal and section forces satisfied at all points along element
    - Constant axial force and linear bending moment in absence of member loads
    - Straightforward to include member loads
- Improve numerical solution by using more integration points per element while maintaining mesh of one element per member

Propped Cantilever

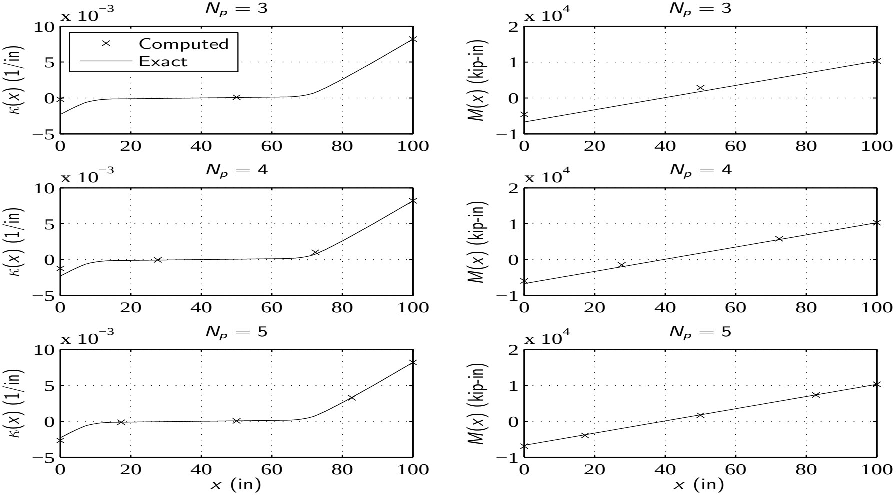
- Constant axial load and increasing moment applied at propped end
- Fiber-discretized section response with strain-hardening stress-strain
- Investigate refinement for increasing number of Gauss-Lobatto point using one element



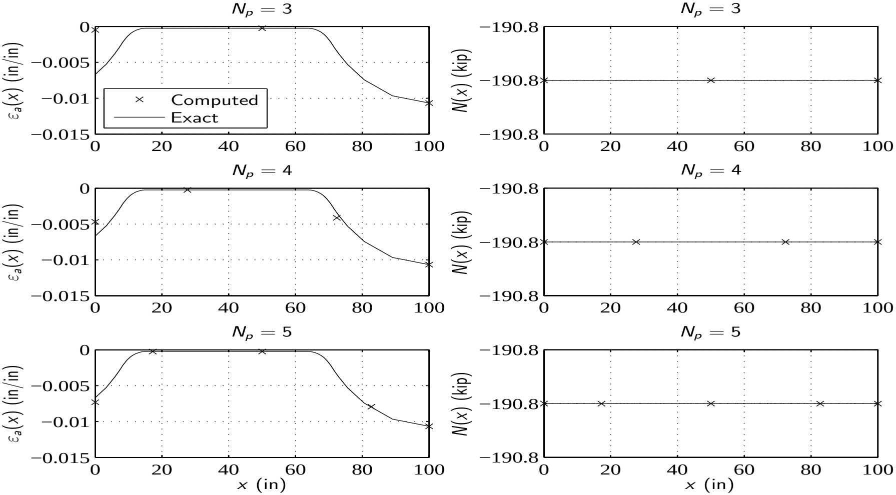
Global Response



Local Flexural Response

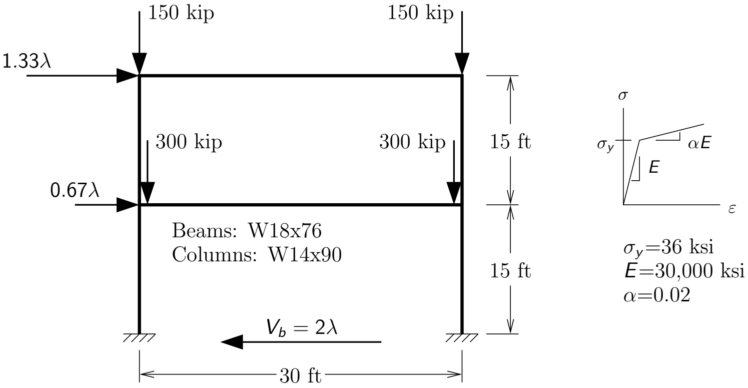


Local Axial Response



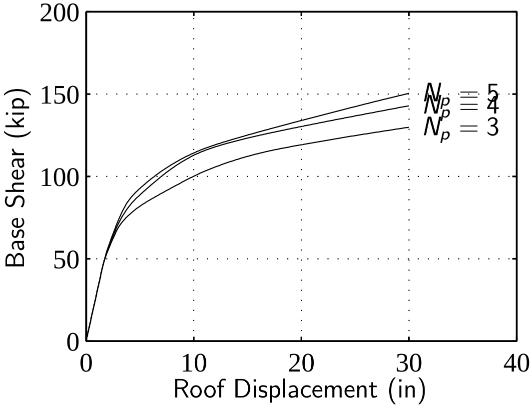
Steel Frame Pushover Analysis

- Investigate refinement of load-displacement response for increasing number of Gauss-Lobatto integration points per element
- Maintain one element per member



Steel Frame Pushover Analysis

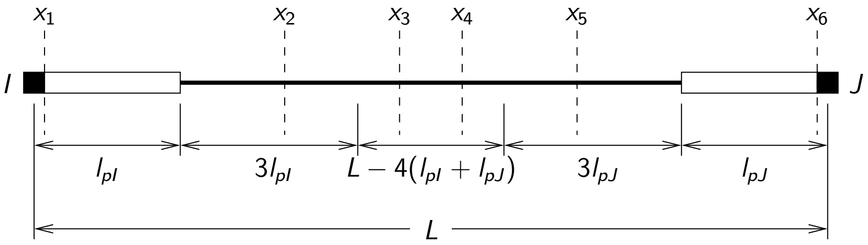
- Same yield point predicted in all cases
- Post-yield stiffness more flexible with fewer integration points



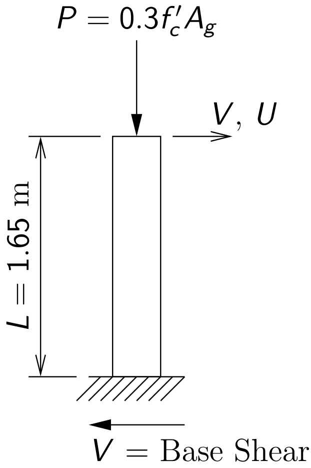
Force-Based Plastic Hinge Frame Element

```
element forceBeamColumn $tag $ndI $ndJ $transfTag HingeRadau $secTagI $lpI
$secTagJ $lpJ $secTagE
or
element beamWithHinges $tag $ndI $ndJ $secTagI $lpI $secTagJ $lpJ $E $A $I
$transfTag
```

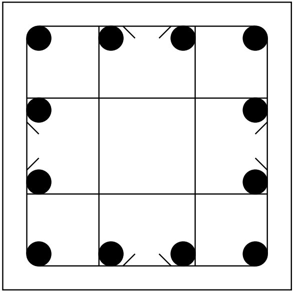
- Control integration weights at element ends
- Important for strain-softening section response



Reinforced Concrete Bridge Pier



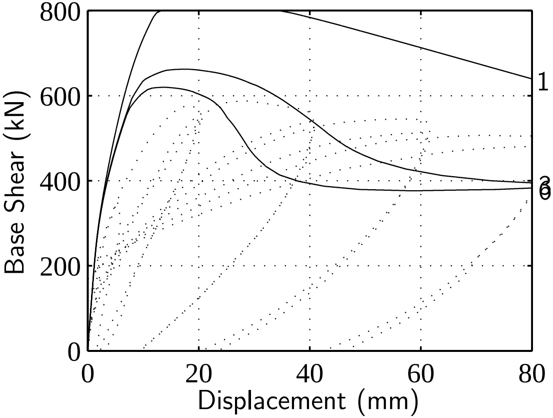
550mm x 550mm square
12 bars, $d_b = 20$ mm
40 mm clear cover



Tanaka and Park (1990)
Specimen 7

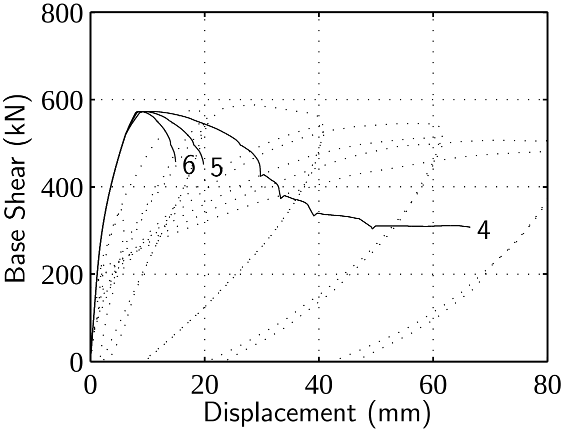
Displacement-Based Elements

- Post-peak response is mesh-dependent
- Function of element length



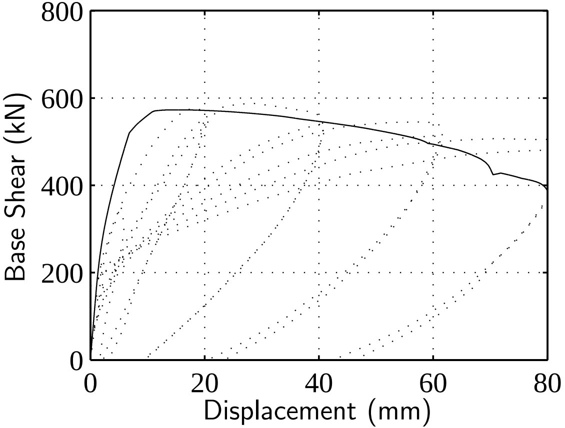
Force-Based Element

- Post-peak response depends on number of integration points
- Function of integration weight at base of column



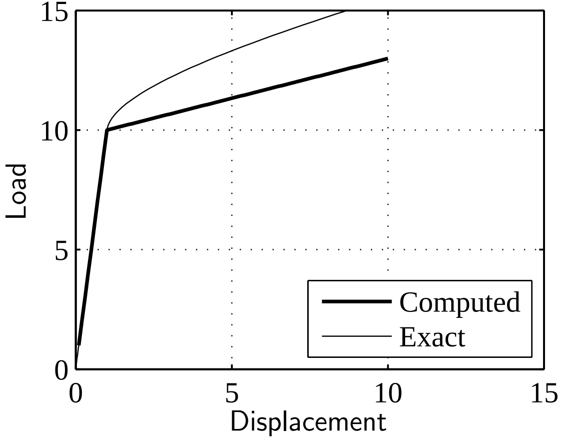
Force-Based Plastic Hinge Element

- Post-peak response controlled by plastic hinge length
- $l_p = 0.22L$ from empirical equation



Drawback to Force-Based Plastic Hinge Element

- For strain-hardening section behavior, post-peak response is too flexible



Modeling Recommendations

There's no silver bullet

Strain-Hardening Section Response

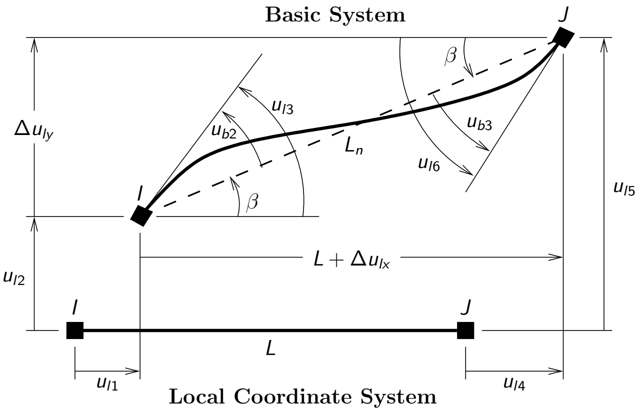
- Use mesh of displacement-based elements
- Use one force-based elements with 4 to 6 Gauss-Lobatto points
- Plastic hinge element not recommended because post-peak response will be too flexible

Strain-Softening Section Response

- Use force-based plastic hinge element
- Response with displacement-based elements is mesh dependent
- Response with Gauss-Lobatto force-based element depends on number of integration points

Geometric Transformation of Element Response

- Element formulation of material nonlinearity *inside* the basic system (free or rigid body displacement modes)
- Element formulation of geometric nonlinearity *outside* the basic system



Geometric Transformation

`geomTransf Linear $tag`

- Small displacement assumptions in local to basic transformation
- Linear transformation of forces and displacements

`geomTransf PDelta $tag`

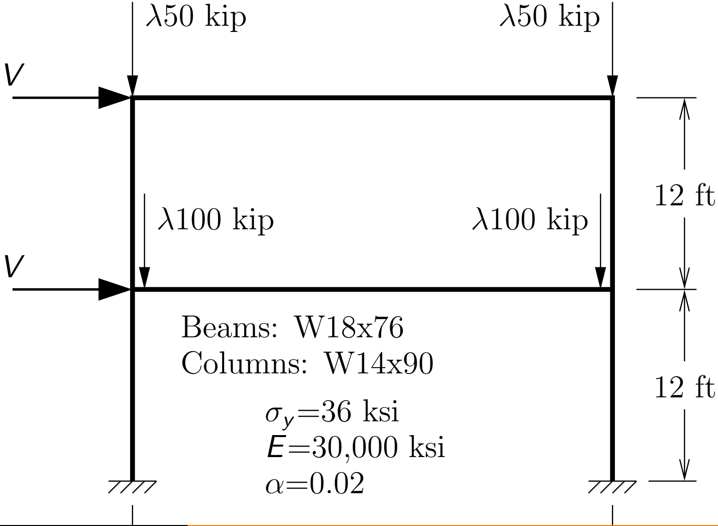
- Small displacement assumption transformation of displacements
- Account for transverse displacement of axial load in equilibrium relationship

`geomTransf Corotational $tag`

- Fully nonlinear transformation of displacements and forces
- Exact in 2D but some approximations in 3D

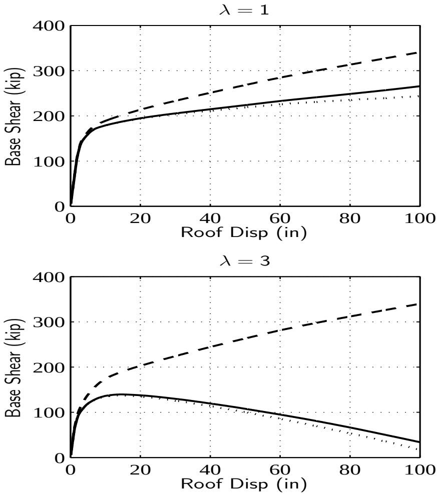
Steel Frame Pushover Analysis

- Examine pushover response for different levels of gravity load



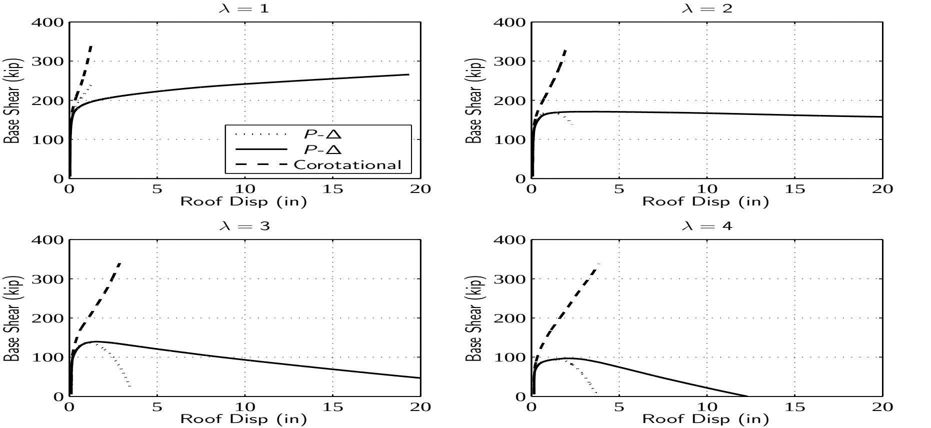
Steel Frame Pushover Analysis

- $P-\Delta$ and Corotational – similar results for lateral displacement



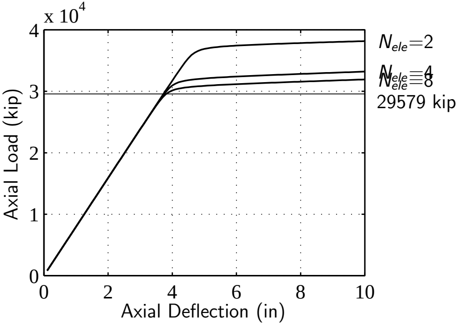
Steel Frame Pushover Analysis

- "Exact" Corotational predicts change in vertical displacement
- Important for collapse prediction and post-buckling capacity



Elastic Buckling

- Use mesh of corotational frame elements to simulate buckling
- Simply-supported W14x90, L=100 in, P_{cr}=29579 kip
- L/r=16.26: short column, but demonstrates point
- Imperfection applied to nodes, $u(t) = 0.1 \sin(\pi x/L)$ in



Concept works well for inelastic buckling too

Summary

- Material and geometric nonlinearity treated separately for frame finite elements in OpenSees
- Only scratching the surface – other element formulations and models of nonlinear stress-strain response
- Other Resources
    - OpenSees wiki
    - OpenSees message board
    - OpenSees YouTube videos
    - Course assignments