

Extending OpenSees for the Particle Finite Element Method

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Outline

- 1 Motivation
- 2 Particle Finite Element Method
 - Introduction to PFEM
 - Basic Equations
 - Fractional Step Method (FSM)
- 3 Implementation in OpenSees
 - Problems of the implementation of FSM
 - Options and solution to the implementation of FSM
 - Example
 - Mesh and Remesh
- 4 Sensitivity Analysis of PFEM
 - Introduction to the sensitivity
 - Computing sensitivity
 - Example

Fluid-Structure Interaction

- Recent natural disasters
- Simulating structural response to wave loading



OSU's Hinsdale Wave Research Lab (www.flickr.com/photos/scottfrey)

- Computational challenges
- The “OpenSees” homophone

Introduction to PFEM

Introduction to PFEM

- Lagrangian formulation
 - The position and physical properties of the particles are described in terms of the material or referential coordinates and time which is normally used in solid mechanics

Introduction to PFEM

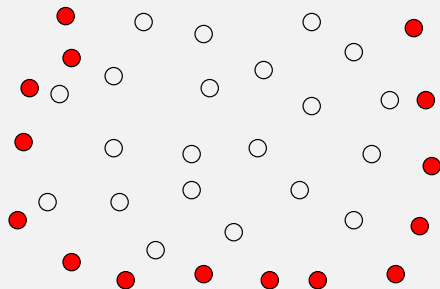
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- Effective for solving free-surface flow problems
 - Changes in the position and physical properties as the material particles move in free-surface are easy to capture
 - Delaunay tessellation and Alpha shape method \Rightarrow mesh
 - Fractional Step Method \Rightarrow fast iterative solver
 - Finite calculus method \Rightarrow stabilize the incompressible condition

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 - Finite calculus method \Rightarrow stabilize the incompressible condition
- Natural for solving solid-fluid interaction problems

Introduction to PFEM

- Particle Based Method
- Finite Element Method
- Particles Moving and Elements Updating



All the physical properties are associated with particles.

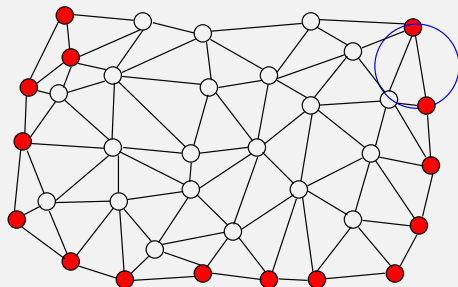
○ Mass, Pressure, Velocity, ...

○ Free Particles

● Fixed Particles

Introduction to PFEM

- Particle Based Method
- **Finite Element Method**
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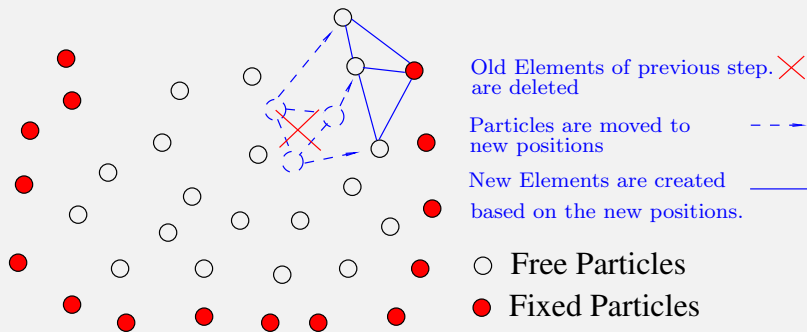


The Elements are created by using the Delaunay Tesselation.
The circumcircle of each element won't contain any other particles.

- Free Particles
- Fixed Particles

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Basic Equations

Basic Equations

- Conservation Laws

Mass:

$$\frac{D\rho}{Dt} + \rho \frac{\partial u_i}{\partial x_i} = 0$$

Momentum:

$$\rho \frac{Du_i}{Dt} = -\frac{\partial}{\partial x_i} P + \frac{\partial}{\partial x_j} \sigma_{ij} + \rho f_i$$

Where,

ρ – Density, u_i – Velocities

x_i – Coordinates, P – Pressure

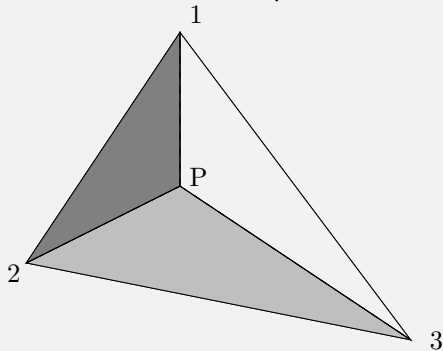
σ_{ij} – Cauchy Stress, f_i – Body forces

Basic Equations

- Linear interpolation (**Unstable** for incompressible continuum)

$$N_i = L_i \quad i = 1, 2, 3 \text{ For 2D}$$

Where, N_i is the shape function, L_i is the area coordinate for node i



For point P, the area coordinates

$$L_1 = \frac{\text{area}P23}{\text{area}123}$$

$$L_2 = \frac{\text{area}P13}{\text{area}123}$$

$$L_3 = \frac{\text{area}P12}{\text{area}123}$$

Basic Equations

- Discretized equations (stabilized by Finite Calculus Method)

$$\mathbf{M}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}$$

where,

$$\mathbf{M} = \begin{bmatrix} \bar{\mathbf{M}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} \bar{\mathbf{K}} & -\mathbf{G} & \mathbf{0} \\ \mathbf{G}^T & \mathbf{L} & \mathbf{Q} \\ \mathbf{0} & \mathbf{Q}^T & \hat{\mathbf{M}} \end{bmatrix}$$

$$\mathbf{F} = [\bar{\mathbf{F}} \quad \mathbf{0} \quad \mathbf{0}]^T$$

$$\mathbf{U} = [\mathbf{v} \quad \mathbf{p}]^T$$

$$\dot{\mathbf{U}} = [\dot{\mathbf{v}} \quad \dot{\mathbf{p}}]^T$$

$\bar{\mathbf{M}}$ – Mass matrix

$\bar{\mathbf{K}}$ – Stiffness matrix

\mathbf{G} – Gradient operator

\mathbf{L} – Laplacian operator

$\mathbf{Q}, \hat{\mathbf{M}}$ – Stabilization matrices

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Blue matrices are related to the stability issues of incompressible fluid.

Fractional Step Method (FSM)

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To overcome this problem, the **fractional step method (FSM)** is used to solve the discretized equations.

Fractional Step Method (FSM)

- Predictor of velocity \mathbf{v}^*

$$\mathbf{v}^* = \mathbf{v}^n - \Delta t \bar{\mathbf{M}}^{-1} [\bar{\mathbf{K}} \mathbf{v}^j - \mathbf{G} \mathbf{p}^j - \mathbf{f}]$$

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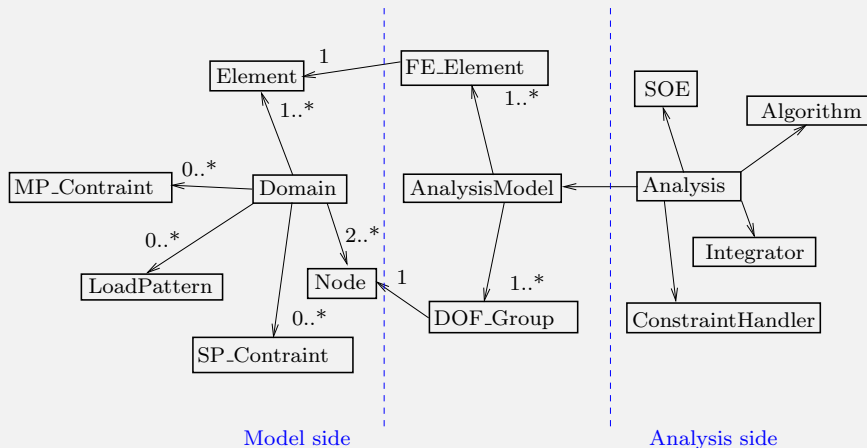
- Pressure gradient π^{j+1}

$$\pi^{j+1} = -\hat{\mathbf{M}}^{-1} \mathbf{Q}^T \mathbf{p}^{j+1}$$

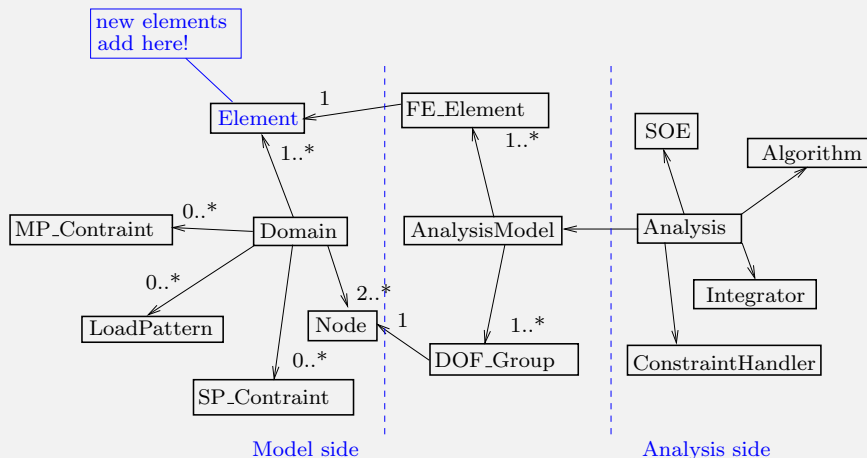
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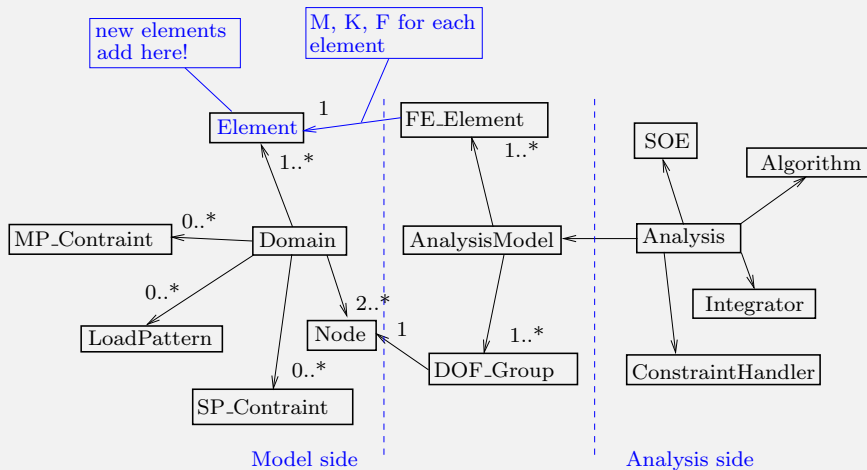
OpenSees class diagram



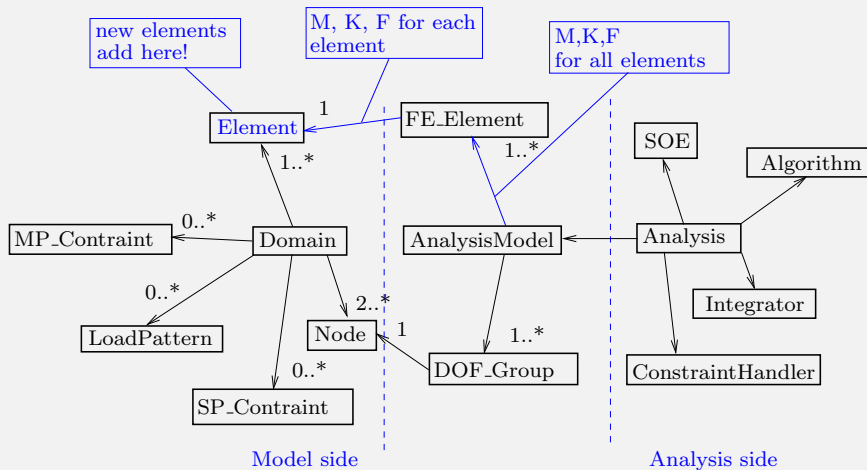
Flow of assembling local matrices to global



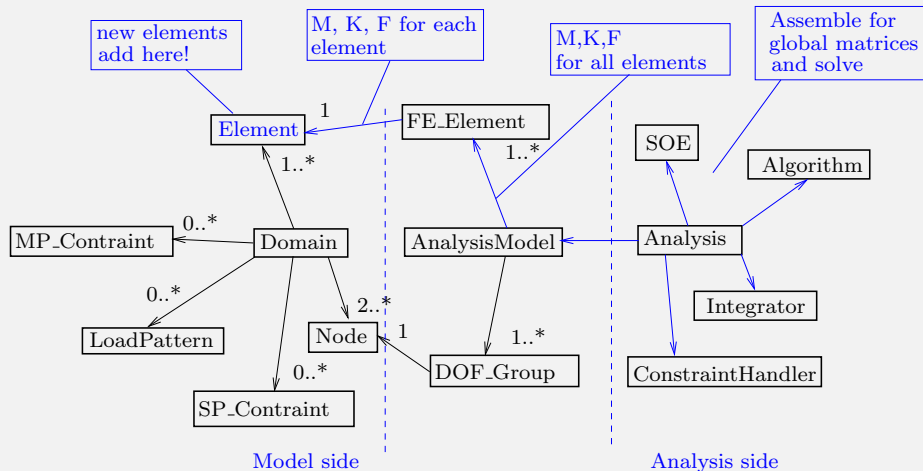
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- Matrices multiplications and inversions are applied on the partitioned matrices. Therefore partitioning of matrices has to be performed at the global level. Three inversions, $\bar{\mathbf{M}}^{-1}$, \mathbf{S}^{-1} and $\hat{\mathbf{M}}^{-1}$ occur in a single iteration while normally there is only one in OpenSees.

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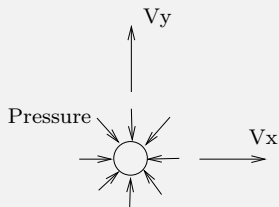
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- From the flow chart shown before, global matrices are only available in the analysis side. **So we can only partition matrices in the analysis side.**

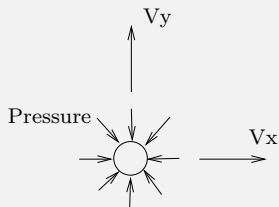
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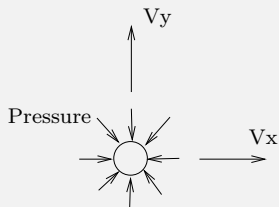
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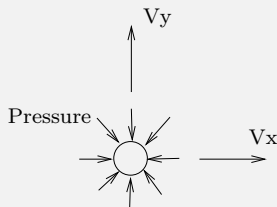
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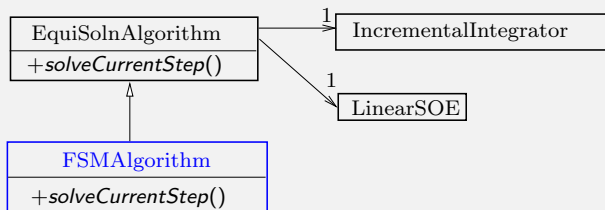
- The philosophy of OpenSees requires model and analysis uncoupled.
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- Model side? Analysis side?

Option 1 : add a new algorithm

The first option is to implement FSM in a new algorithm
FSMAlgorithm,

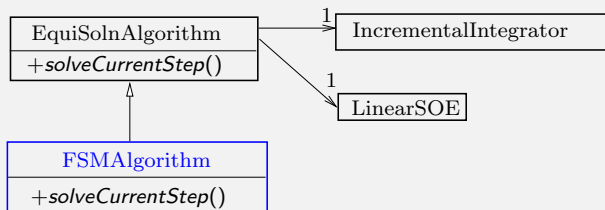
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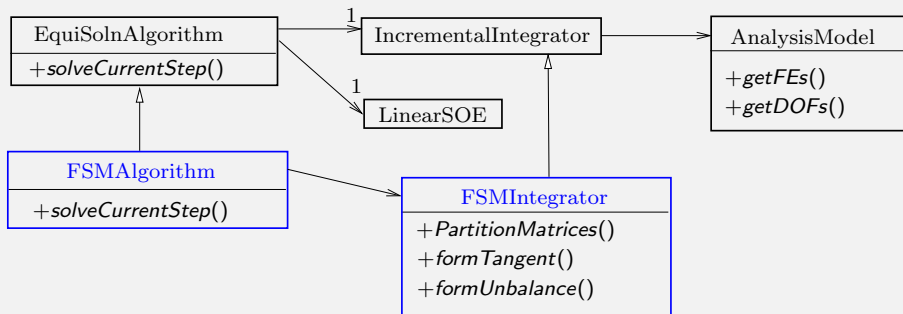
We want use **FSMAlgorithm** to partition matrices in the global level. But **FSMAlgorithm** **does not have the DOFs information**. To get the information, another new class **PFEMIntegrator** has to be added also.

Option 2 : add a new algorithm and integrator

The second option is to create both an algorithm and integrator,

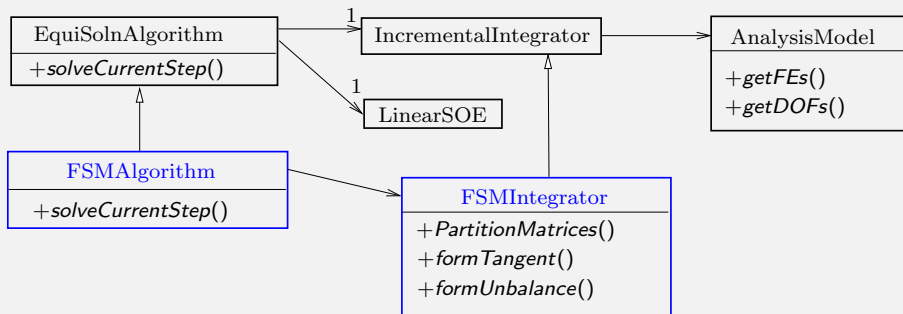
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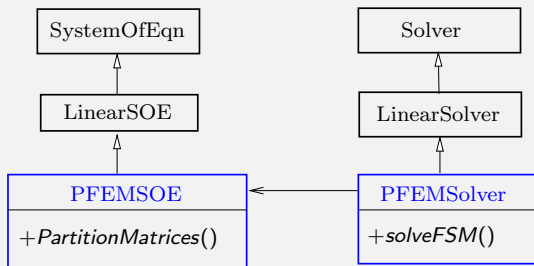
However, this implementation makes the algorithm and integrator **too specialized for FSM** and needs **new methods in the interface** of the FSM algorithm and integrator. The FSM algorithm and integrator are **coupled to the model information**.

Option 3: Add a SOE and solver

The third option is to add a LinearSOE and Solver,

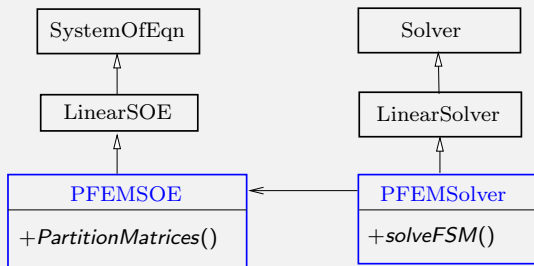
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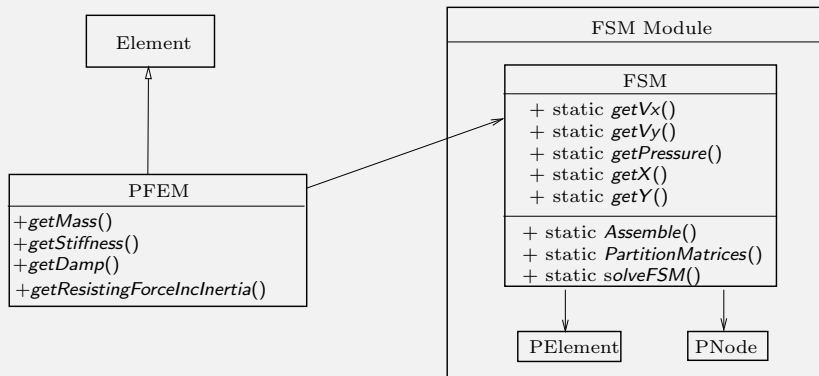
This option has the same problem with option 1 that **neither LinearSOE nor Solver knows the DOFs information**. They cannot partition without knowing IDs and **only work with each other**.

A model side solution: Adapter

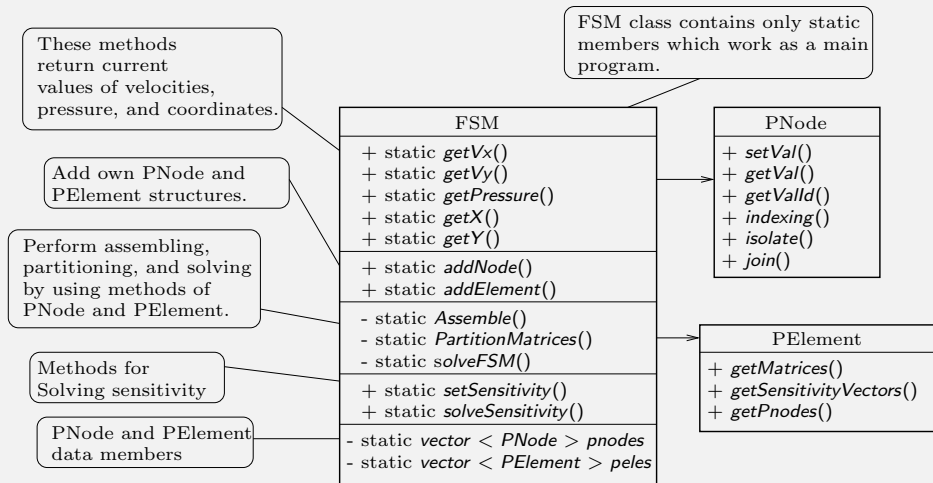
The solution is a model side implementation, not an analysis side implementation.

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- Pressures are hidden from OpenSees. PFEM elements can be treated uniformly as solid elements in OpenSees.

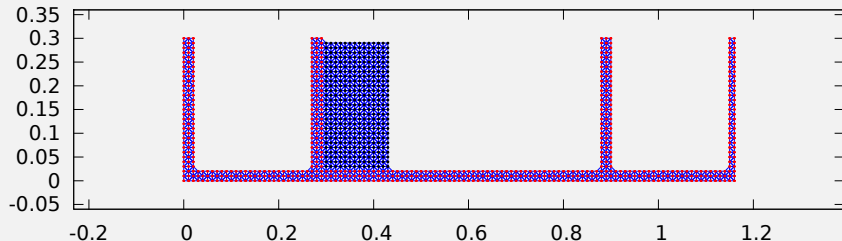
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- PFEM works as a wrapper to ask FSM for information needed.
- Pressures are hidden from OpenSees. PFEM elements can be treated uniformly as solid elements in OpenSees.
- PNode and PElement are standalone structures used by FSM for storing its nodes and elements

Water column collapse

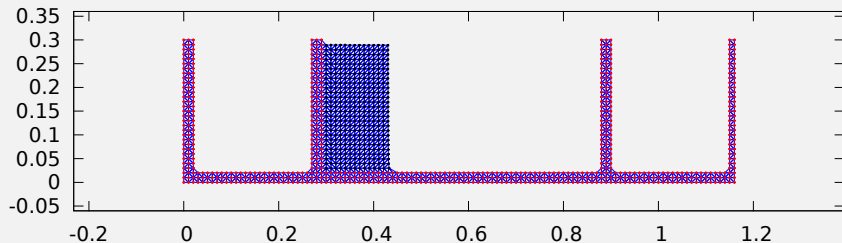
- A standard test problem of PFEM, involving large domain changing and strong nonlinearity.
- A water column is kept in a container by an unseen vertical board. Start by removing the board.

Time = 0.0 sec, Color = default



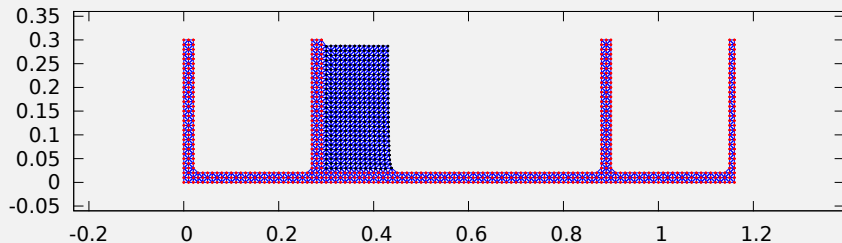
Water column collapse

Time = 0.01 sec, Color = default



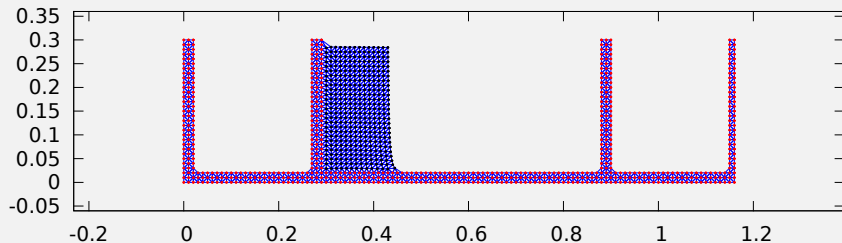
Water column collapse

Time = 0.02 sec, Color = default

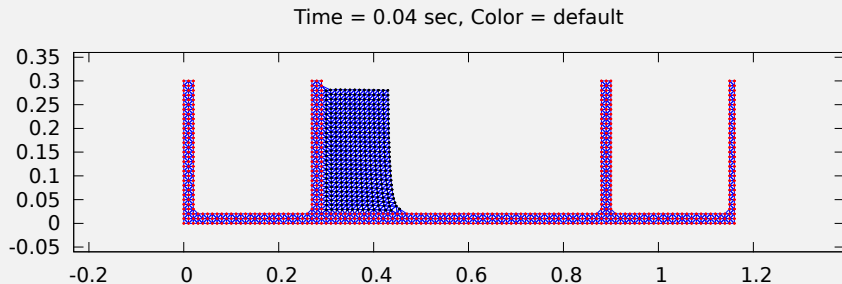


Water column collapse

Time = 0.03 sec, Color = default

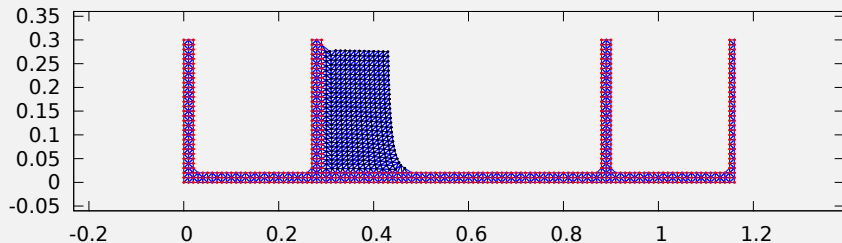


Water column collapse



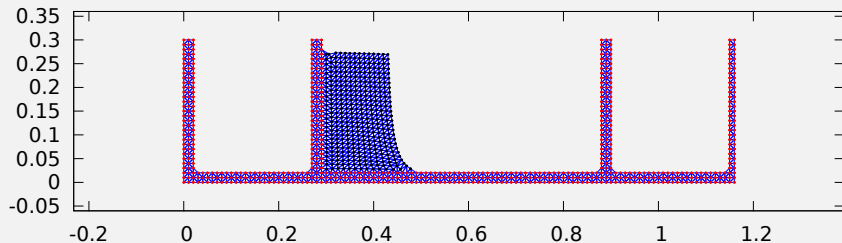
Water column collapse

Time = 0.05 sec, Color = default



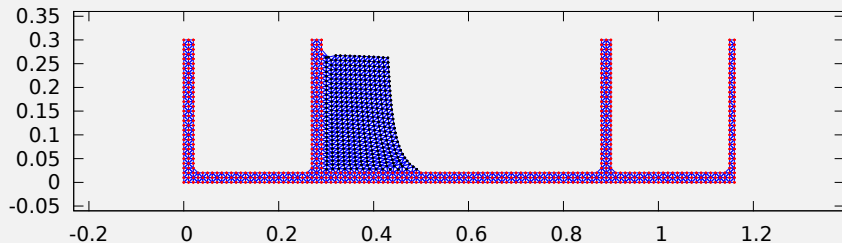
Water column collapse

Time = 0.06 sec, Color = default



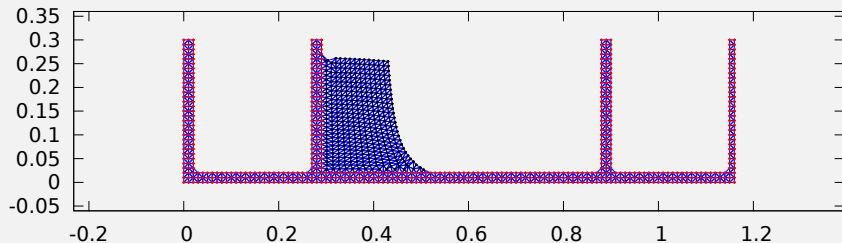
Water column collapse

Time = 0.07 sec, Color = default



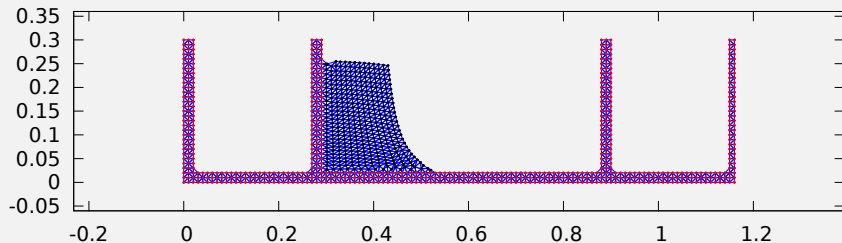
Water column collapse

Time = 0.08 sec, Color = default



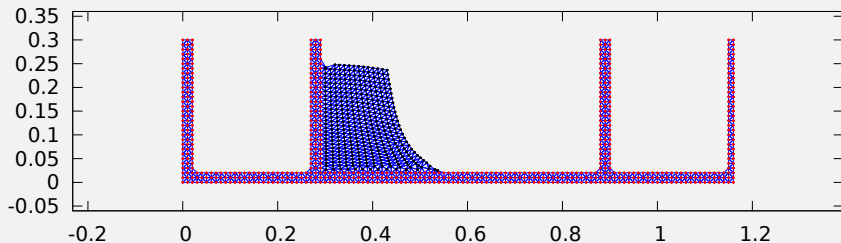
Water column collapse

Time = 0.09 sec, Color = default



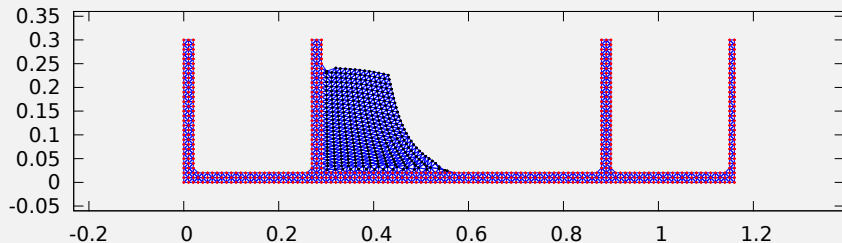
Water column collapse

Time = 0.1 sec, Color = default



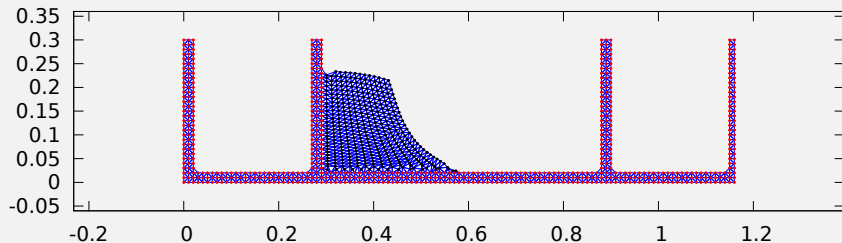
Water column collapse

Time = 0.11 sec, Color = default



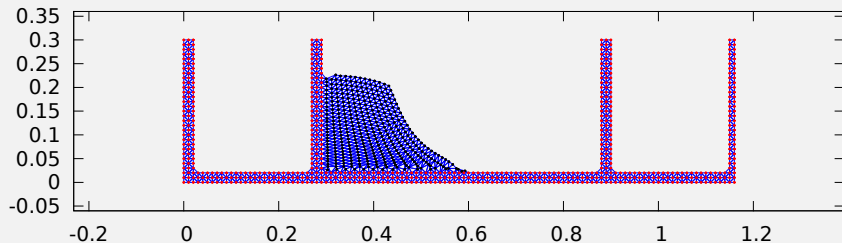
Water column collapse

Time = 0.12 sec, Color = default



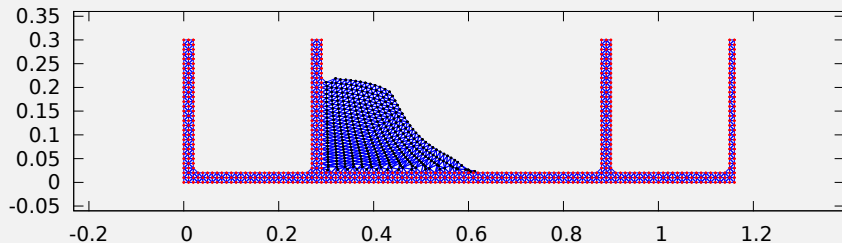
Water column collapse

Time = 0.13 sec, Color = default



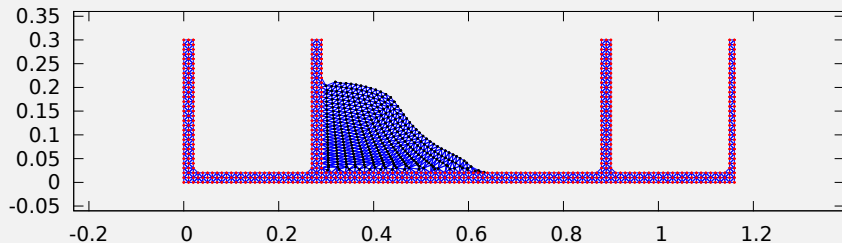
Water column collapse

Time = 0.14 sec, Color = default

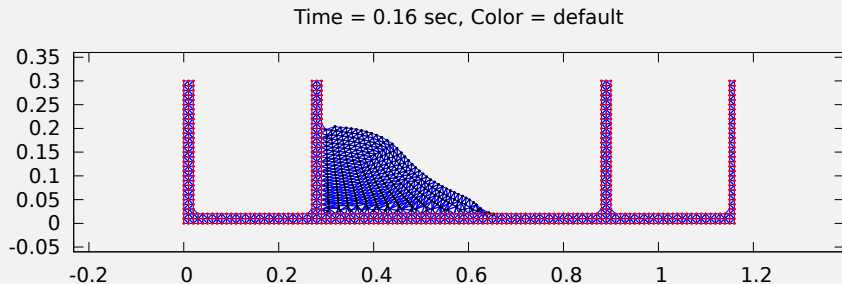


Water column collapse

Time = 0.15 sec, Color = default

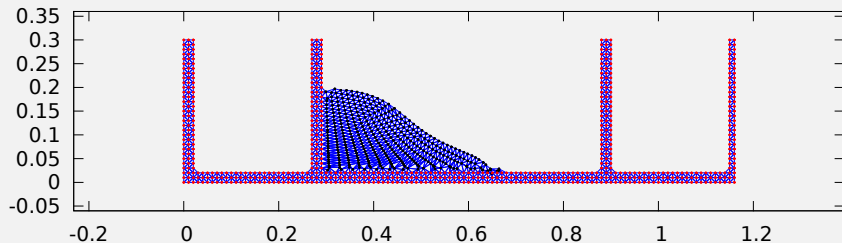


Water column collapse



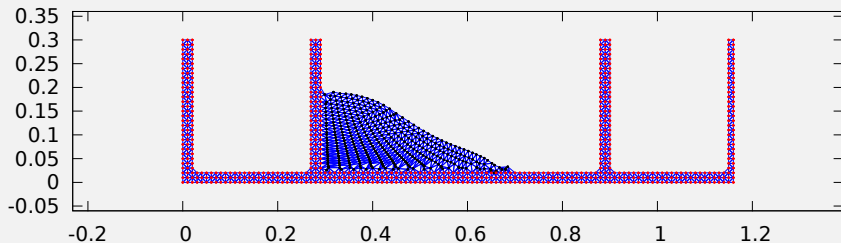
Water column collapse

Time = 0.17 sec, Color = default



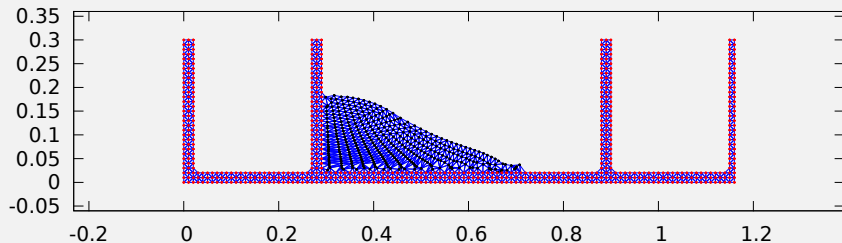
Water column collapse

Time = 0.18 sec, Color = default



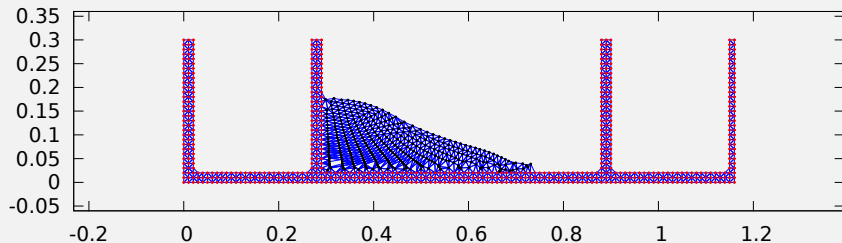
Water column collapse

Time = 0.19 sec, Color = default



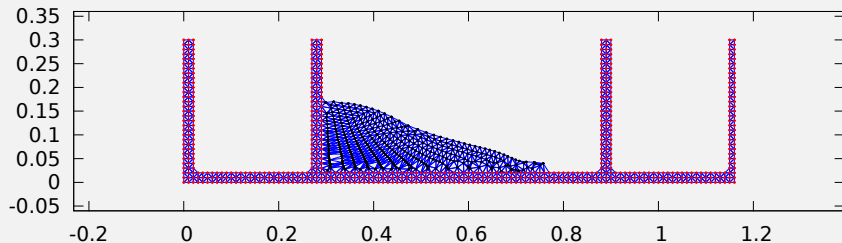
Water column collapse

Time = 0.2 sec, Color = default



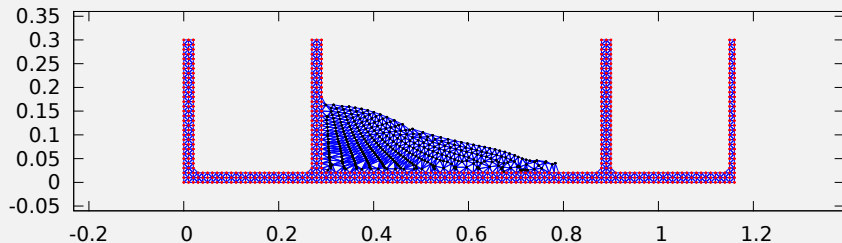
Water column collapse

Time = 0.21 sec, Color = default



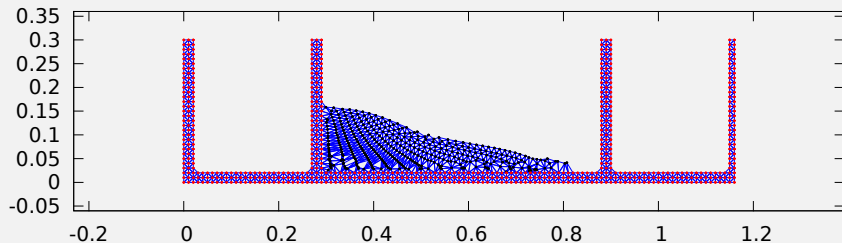
Water column collapse

Time = 0.22 sec, Color = default



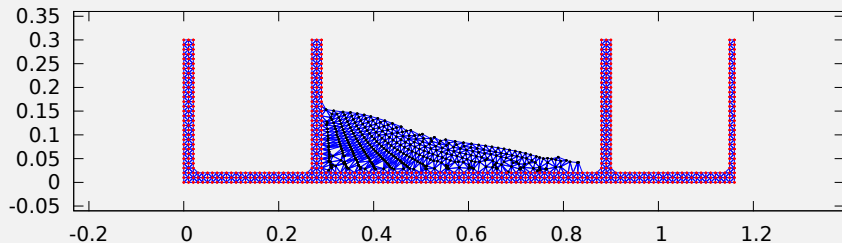
Water column collapse

Time = 0.23 sec, Color = default



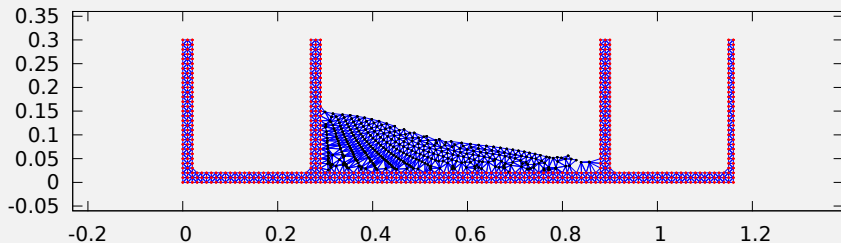
Water column collapse

Time = 0.24 sec, Color = default



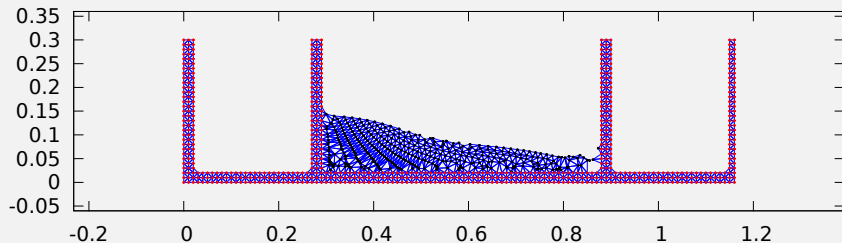
Water column collapse

Time = 0.25 sec, Color = default



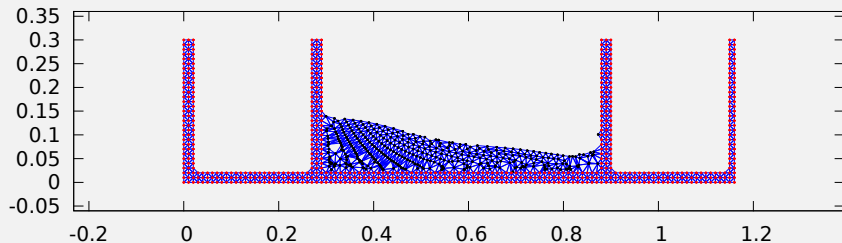
Water column collapse

Time = 0.26 sec, Color = default



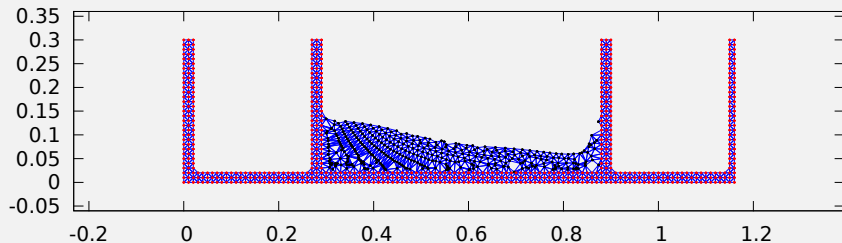
Water column collapse

Time = 0.27 sec, Color = default



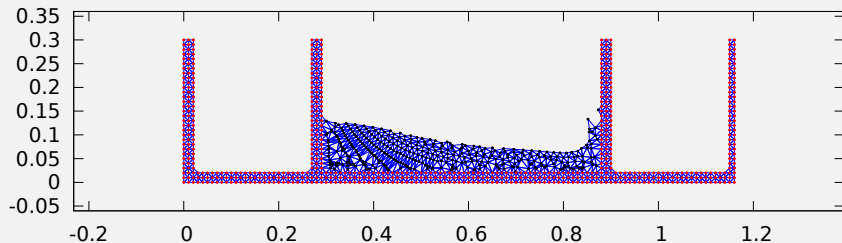
Water column collapse

Time = 0.28 sec, Color = default



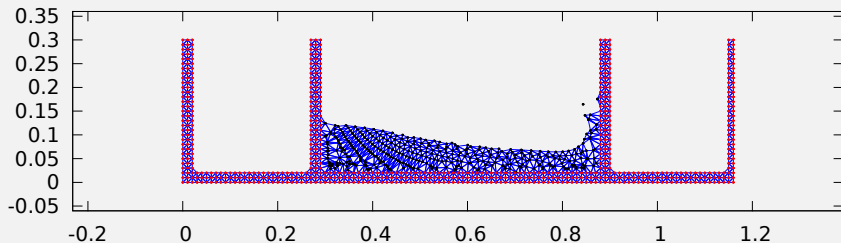
Water column collapse

Time = 0.29 sec, Color = default



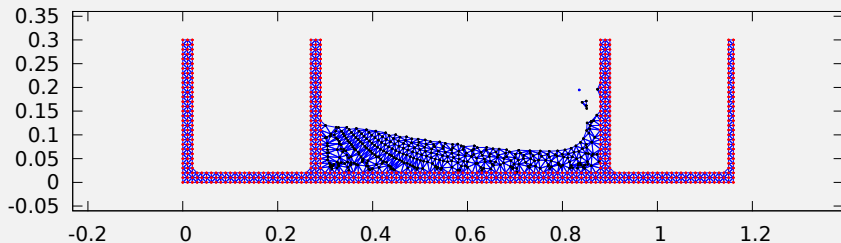
Water column collapse

Time = 0.3 sec, Color = default



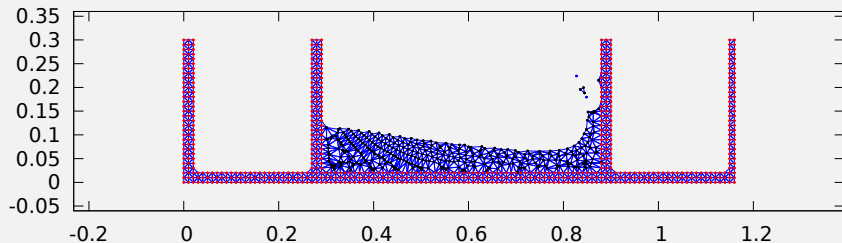
Water column collapse

Time = 0.31 sec, Color = default



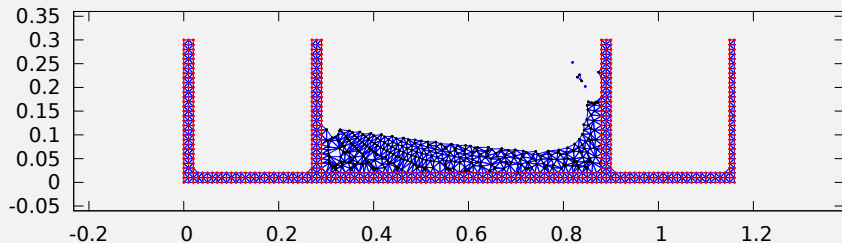
Water column collapse

Time = 0.32 sec, Color = default



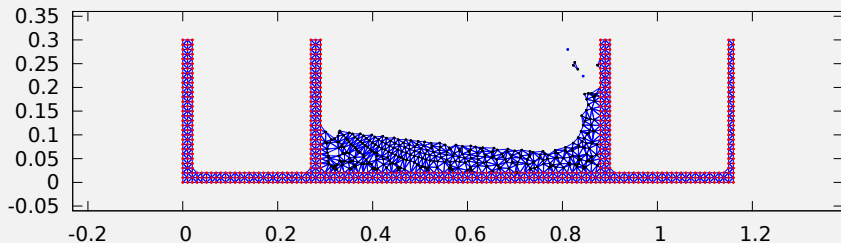
Water column collapse

Time = 0.33 sec, Color = default



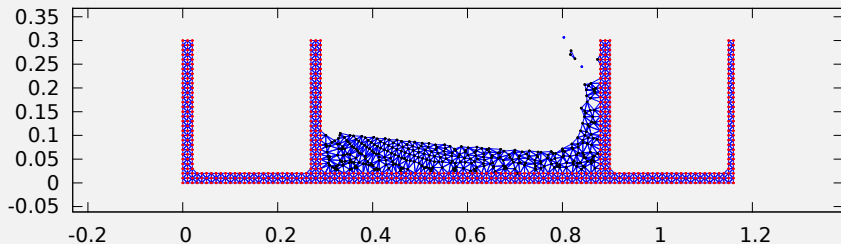
Water column collapse

Time = 0.34 sec, Color = default



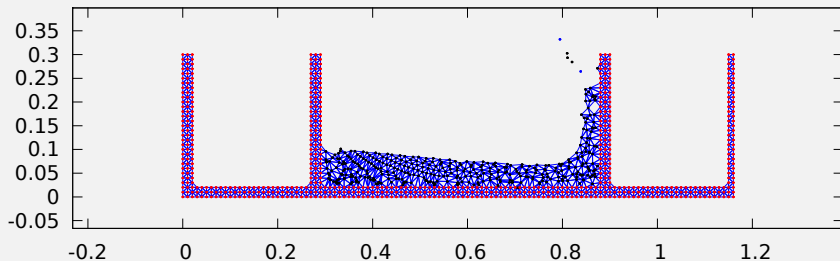
Water column collapse

Time = 0.35000000000000003 sec, Color = default



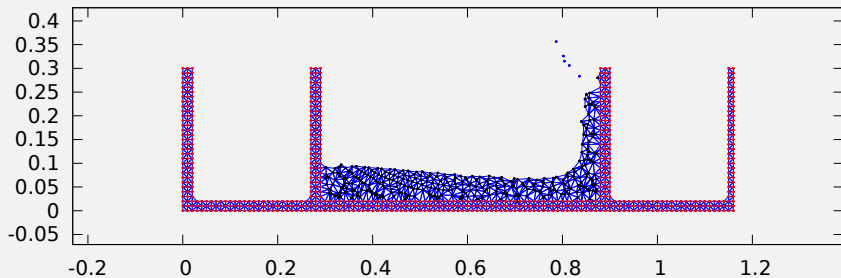
Water column collapse

Time = 0.36 sec, Color = default



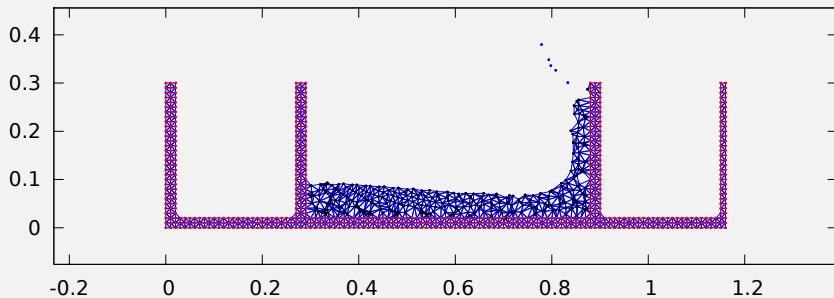
Water column collapse

Time = 0.37 sec, Color = default



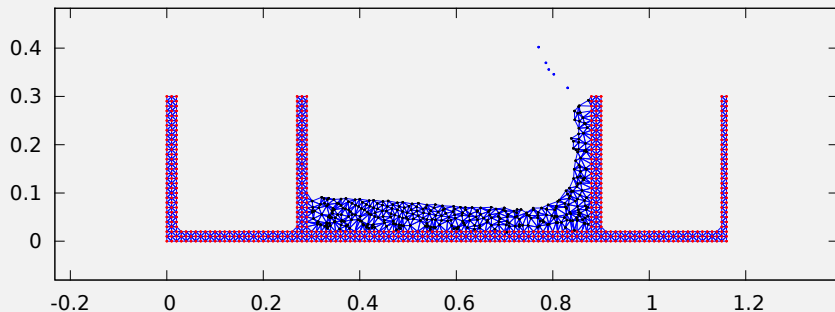
Water column collapse

Time = 0.38 sec, Color = default

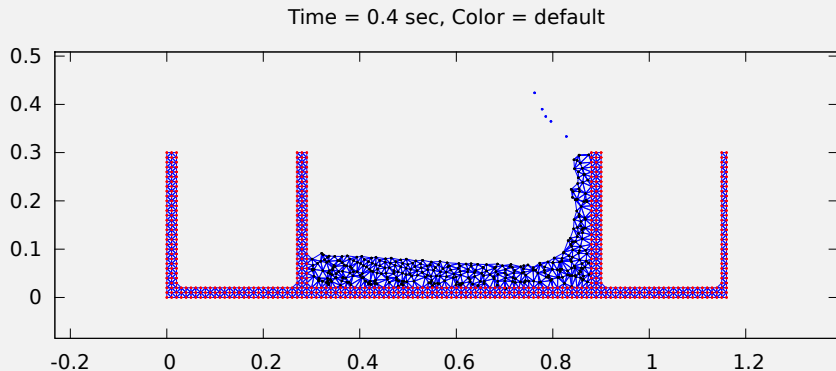


Water column collapse

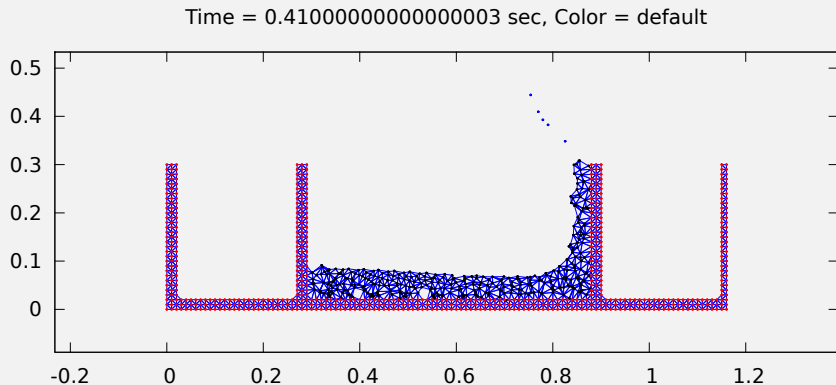
Time = 0.39 sec, Color = default



Water column collapse

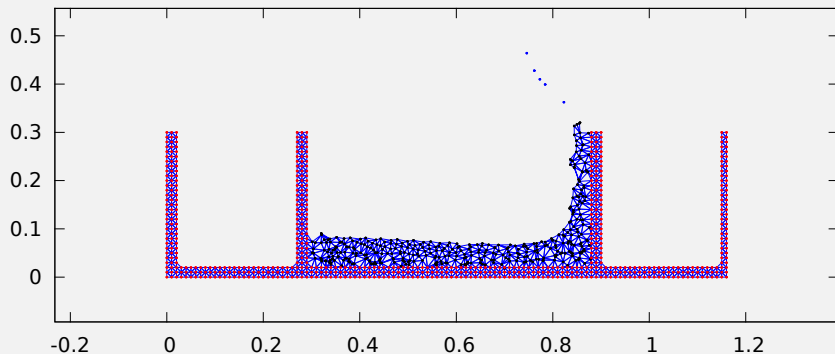


Water column collapse



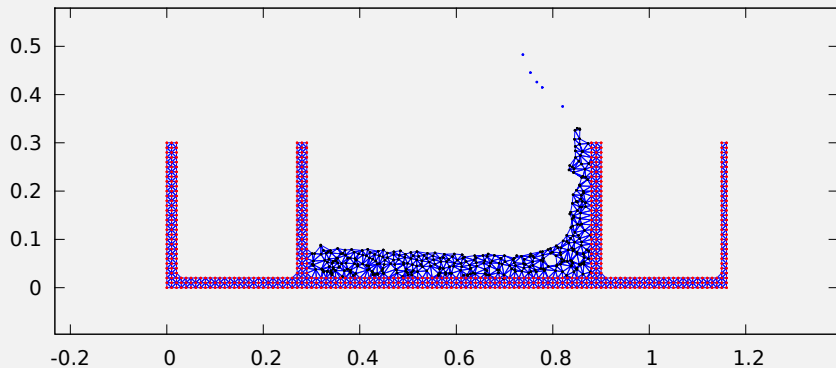
Water column collapse

Time = 0.42 sec, Color = default

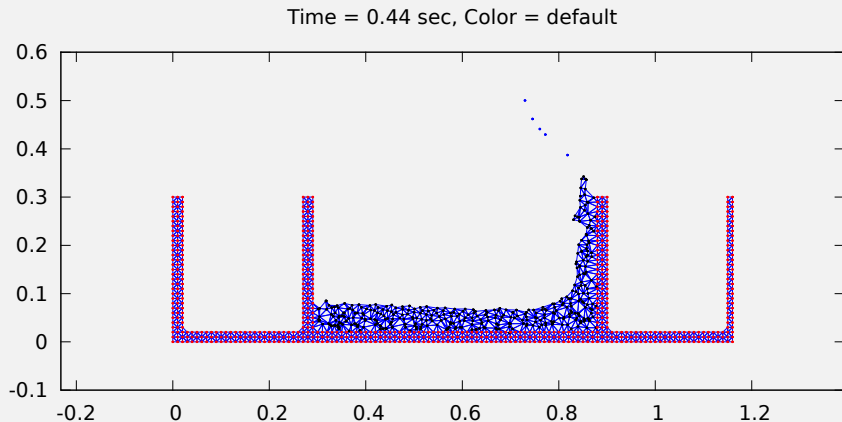


Water column collapse

Time = 0.43 sec, Color = default

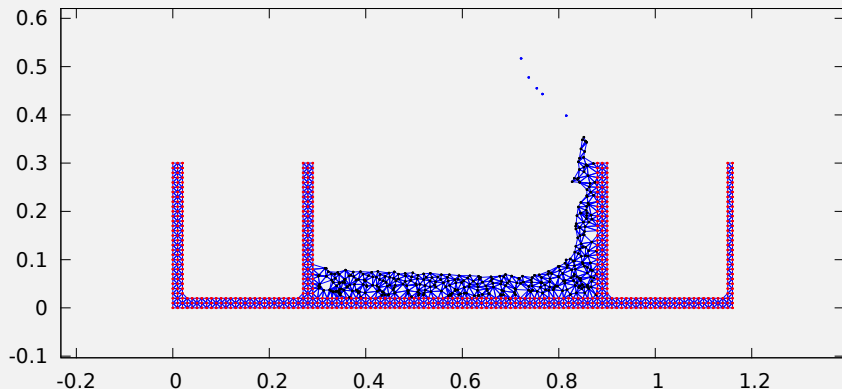


Water column collapse



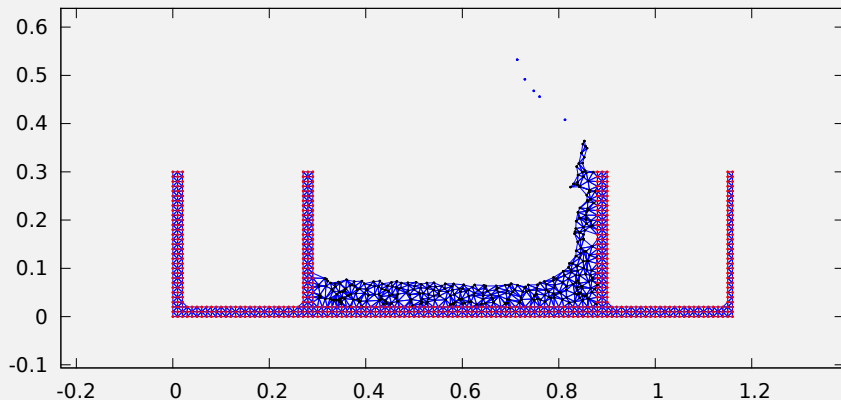
Water column collapse

Time = 0.45 sec, Color = default



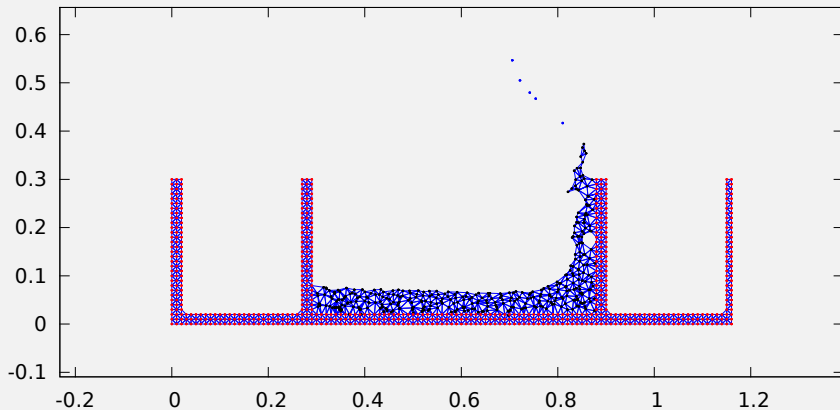
Water column collapse

Time = 0.46 sec, Color = default



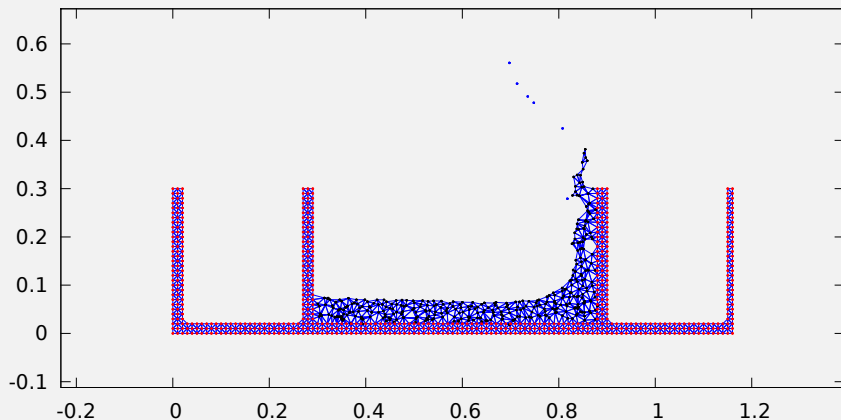
Water column collapse

Time = 0.47000000000000003 sec, Color = default



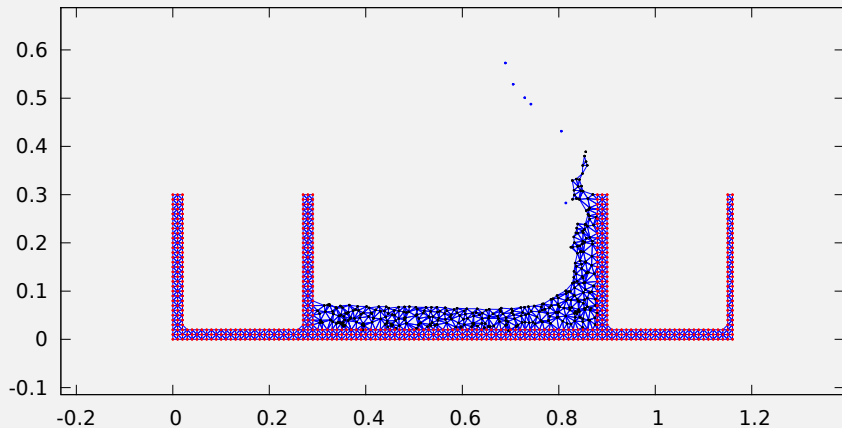
Water column collapse

Time = 0.48 sec, Color = default

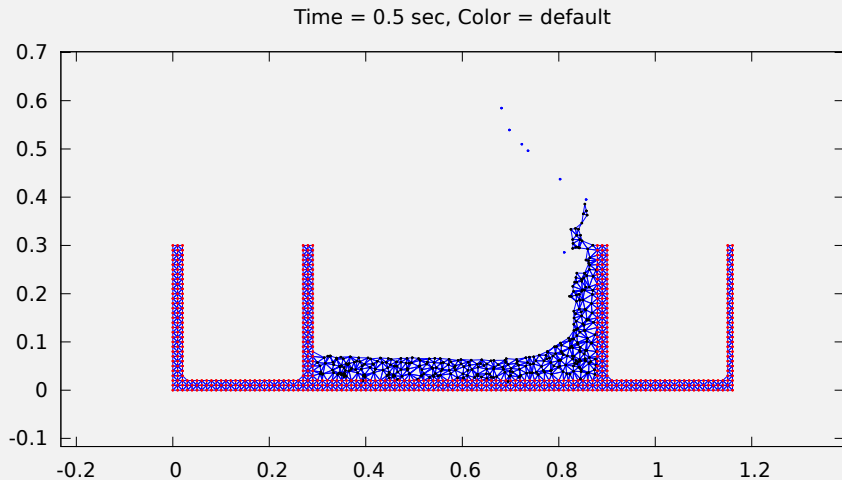


Water column collapse

Time = 0.49 sec, Color = default

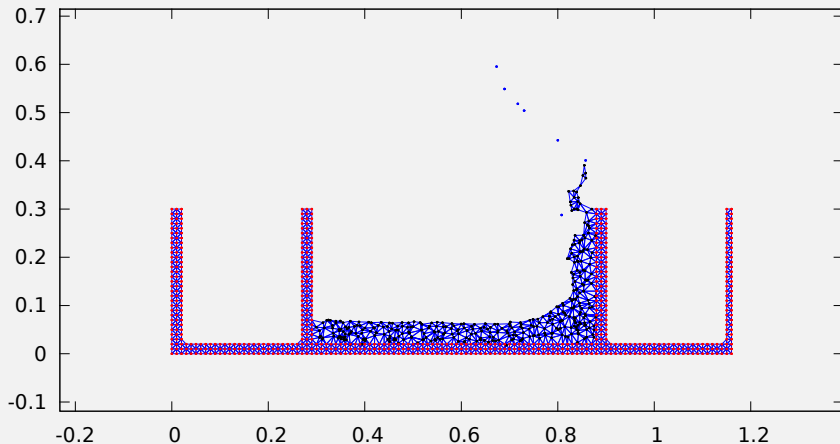


Water column collapse



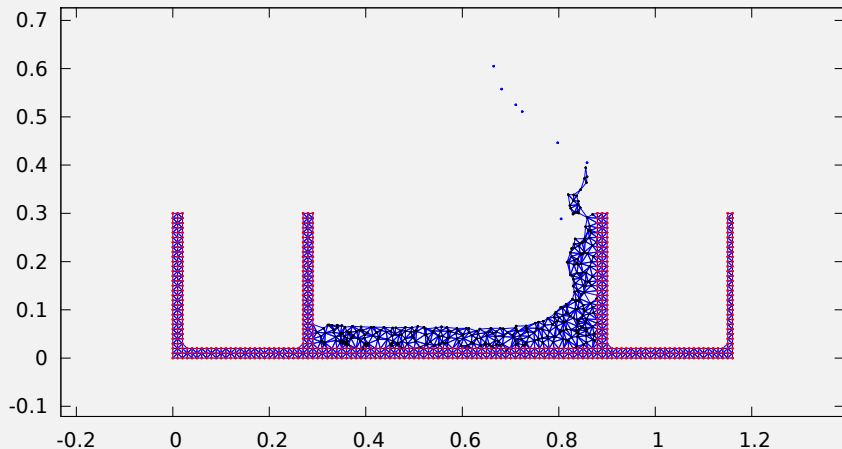
Water column collapse

Time = 0.51 sec, Color = default



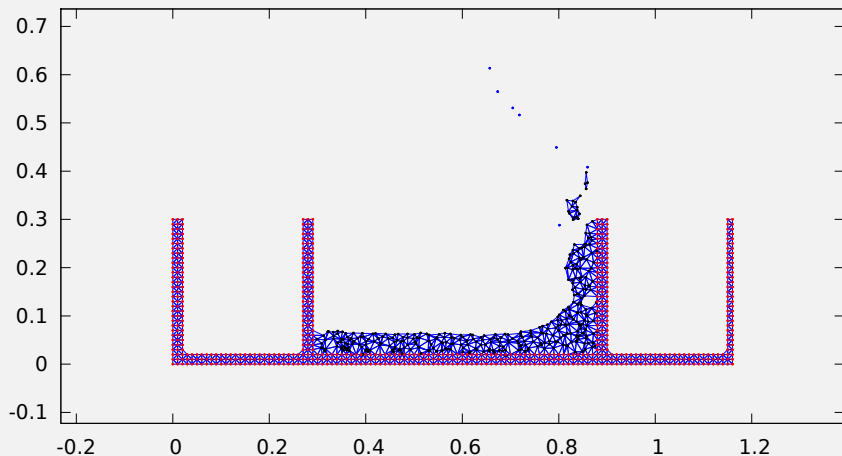
Water column collapse

Time = 0.52 sec, Color = default



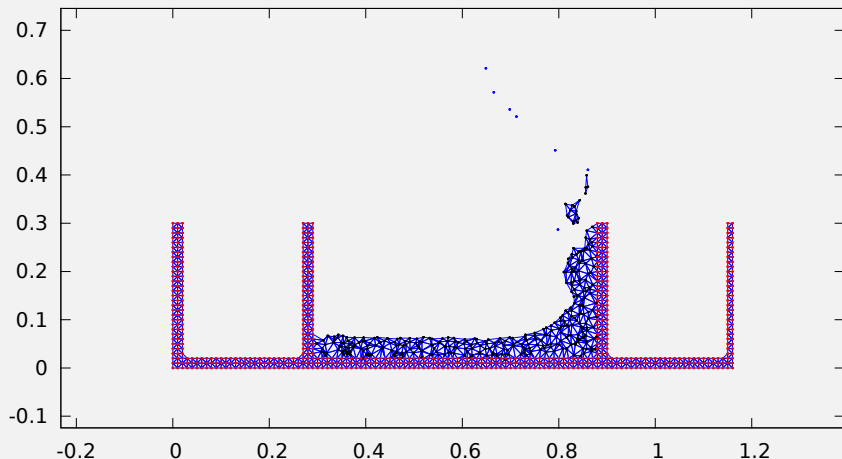
Water column collapse

Time = 0.53 sec, Color = default



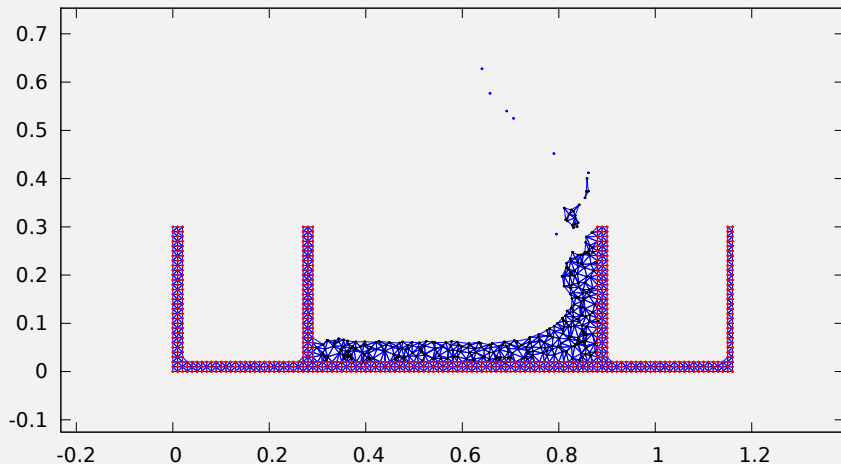
Water column collapse

Time = 0.54 sec, Color = default



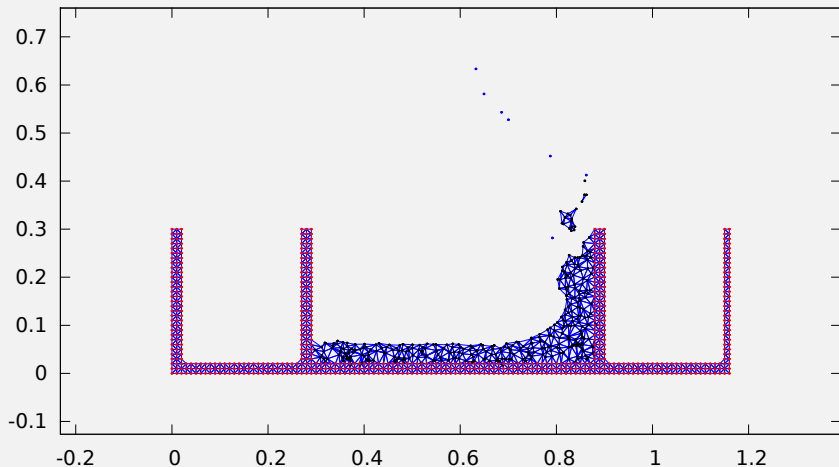
Water column collapse

Time = 0.55 sec, Color = default



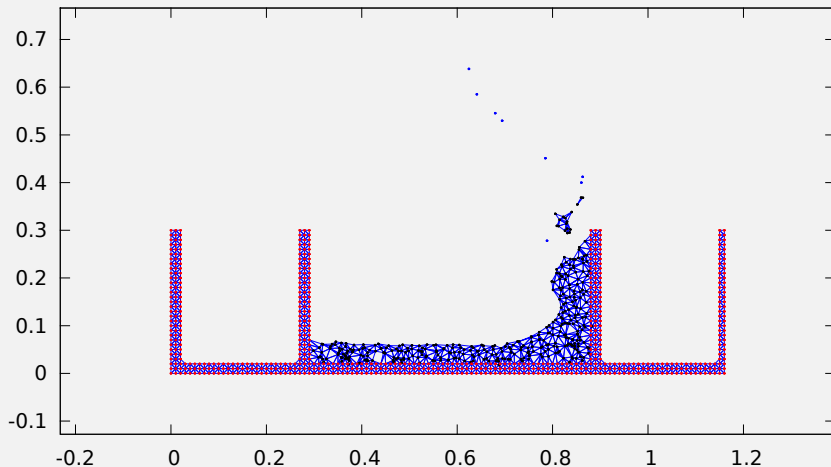
Water column collapse

Time = 0.56 sec, Color = default



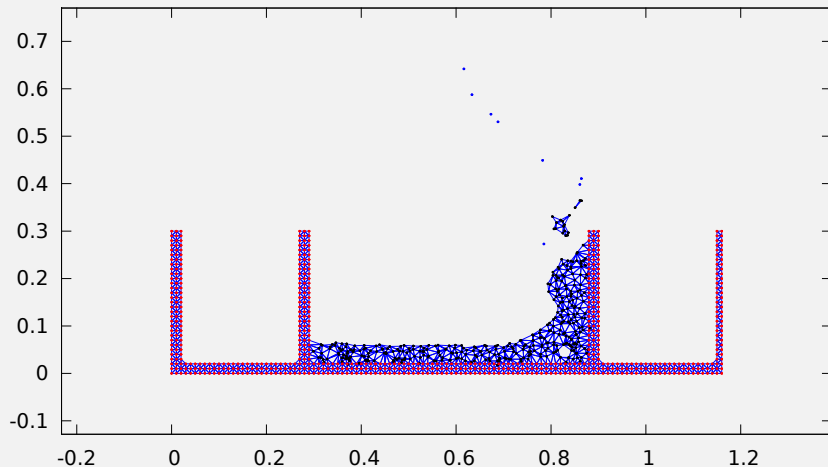
Water column collapse

Time = 0.5700000000000001 sec, Color = default



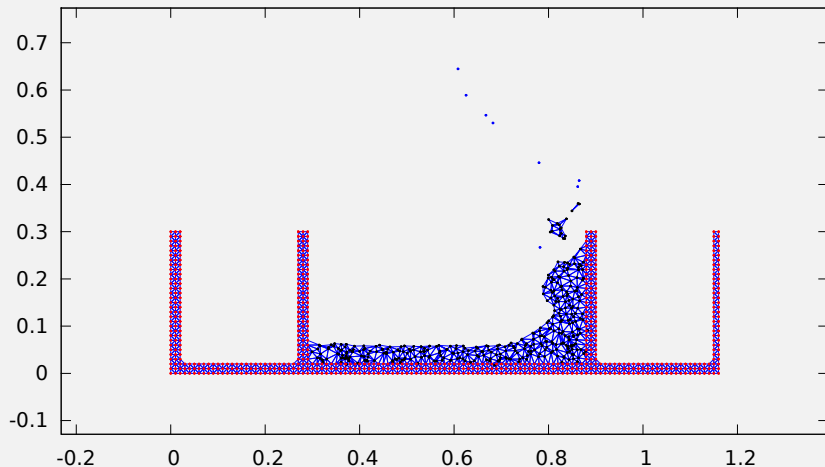
Water column collapse

Time = 0.58 sec, Color = default



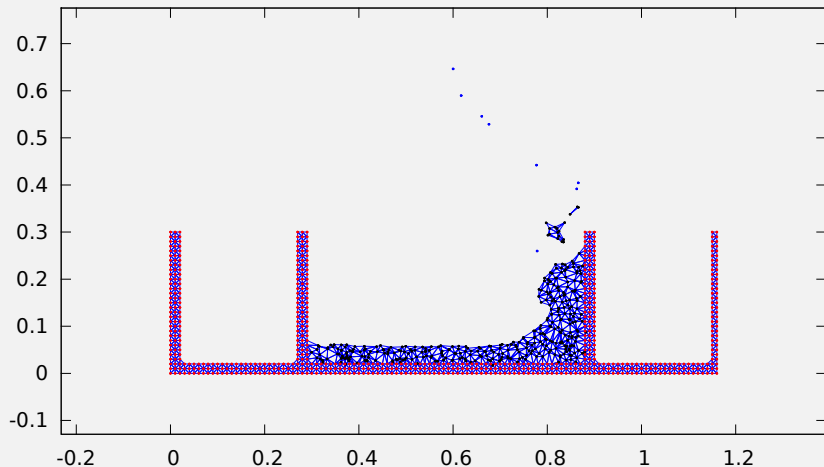
Water column collapse

Time = 0.59 sec, Color = default



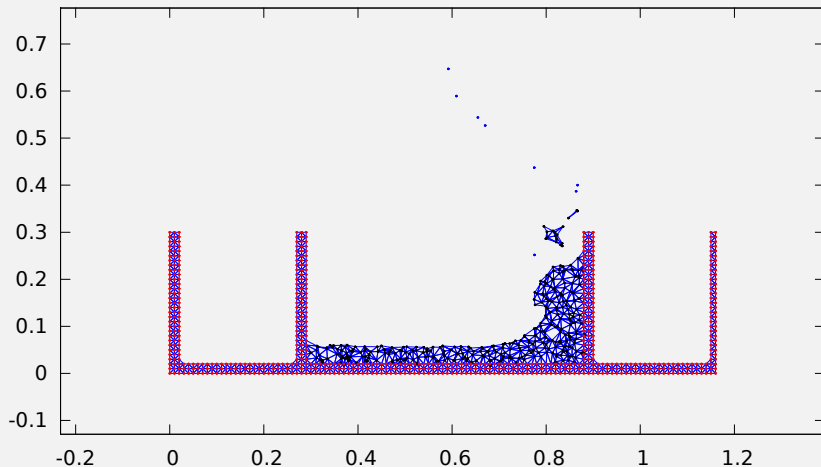
Water column collapse

Time = 0.6 sec, Color = default



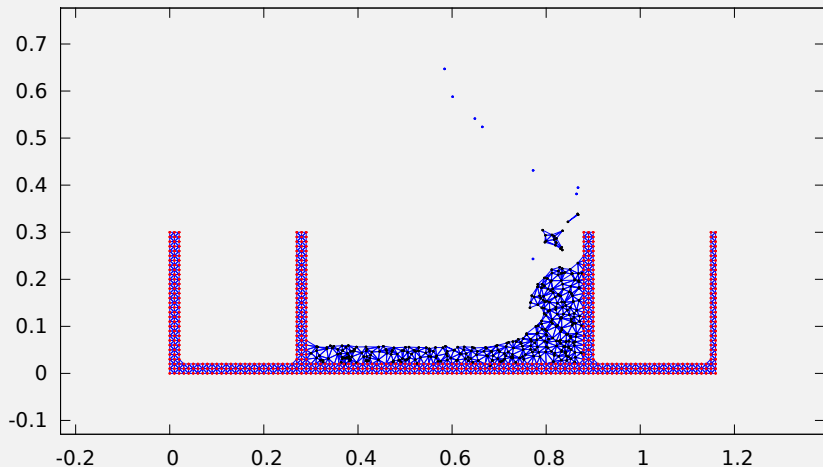
Water column collapse

Time = 0.61 sec, Color = default



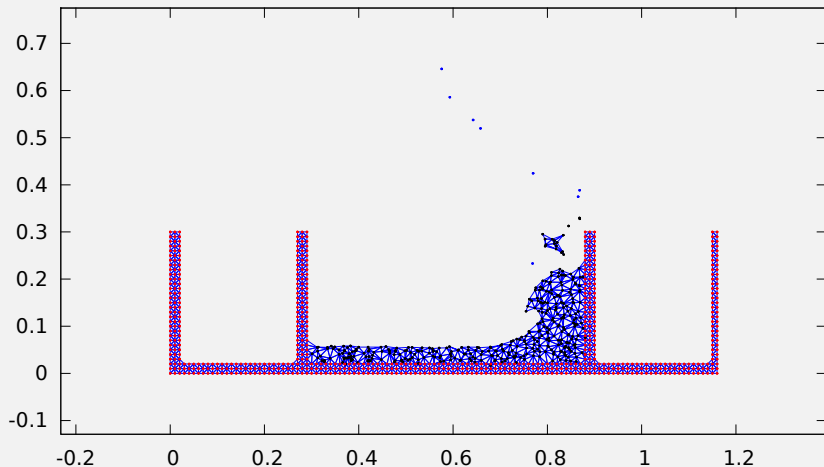
Water column collapse

Time = 0.62 sec, Color = default



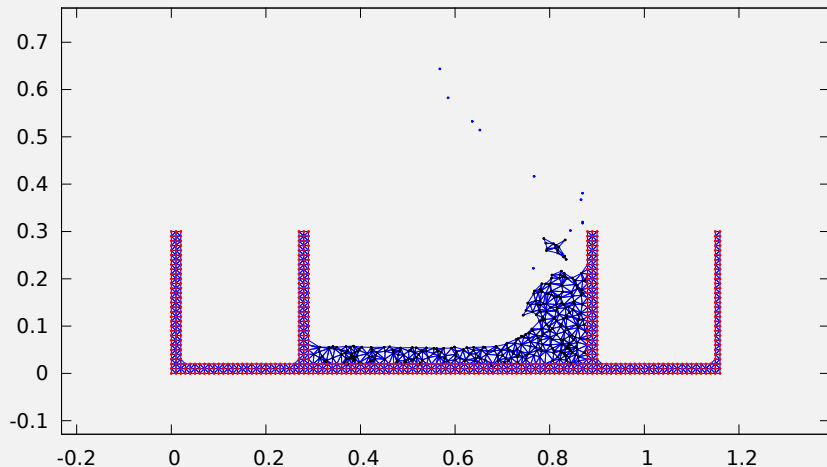
Water column collapse

Time = 0.63 sec, Color = default



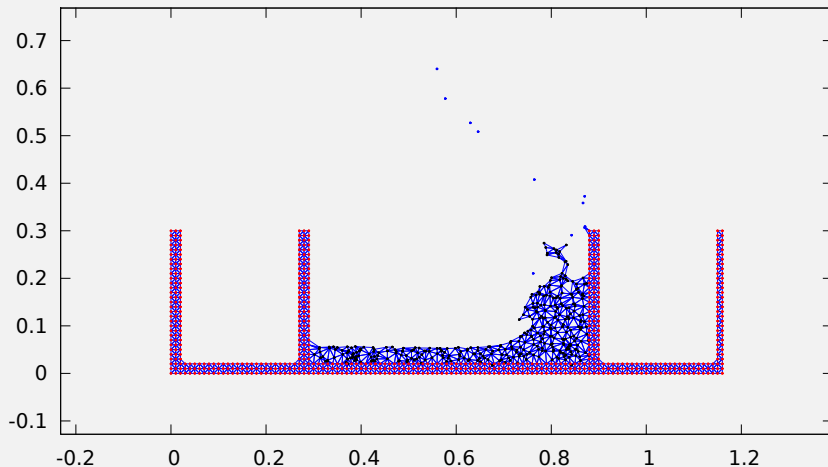
Water column collapse

Time = 0.64 sec, Color = default



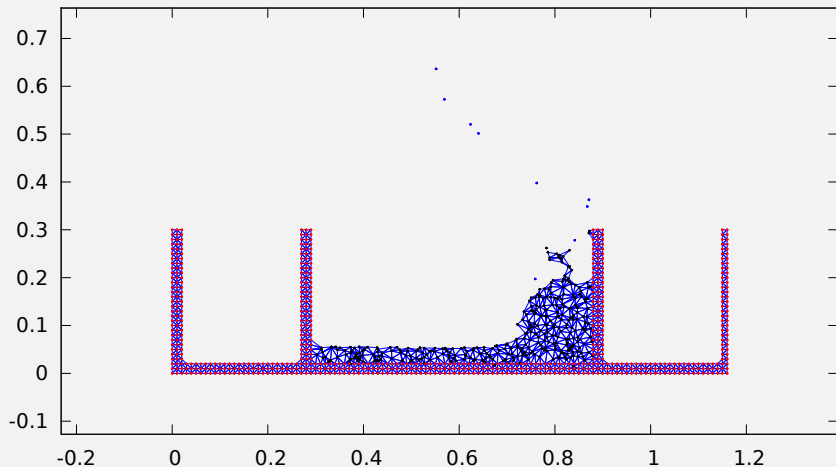
Water column collapse

Time = 0.65 sec, Color = default



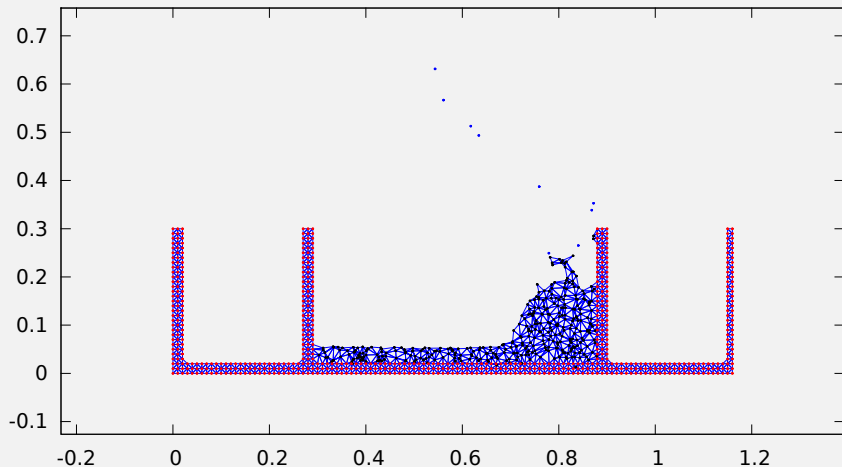
Water column collapse

Time = 0.66 sec, Color = default



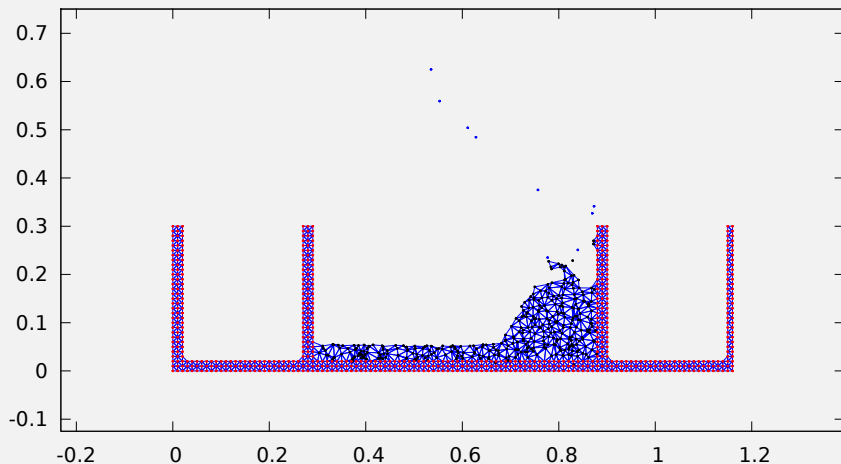
Water column collapse

Time = 0.67 sec, Color = default



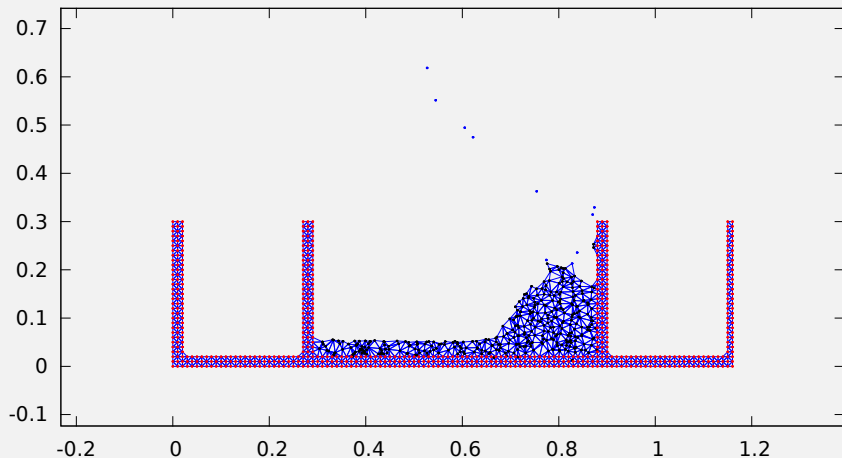
Water column collapse

Time = 0.68 sec, Color = default



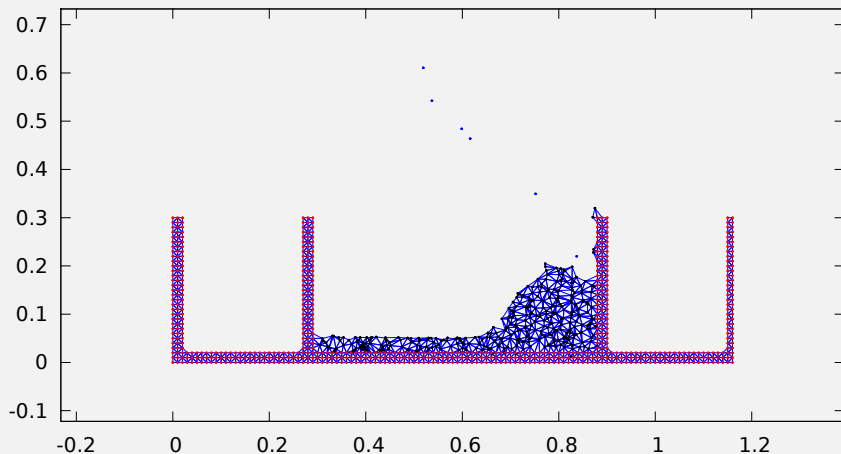
Water column collapse

Time = 0.6900000000000001 sec, Color = default



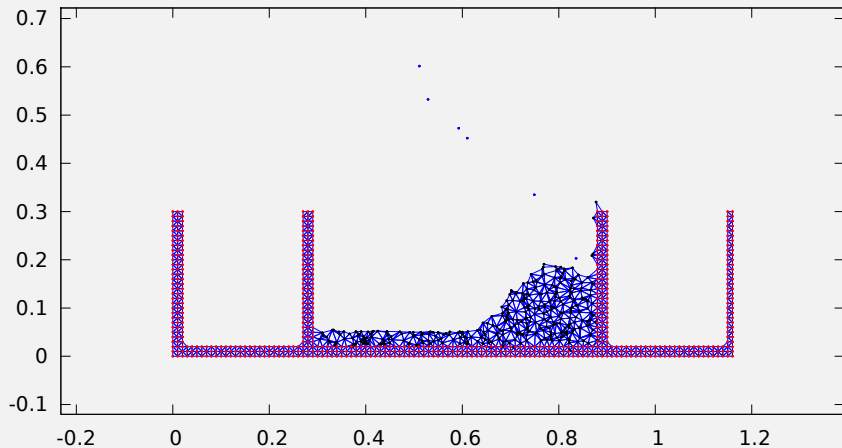
Water column collapse

Time = 0.7000000000000001 sec, Color = default



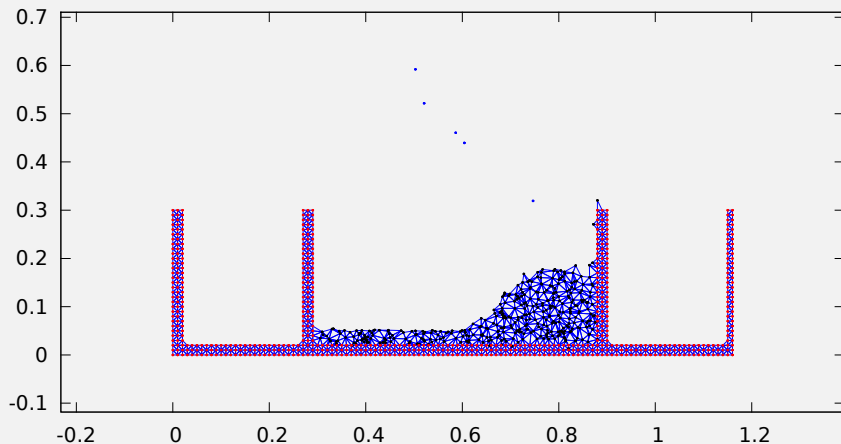
Water column collapse

Time = 0.71 sec, Color = default



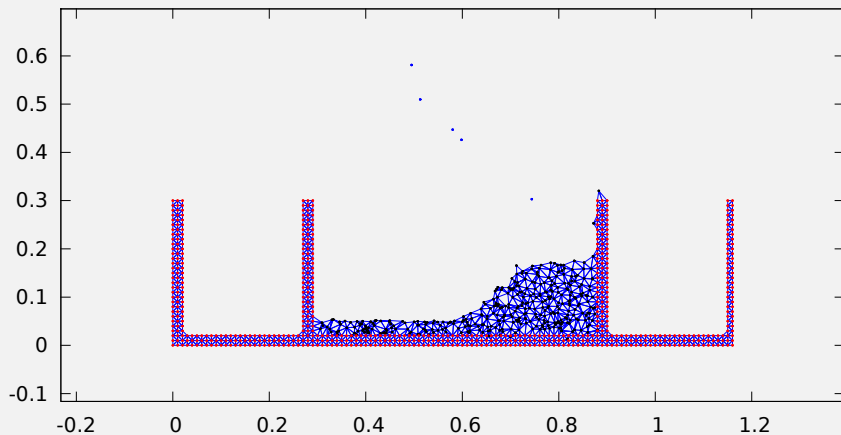
Water column collapse

Time = 0.72 sec, Color = default



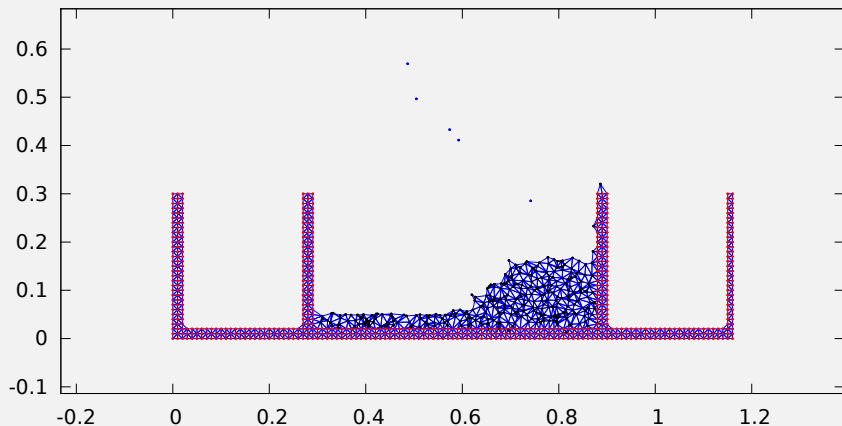
Water column collapse

Time = 0.73 sec, Color = default



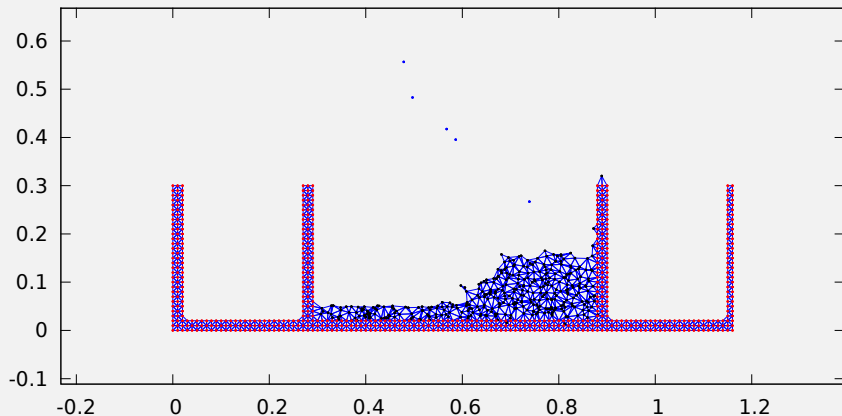
Water column collapse

Time = 0.74 sec, Color = default



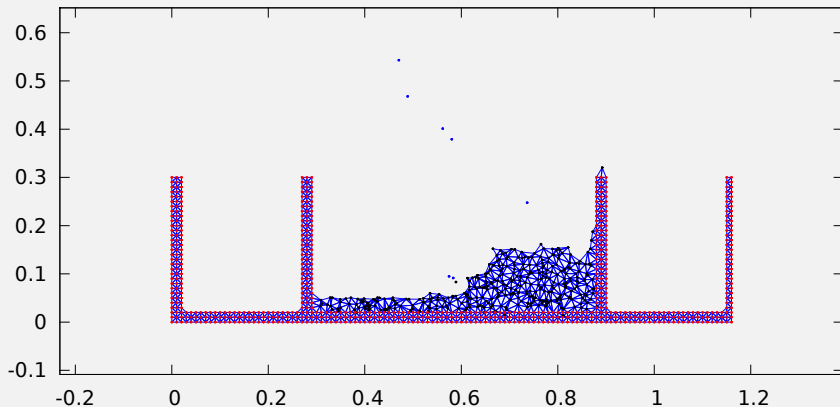
Water column collapse

Time = 0.75 sec, Color = default



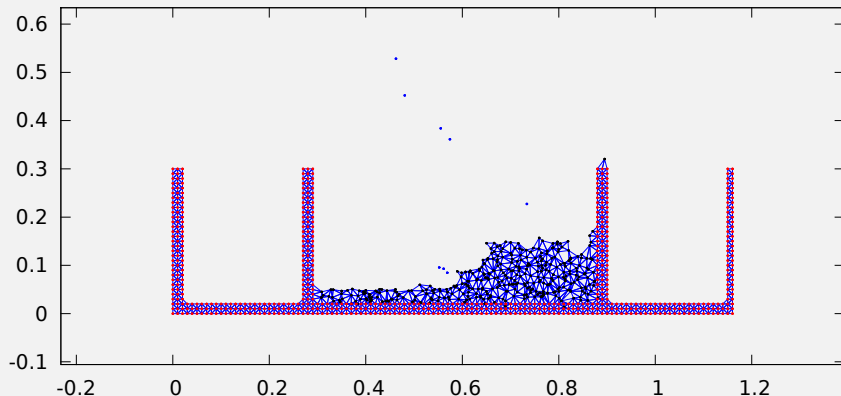
Water column collapse

Time = 0.76 sec, Color = default



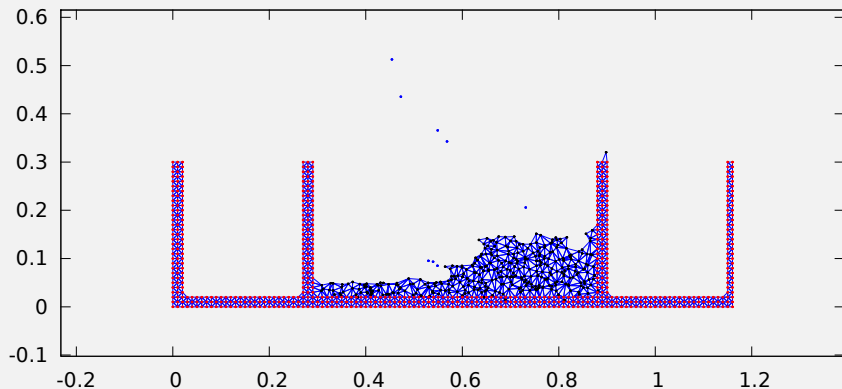
Water column collapse

Time = 0.77 sec, Color = default



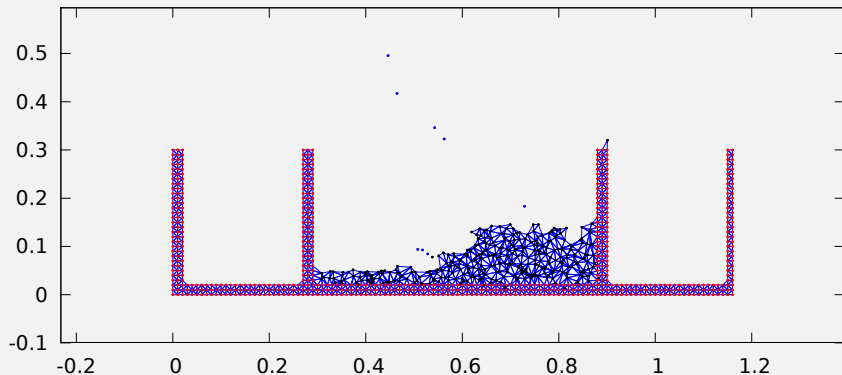
Water column collapse

Time = 0.78 sec, Color = default



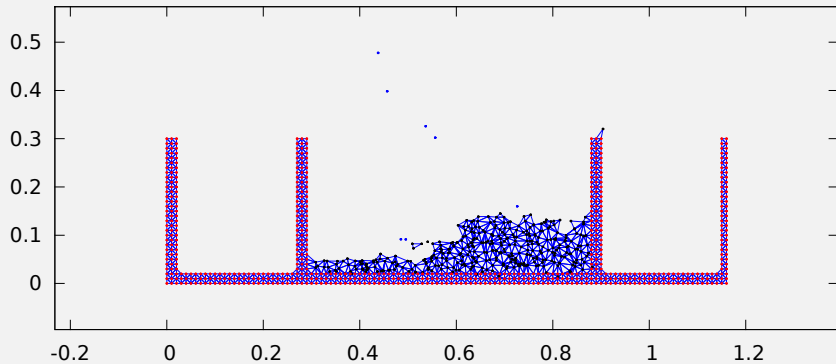
Water column collapse

Time = 0.79 sec, Color = default

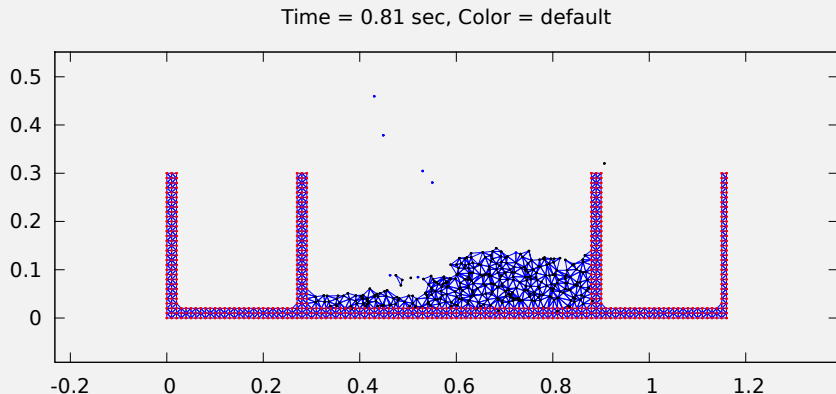


Water column collapse

Time = 0.8 sec, Color = default

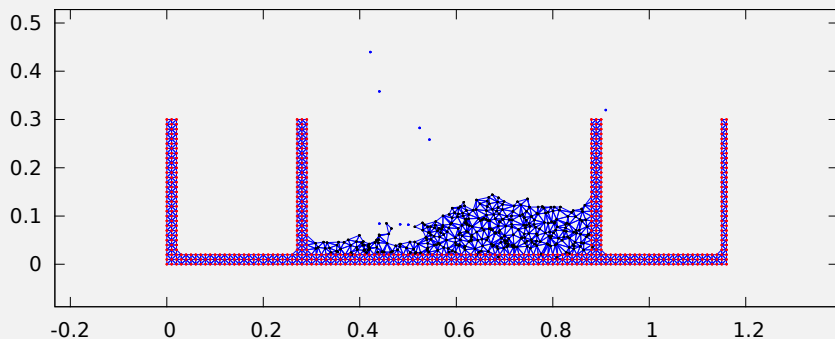


Water column collapse

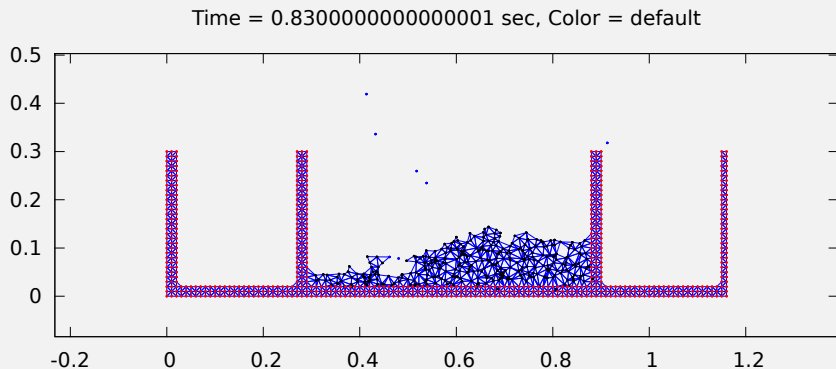


Water column collapse

Time = 0.8200000000000001 sec, Color = default

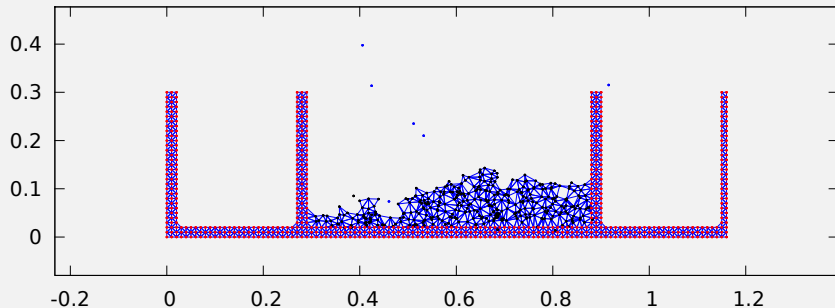


Water column collapse



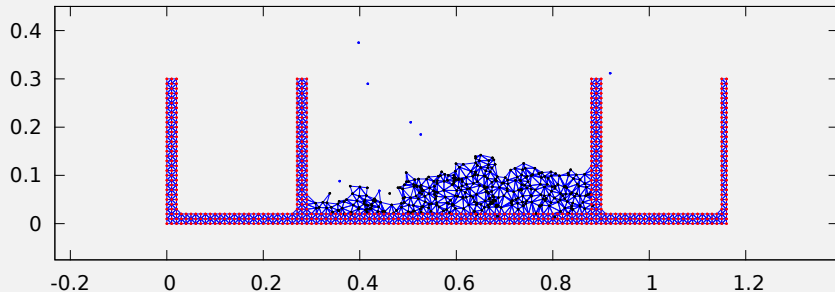
Water column collapse

Time = 0.84 sec, Color = default



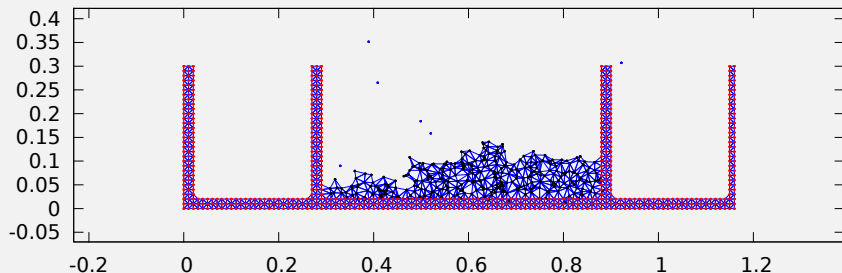
Water column collapse

Time = 0.85 sec, Color = default



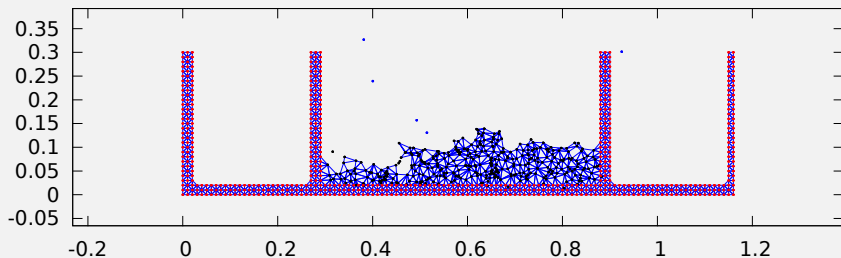
Water column collapse

Time = 0.86 sec, Color = default



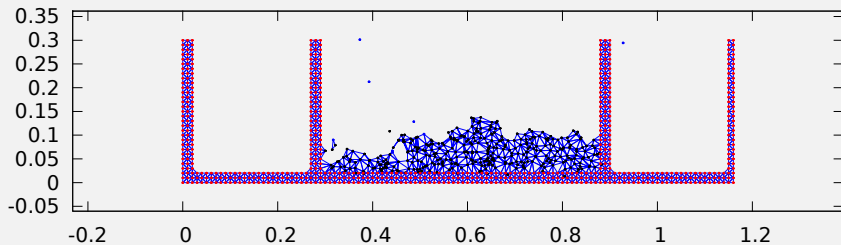
Water column collapse

Time = 0.87 sec, Color = default

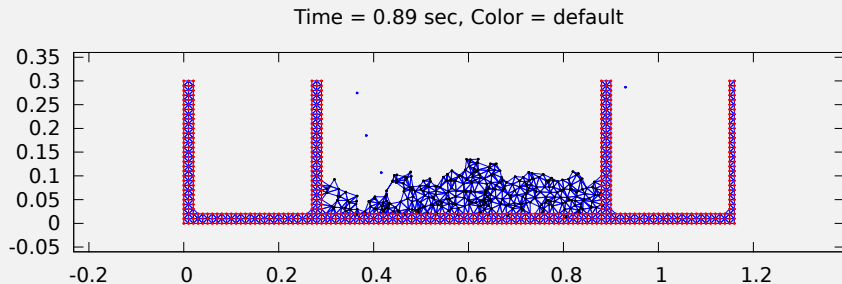


Water column collapse

Time = 0.88 sec, Color = default

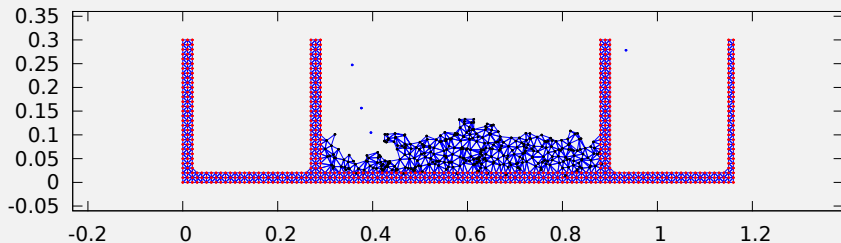


Water column collapse



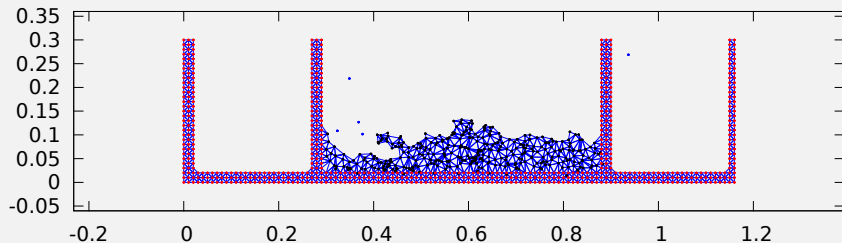
Water column collapse

Time = 0.9 sec, Color = default



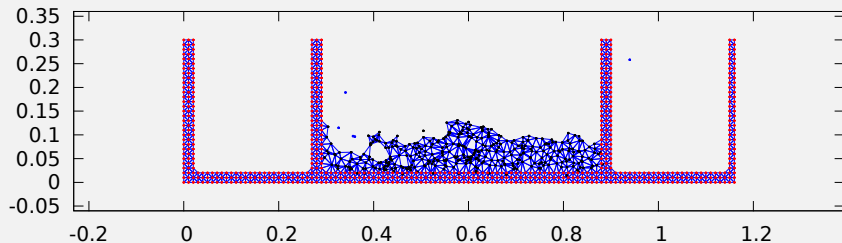
Water column collapse

Time = 0.91 sec, Color = default



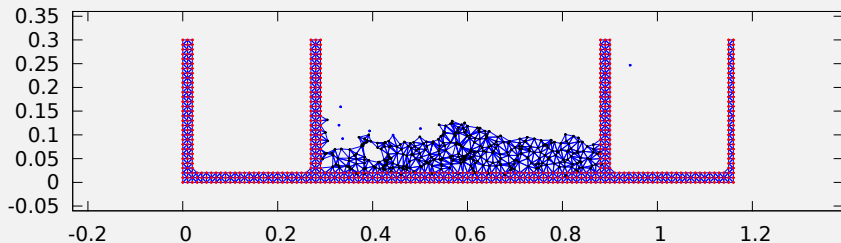
Water column collapse

Time = 0.92 sec, Color = default



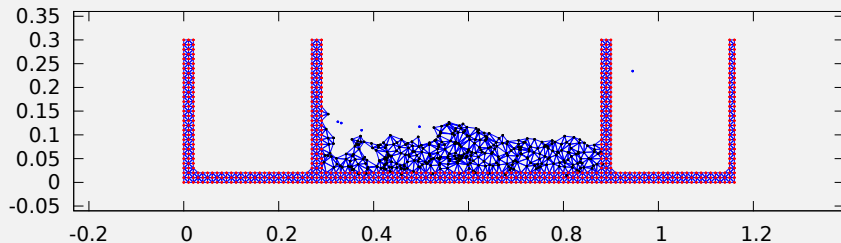
Water column collapse

Time = 0.93 sec, Color = default



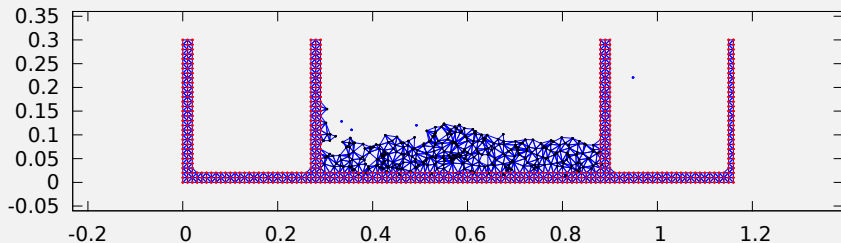
Water column collapse

Time = 0.9400000000000001 sec, Color = default



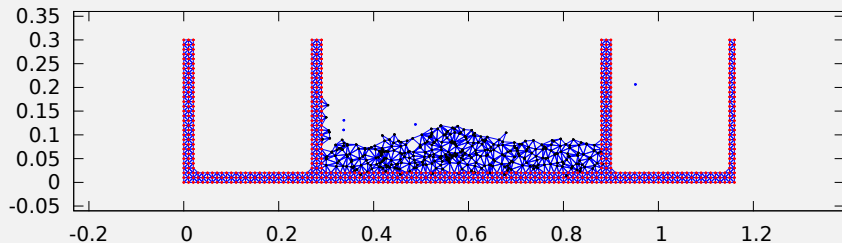
Water column collapse

Time = 0.9500000000000001 sec, Color = default



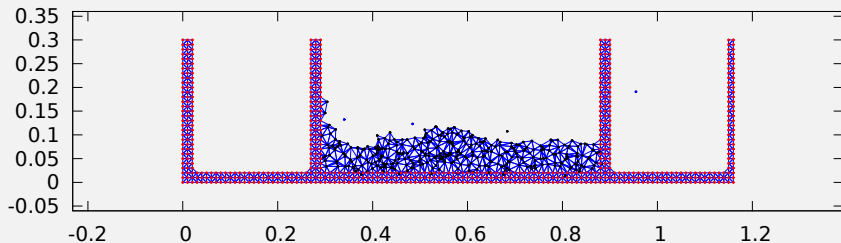
Water column collapse

Time = 0.96 sec, Color = default

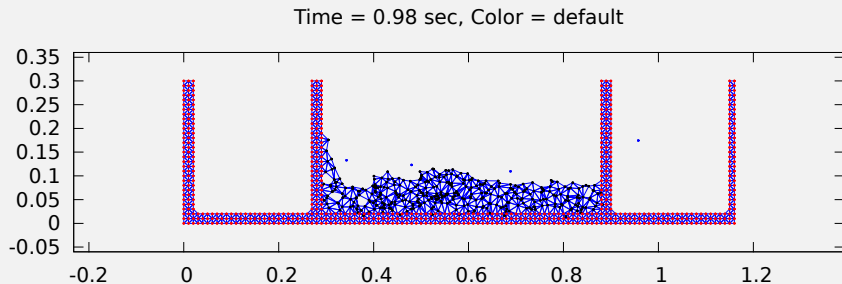


Water column collapse

Time = 0.97 sec, Color = default

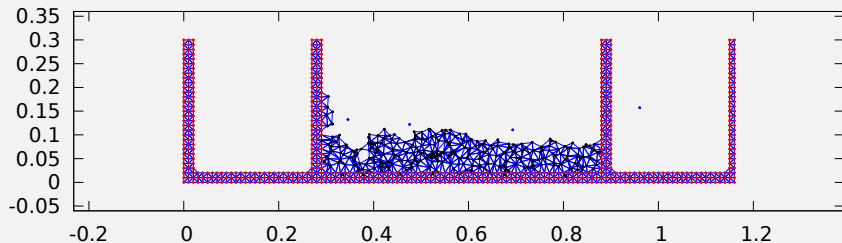


Water column collapse



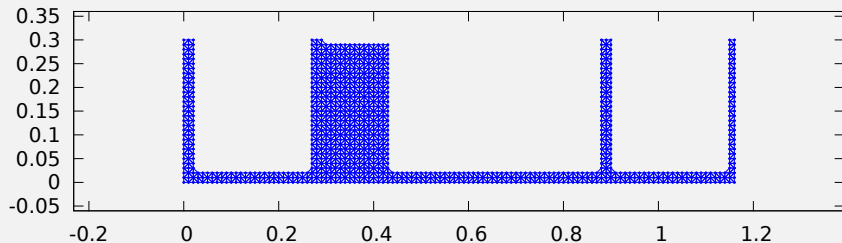
Water column collapse

Time = 0.99 sec, Color = default



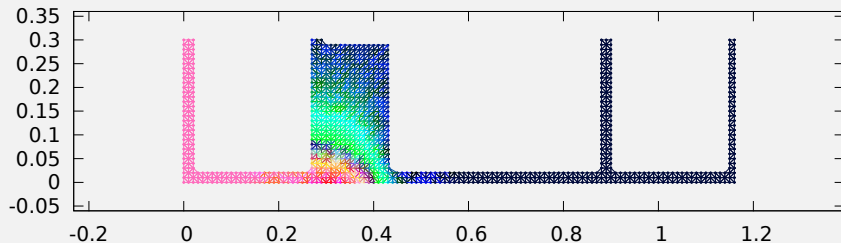
Water column collapse : pressure distribution

Time = 0.0 sec, Color = pressure (red = 0, blue = 0)



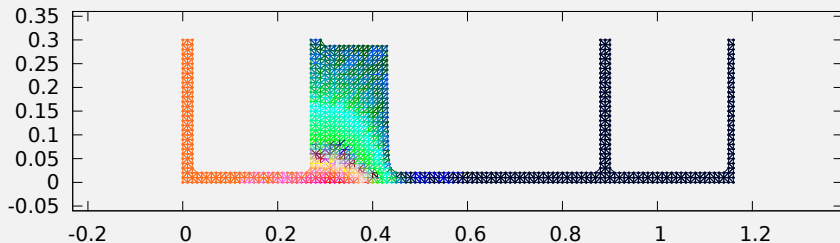
Water column collapse : pressure distribution

Time = 0.01 sec, Color = pressure (red = 1102.96, blue = -29.5204)



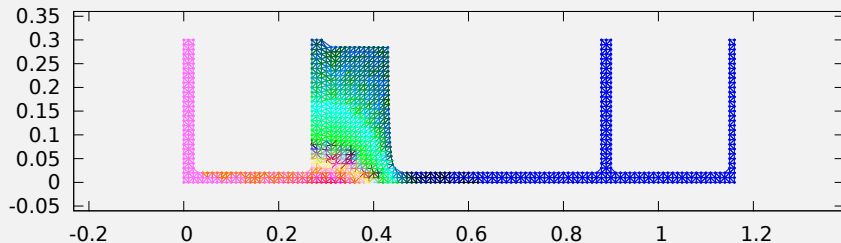
Water column collapse : pressure distribution

Time = 0.02 sec, Color = pressure (red = 1200.88, blue = -106.968)



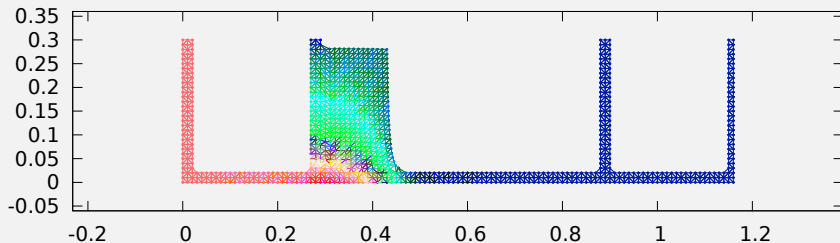
Water column collapse : pressure distribution

Time = 0.03 sec, Color = pressure (red = 1273.64, blue = -115.261)



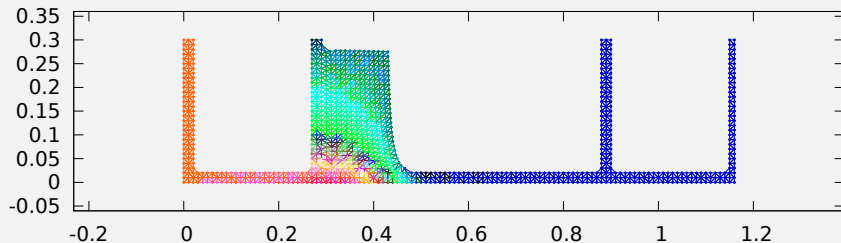
Water column collapse : pressure distribution

Time = 0.04 sec, Color = pressure (red = 1369.2, blue = -164.831)



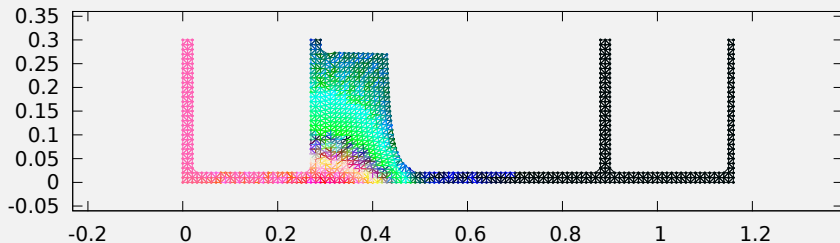
Water column collapse : pressure distribution

Time = 0.05 sec, Color = pressure (red = 1475.41, blue = -196.176)



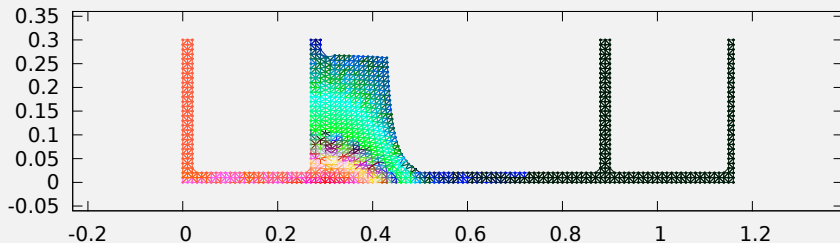
Water column collapse : pressure distribution

Time = 0.06 sec, Color = pressure (red = 1583.88, blue = -169.836)



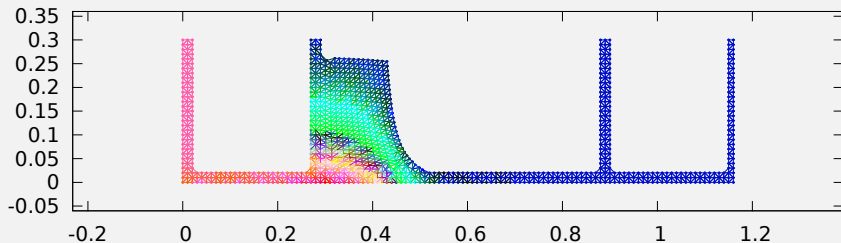
Water column collapse : pressure distribution

Time = 0.07 sec, Color = pressure (red = 1697.72, blue = -152.559)



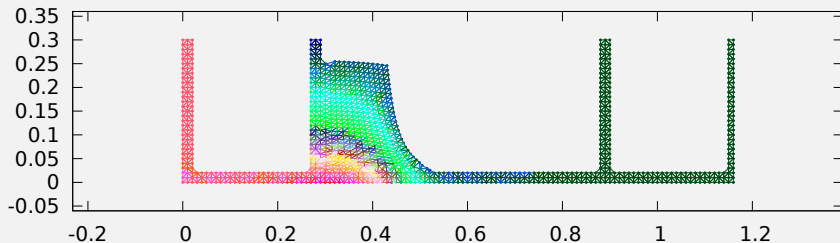
Water column collapse : pressure distribution

Time = 0.08 sec, Color = pressure (red = 1826.05, blue = -62.5572)



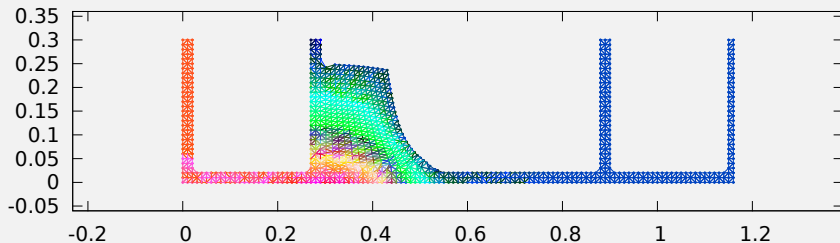
Water column collapse : pressure distribution

Time = 0.09 sec, Color = pressure (red = 1953.14, blue = -165.744)



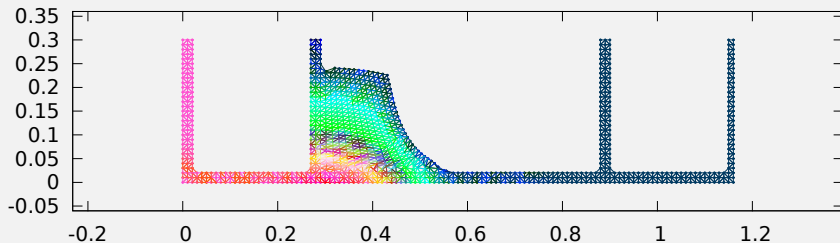
Water column collapse : pressure distribution

Time = 0.1 sec, Color = pressure (red = 2062.04, blue = -128.506)



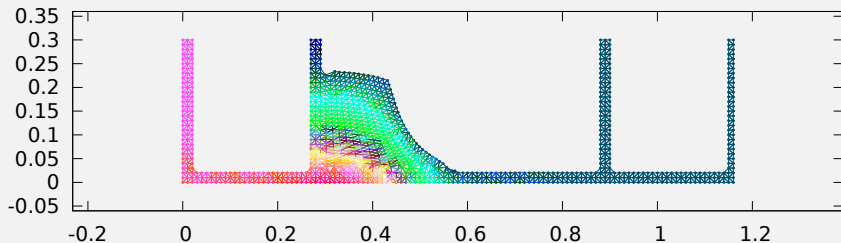
Water column collapse : pressure distribution

Time = 0.11 sec, Color = pressure (red = 2185.27, blue = -96.5532)



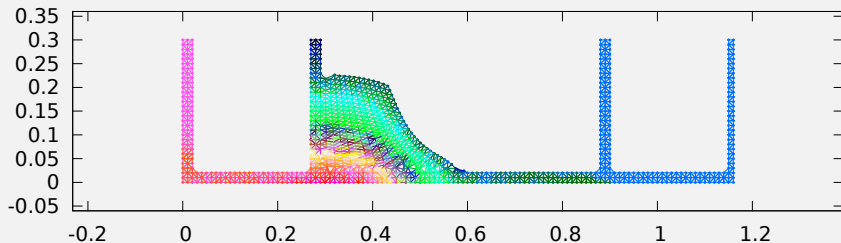
Water column collapse : pressure distribution

Time = 0.12 sec, Color = pressure (red = 2271.75, blue = -170.883)



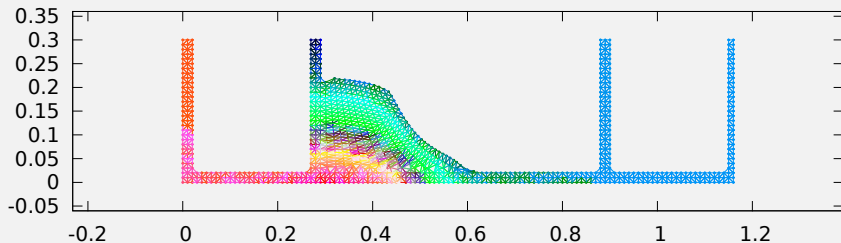
Water column collapse : pressure distribution

Time = 0.13 sec, Color = pressure (red = 2367.68, blue = -213.763)



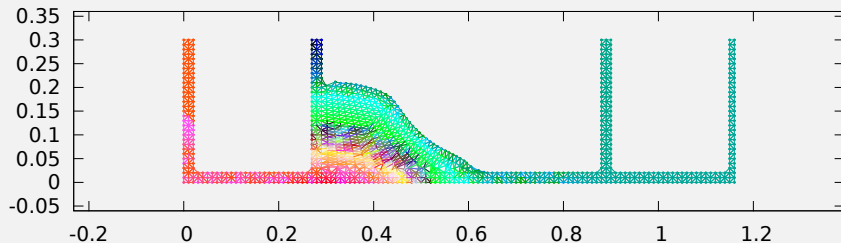
Water column collapse : pressure distribution

Time = 0.14 sec, Color = pressure (red = 2343.74, blue = -304.26)



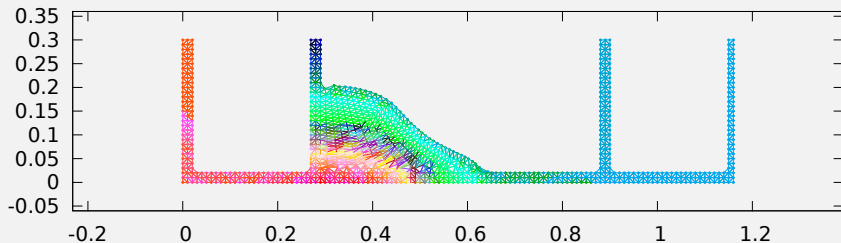
Water column collapse : pressure distribution

Time = 0.15 sec, Color = pressure (red = 2405.87, blue = -418.052)



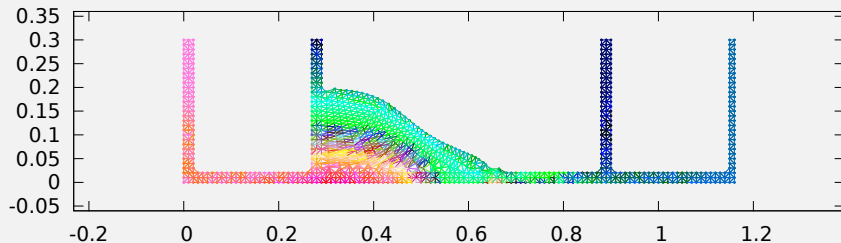
Water column collapse : pressure distribution

Time = 0.16 sec, Color = pressure (red = 2341.86, blue = -448.349)



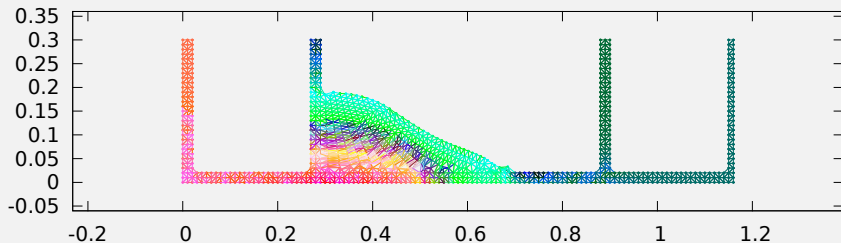
Water column collapse : pressure distribution

Time = 0.17 sec, Color = pressure (red = 2288.05, blue = -454.759)



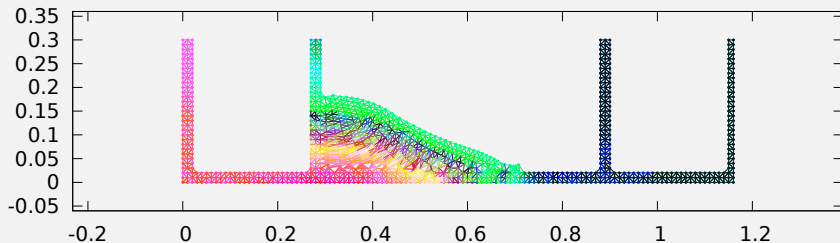
Water column collapse : pressure distribution

Time = 0.18 sec, Color = pressure (red = 2246.45, blue = -726.423)



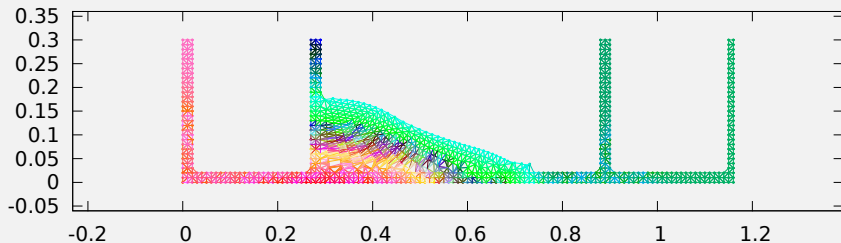
Water column collapse : pressure distribution

Time = 0.19 sec, Color = pressure (red = 2136.04, blue = -1236.06)



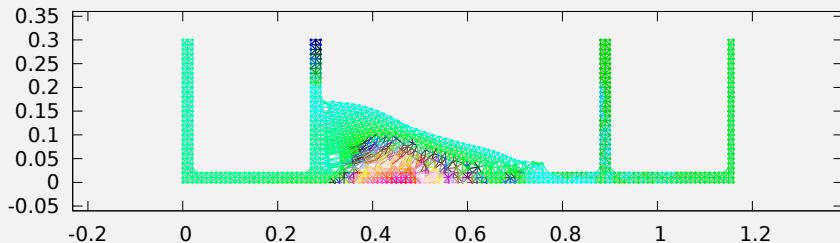
Water column collapse : pressure distribution

Time = 0.2 sec, Color = pressure (red = 1999.51, blue = -753.213)



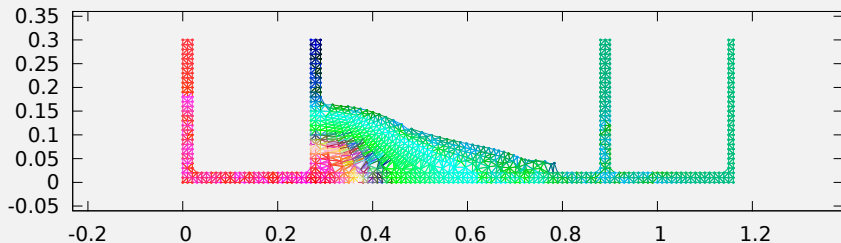
Water column collapse : pressure distribution

Time = 0.21 sec, Color = pressure (red = 1533.99, blue = -587.804)



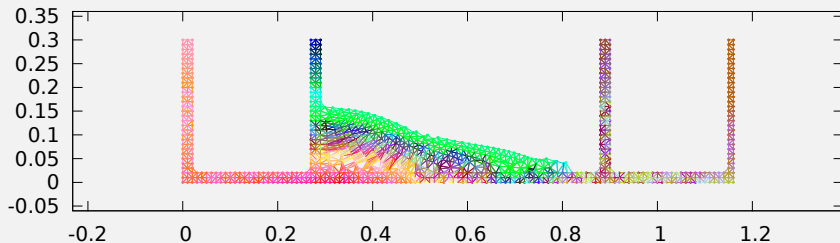
Water column collapse : pressure distribution

Time = 0.22 sec, Color = pressure (red = 5925.9, blue = -1073.56)



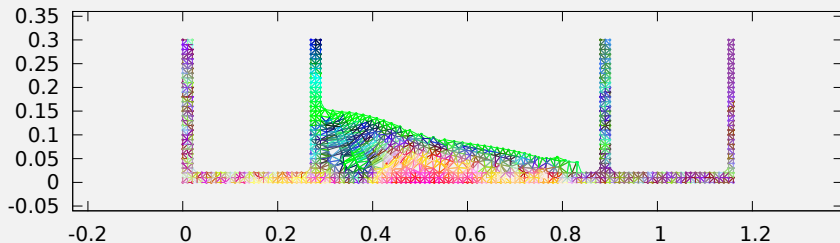
Water column collapse : pressure distribution

Time = 0.23 sec, Color = pressure (red = 1792.83, blue = -1067.49)



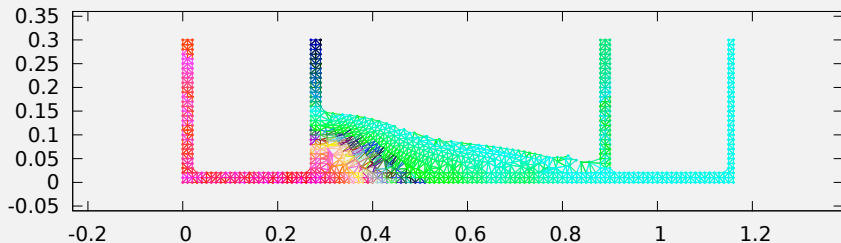
Water column collapse : pressure distribution

Time = 0.24 sec, Color = pressure (red = 1057.15, blue = -957.576)



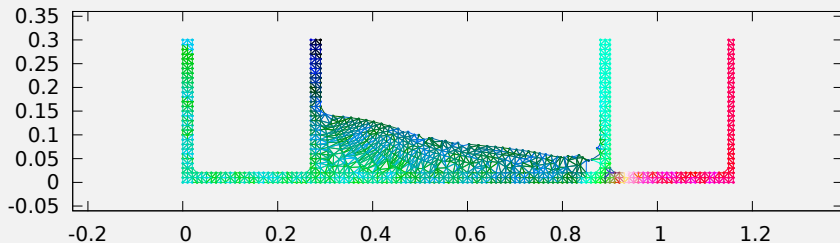
Water column collapse : pressure distribution

Time = 0.25 sec, Color = pressure (red = 4012.49, blue = -1288.49)



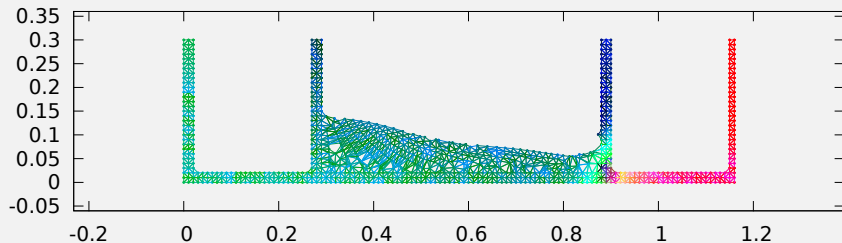
Water column collapse : pressure distribution

Time = 0.26 sec, Color = pressure (red = 11783.5, blue = -1426.04)



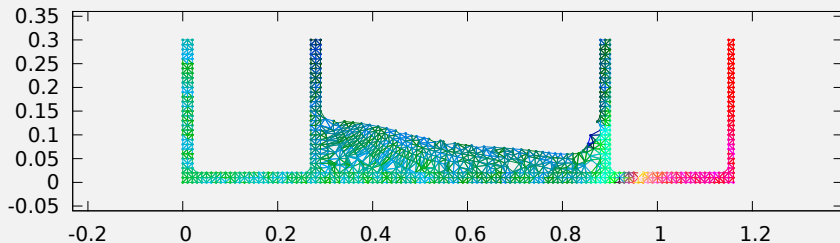
Water column collapse : pressure distribution

Time = 0.27 sec, Color = pressure (red = 22183.6, blue = -3059.62)



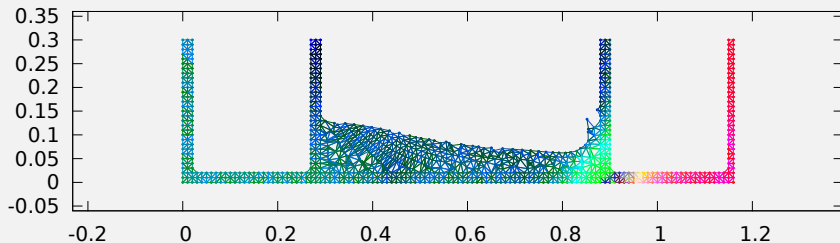
Water column collapse : pressure distribution

Time = 0.28 sec, Color = pressure (red = 16134.7, blue = -2267.34)



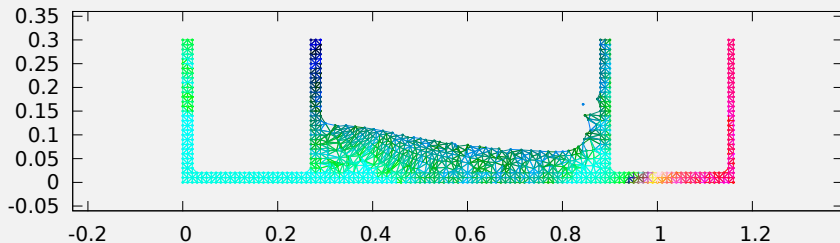
Water column collapse : pressure distribution

Time = 0.29 sec, Color = pressure (red = 19109, blue = -1619.16)



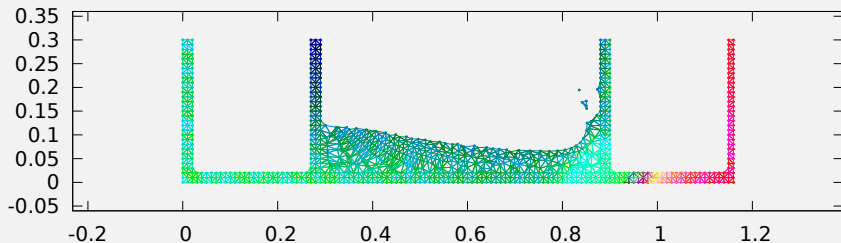
Water column collapse : pressure distribution

Time = 0.3 sec, Color = pressure (red = 11661.9, blue = -1758.07)



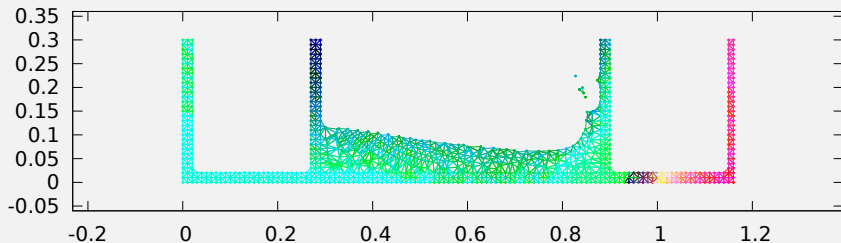
Water column collapse : pressure distribution

Time = 0.31 sec, Color = pressure (red = 11805.6, blue = -1758.52)



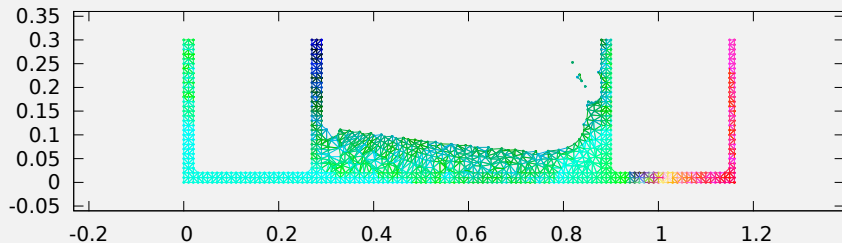
Water column collapse : pressure distribution

Time = 0.32 sec, Color = pressure (red = 9923.7, blue = -2074.36)



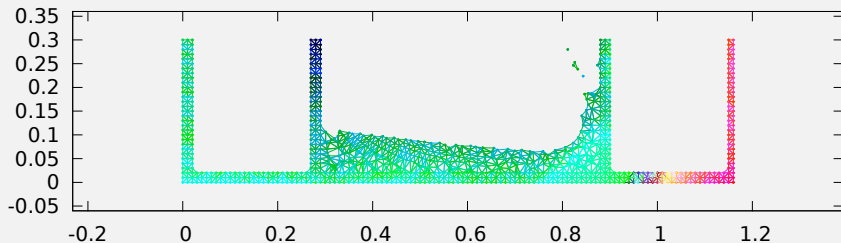
Water column collapse : pressure distribution

Time = 0.33 sec, Color = pressure (red = 10050.5, blue = -2021.12)



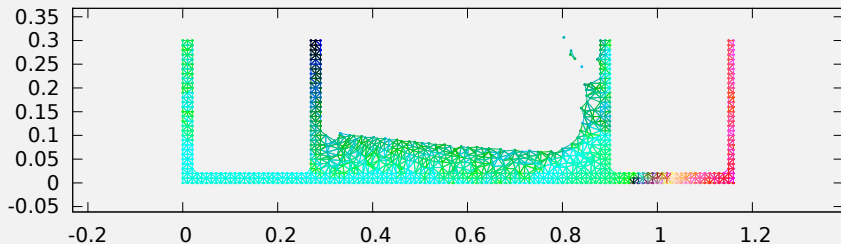
Water column collapse : pressure distribution

Time = 0.34 sec, Color = pressure (red = 9346.59, blue = -1909.95)



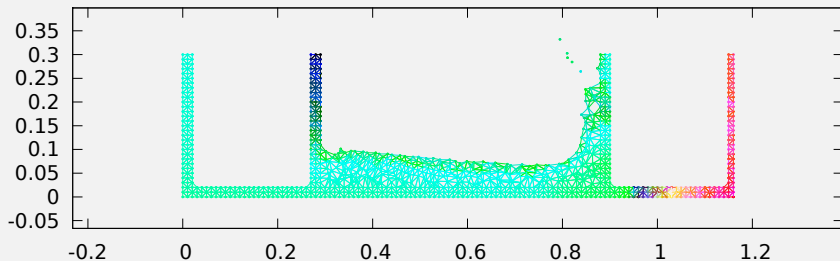
Water column collapse : pressure distribution

Time = 0.35000000000000003 sec, Color = pressure (red = 8518.41, blue = -1876.41)



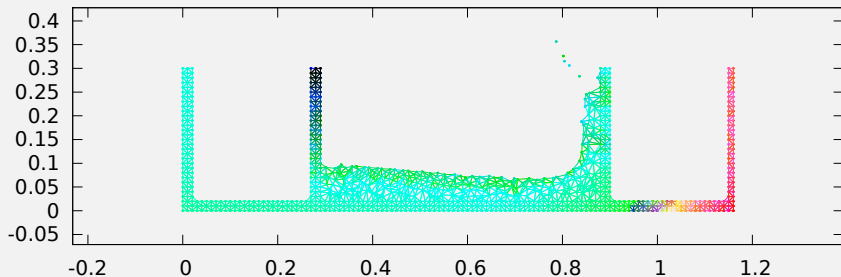
Water column collapse : pressure distribution

Time = 0.36 sec, Color = pressure (red = 7849.56, blue = -2324.51)



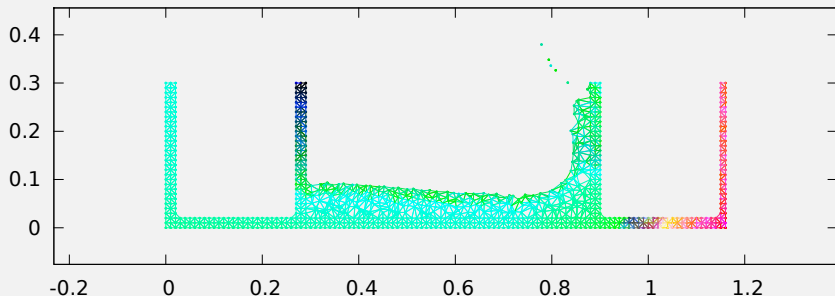
Water column collapse : pressure distribution

Time = 0.37 sec, Color = pressure (red = 7566.75, blue = -2136.51)



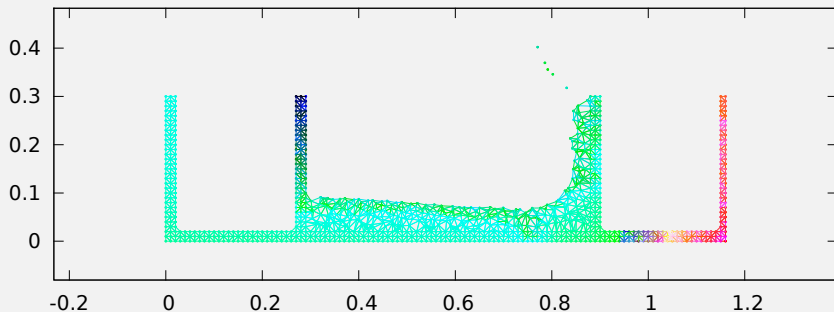
Water column collapse : pressure distribution

Time = 0.38 sec, Color = pressure (red = 7484.74, blue = -2186.44)

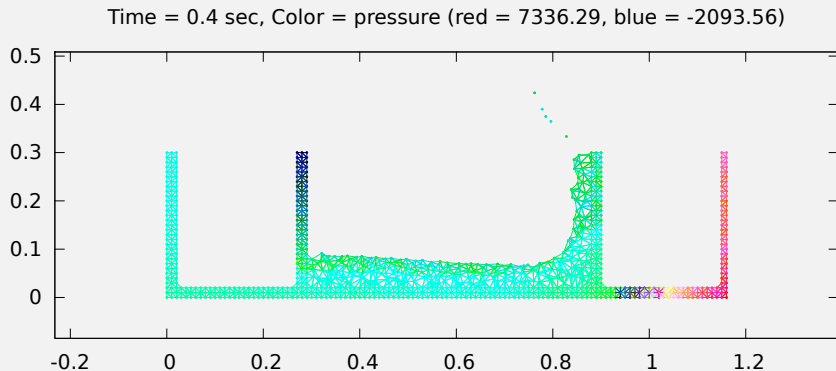


Water column collapse : pressure distribution

Time = 0.39 sec, Color = pressure (red = 7291.57, blue = -2132.48)

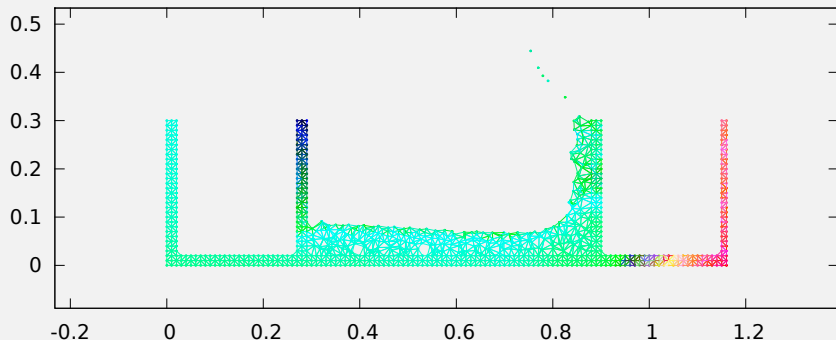


Water column collapse : pressure distribution



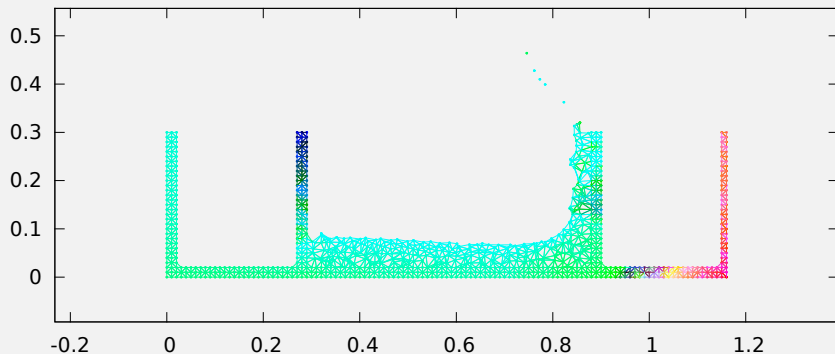
Water column collapse : pressure distribution

Time = 0.41000000000000003 sec, Color = pressure (red = 6816.77, blue = -2134.82)



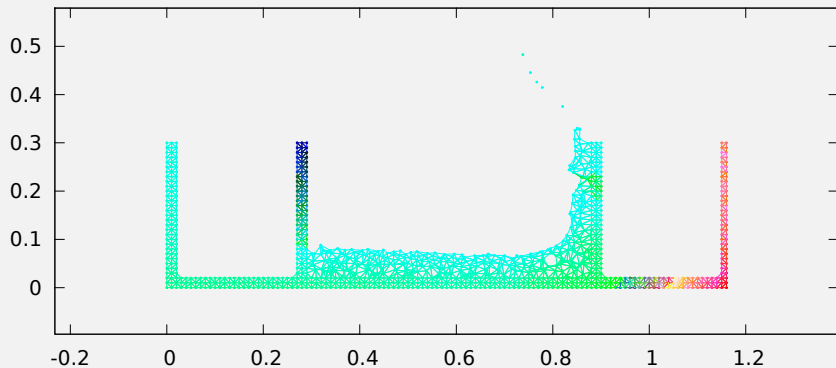
Water column collapse : pressure distribution

Time = 0.42 sec, Color = pressure (red = 6547.08, blue = -2218.52)

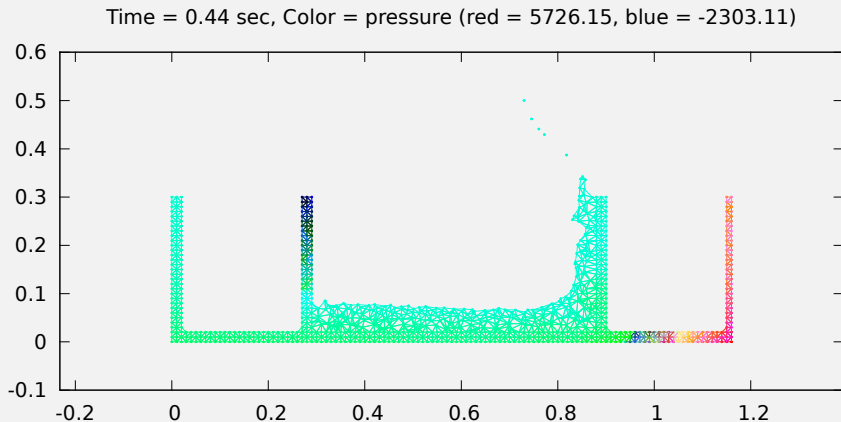


Water column collapse : pressure distribution

Time = 0.43 sec, Color = pressure (red = 6308.81, blue = -2200.94)

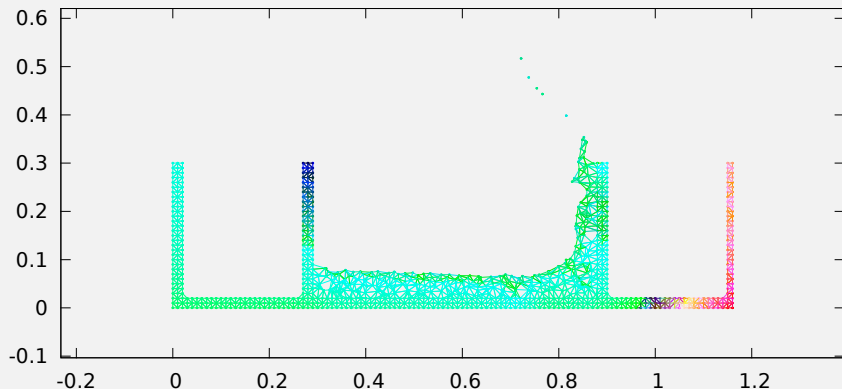


Water column collapse : pressure distribution



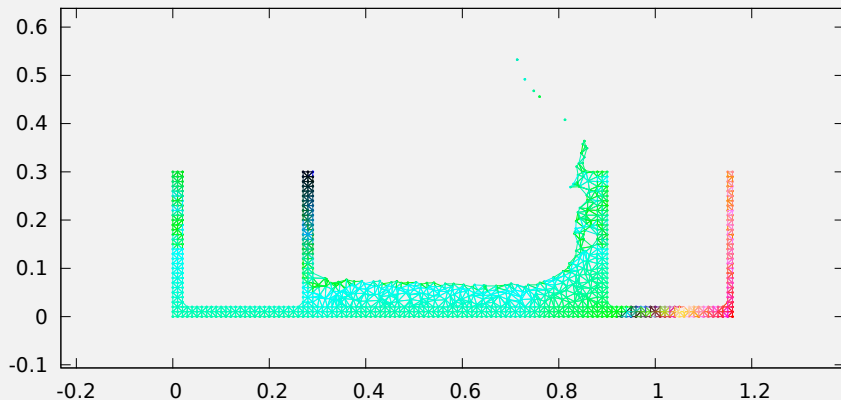
Water column collapse : pressure distribution

Time = 0.45 sec, Color = pressure (red = 5388.83, blue = -1600.64)



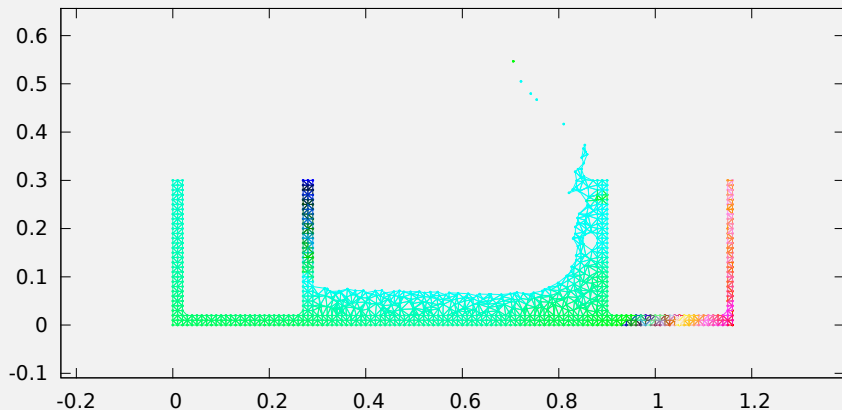
Water column collapse : pressure distribution

Time = 0.46 sec, Color = pressure (red = 5973.55, blue = -1903.91)



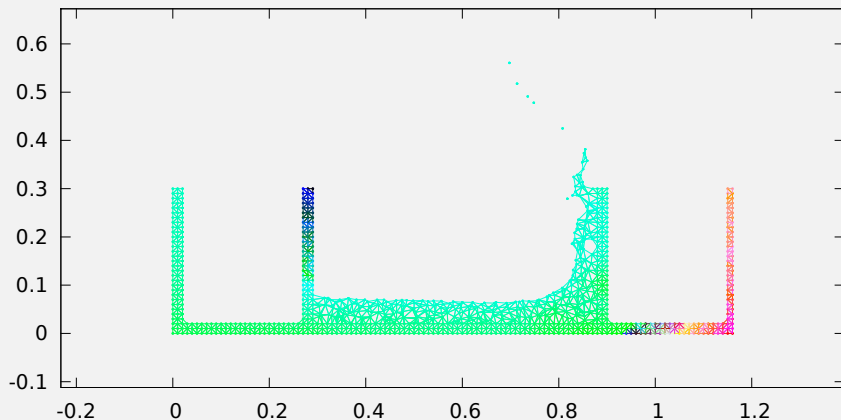
Water column collapse : pressure distribution

Time = 0.47000000000000003 sec, Color = pressure (red = 5618.24, blue = -1924.57)



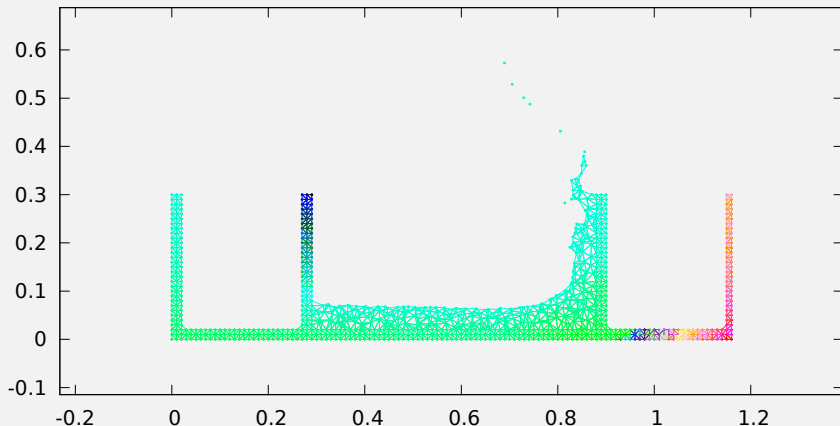
Water column collapse : pressure distribution

Time = 0.48 sec, Color = pressure (red = 5259.63, blue = -2152.24)

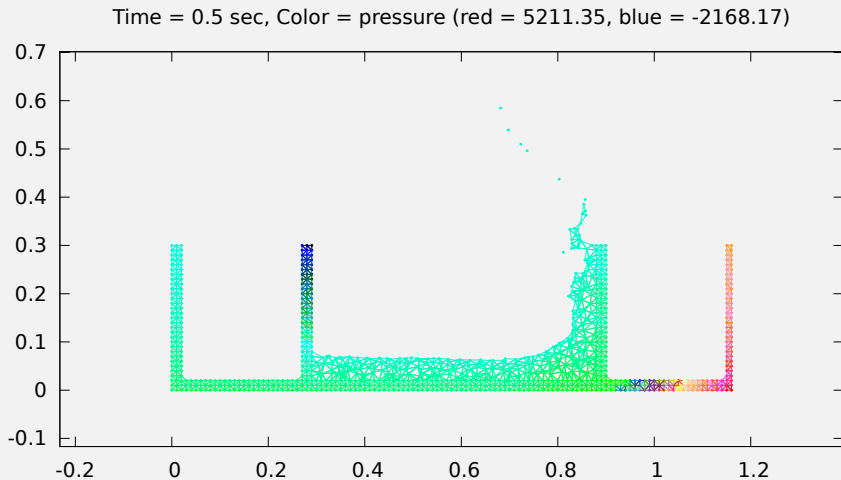


Water column collapse : pressure distribution

Time = 0.49 sec, Color = pressure (red = 5239.28, blue = -2156.92)

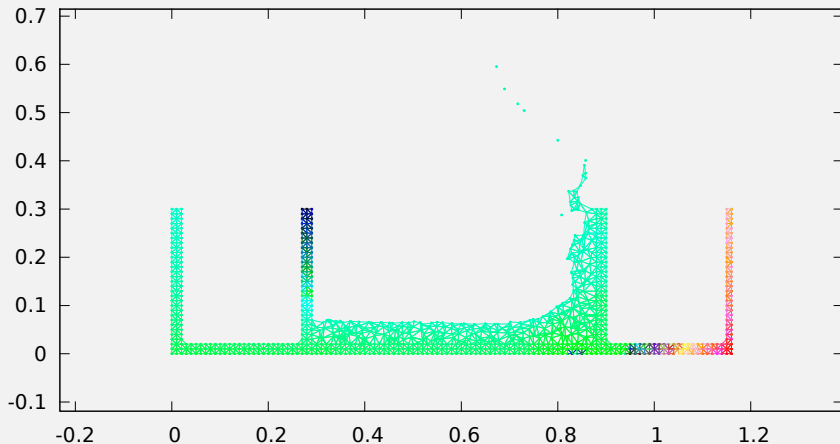


Water column collapse : pressure distribution



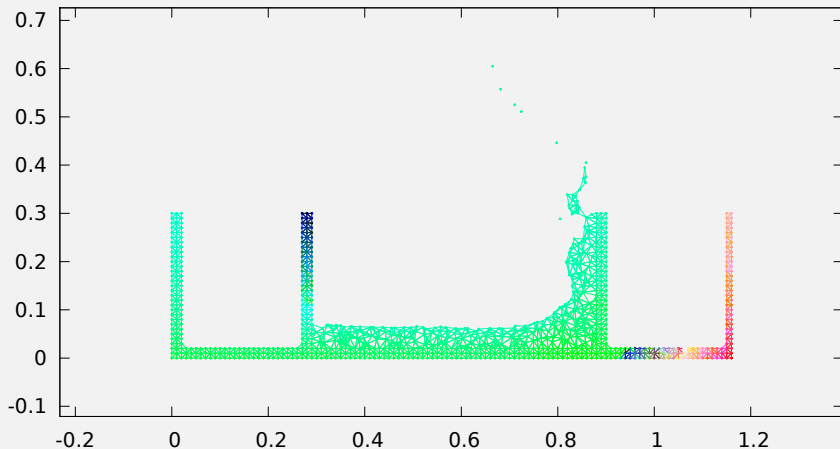
Water column collapse : pressure distribution

Time = 0.51 sec, Color = pressure (red = 4815.69, blue = -2207.34)



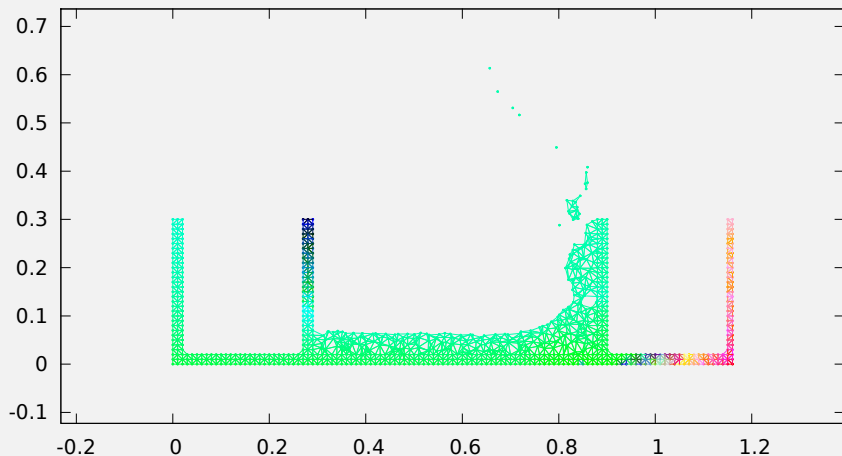
Water column collapse : pressure distribution

Time = 0.52 sec, Color = pressure (red = 4745.29, blue = -2286.91)



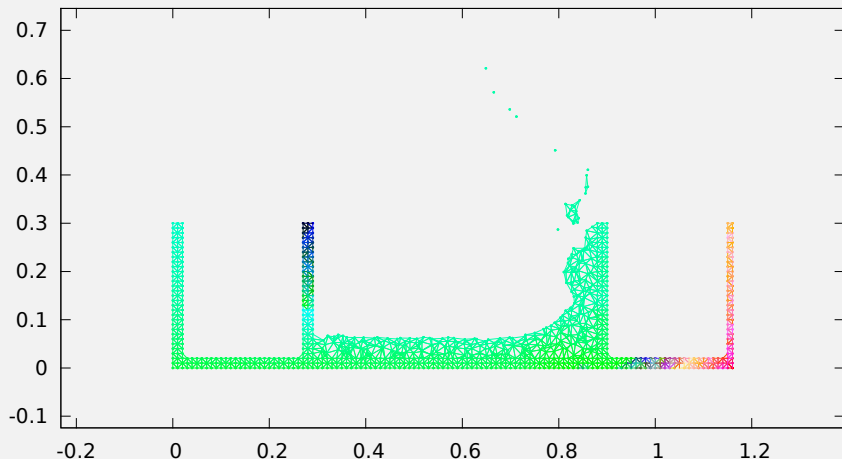
Water column collapse : pressure distribution

Time = 0.53 sec, Color = pressure (red = 4658.03, blue = -2298.47)



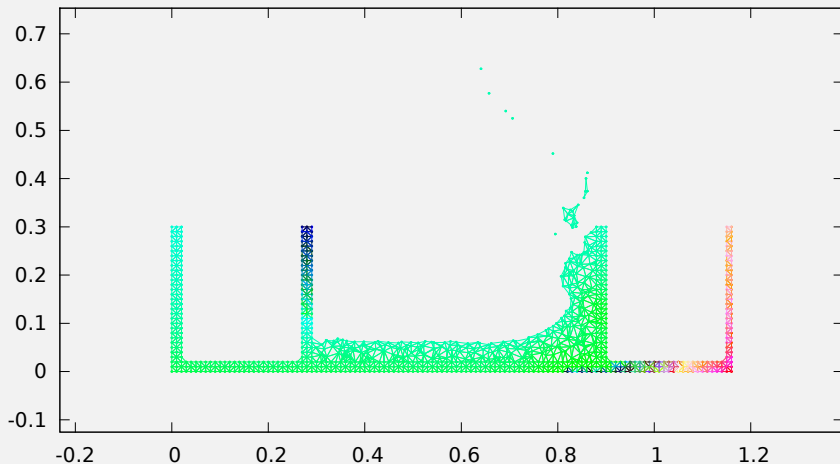
Water column collapse : pressure distribution

Time = 0.54 sec, Color = pressure (red = 4563.63, blue = -2290.13)



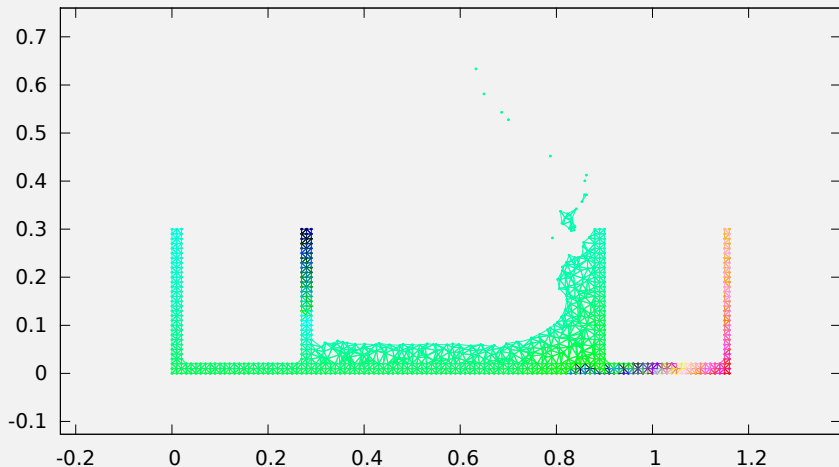
Water column collapse : pressure distribution

Time = 0.55 sec, Color = pressure (red = 4737.83, blue = -2298.14)



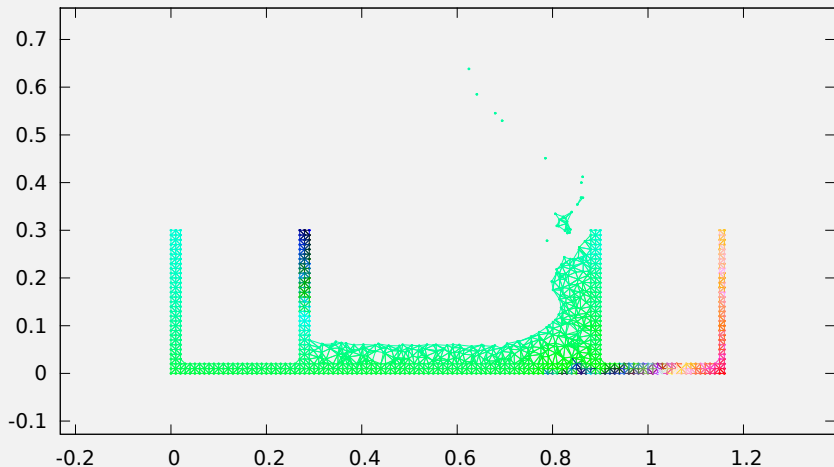
Water column collapse : pressure distribution

Time = 0.56 sec, Color = pressure (red = 4709.26, blue = -2297.82)



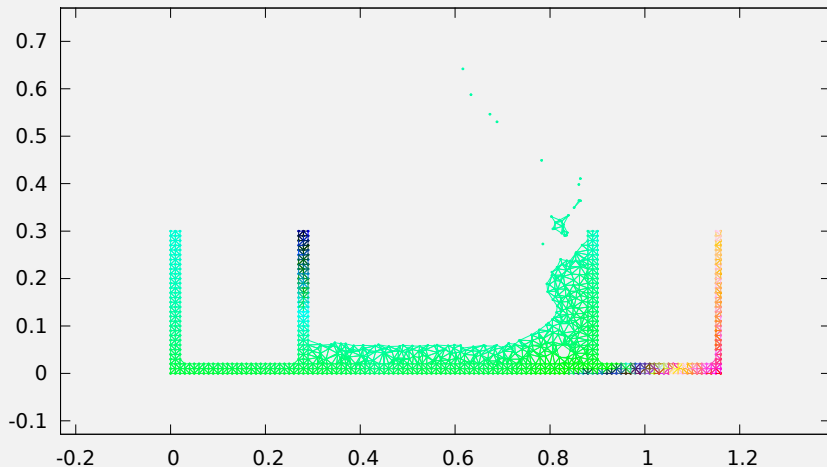
Water column collapse : pressure distribution

Time = 0.5700000000000001 sec, Color = pressure (red = 4424.12, blue = -2300.15)



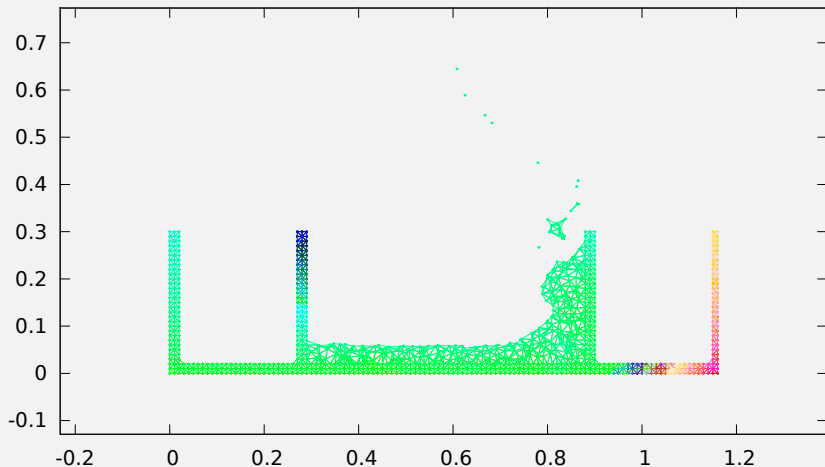
Water column collapse : pressure distribution

Time = 0.58 sec, Color = pressure (red = 4274.07, blue = -2177.58)



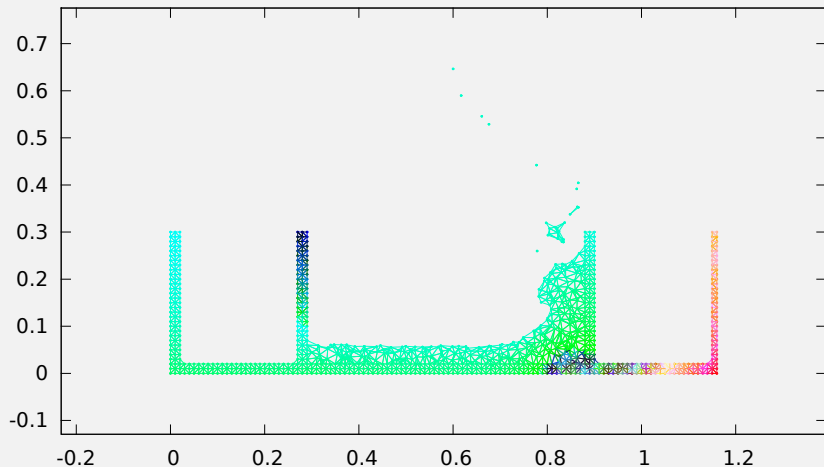
Water column collapse : pressure distribution

Time = 0.59 sec, Color = pressure (red = 3894.7, blue = -2197.36)



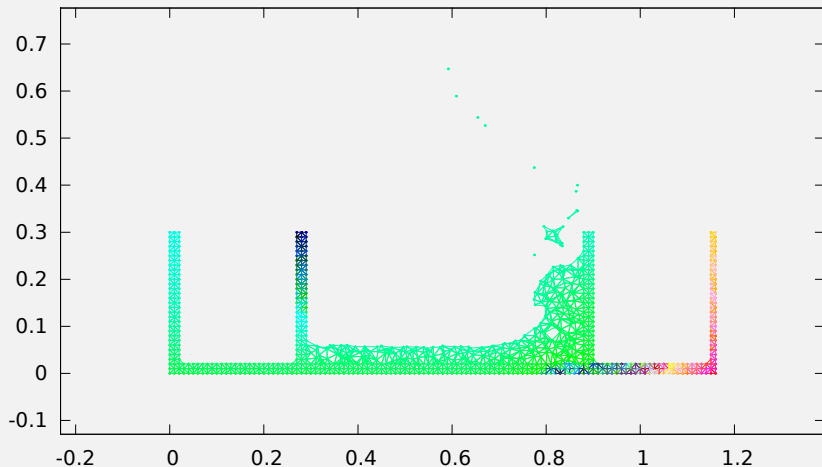
Water column collapse : pressure distribution

Time = 0.6 sec, Color = pressure (red = 5130.32, blue = -2224.69)



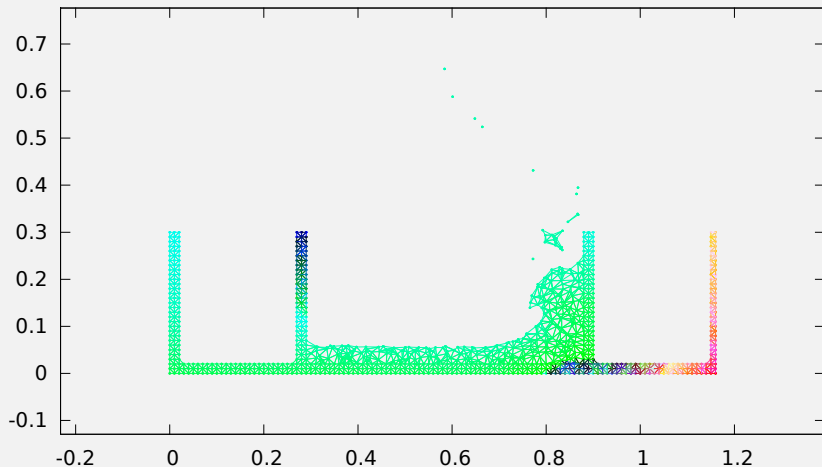
Water column collapse : pressure distribution

Time = 0.61 sec, Color = pressure (red = 4376.03, blue = -2221.9)



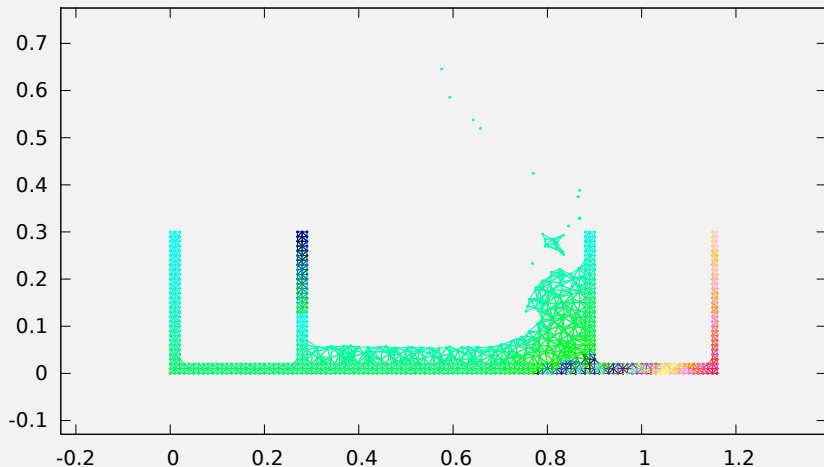
Water column collapse : pressure distribution

Time = 0.62 sec, Color = pressure (red = 4441.6, blue = -2186.75)



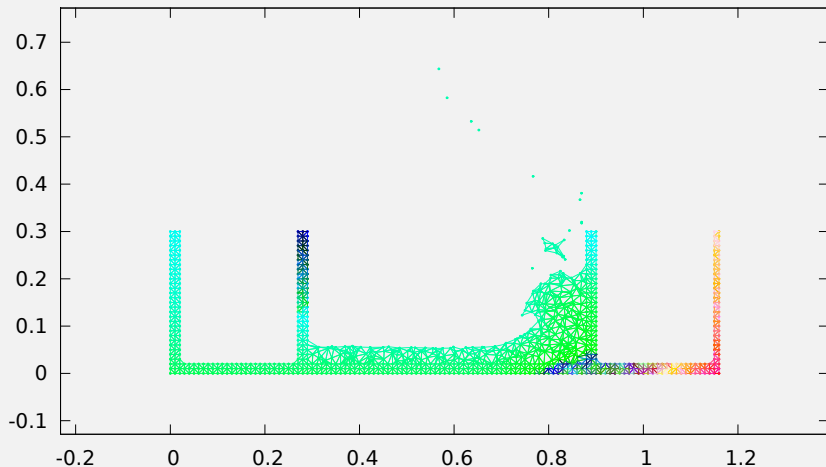
Water column collapse : pressure distribution

Time = 0.63 sec, Color = pressure (red = 4576.19, blue = -2186.77)



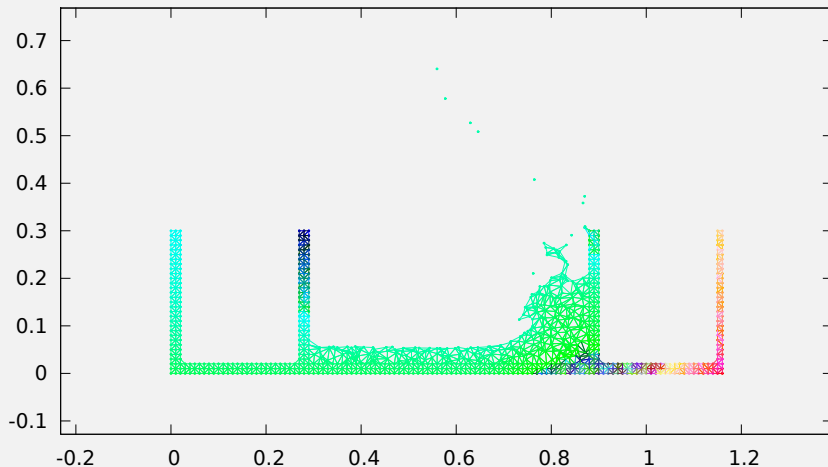
Water column collapse : pressure distribution

Time = 0.64 sec, Color = pressure (red = 4515.59, blue = -2206.6)



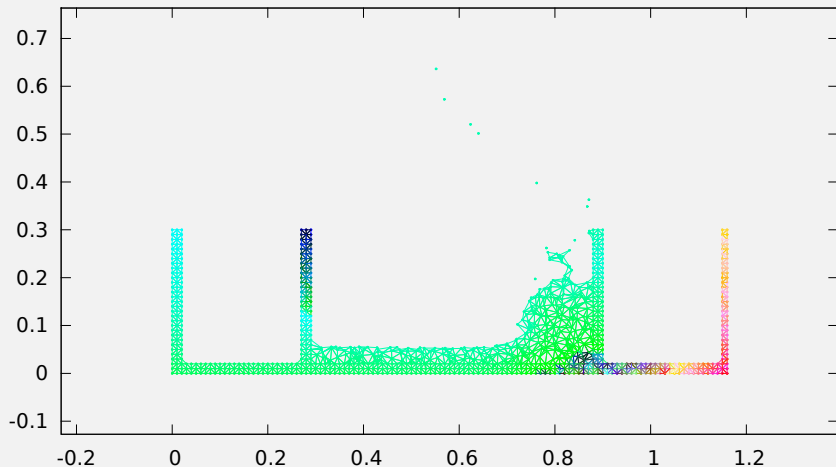
Water column collapse : pressure distribution

Time = 0.65 sec, Color = pressure (red = 4570.2, blue = -2250.04)



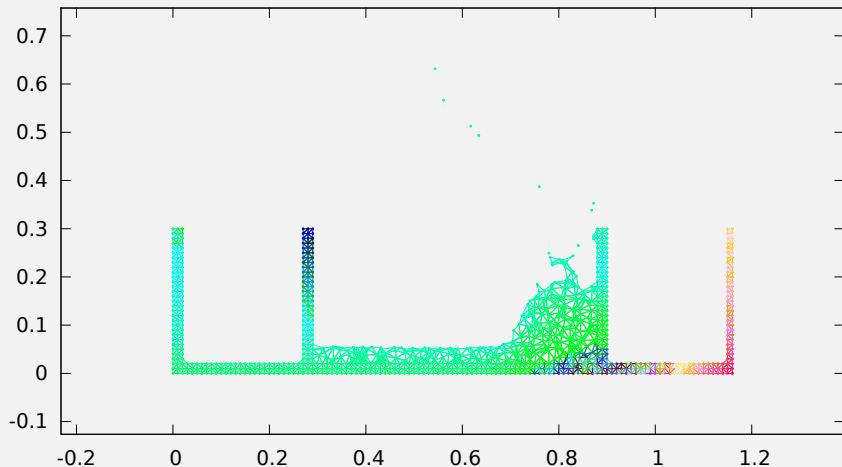
Water column collapse : pressure distribution

Time = 0.66 sec, Color = pressure (red = 4581.2, blue = -2208.92)



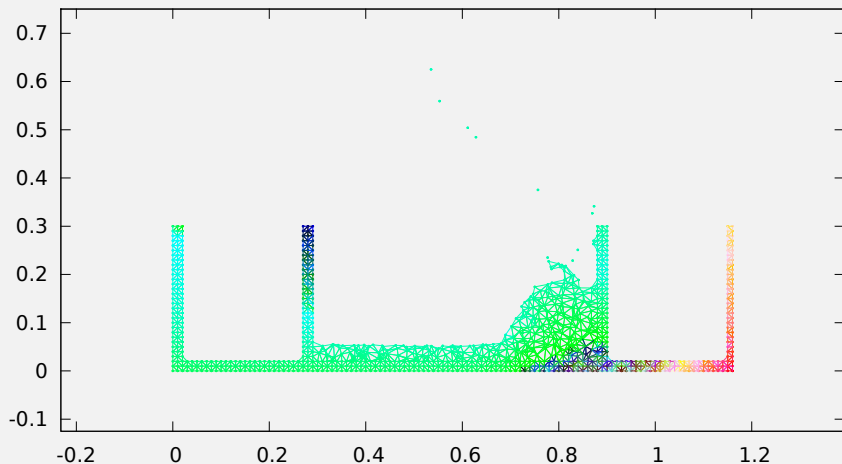
Water column collapse : pressure distribution

Time = 0.67 sec, Color = pressure (red = 4680.22, blue = -2182.7)



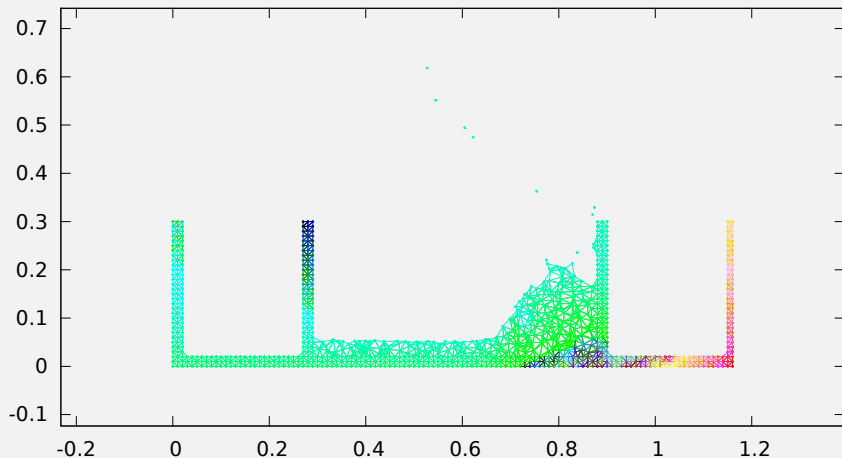
Water column collapse : pressure distribution

Time = 0.68 sec, Color = pressure (red = 4540.25, blue = -2195.05)



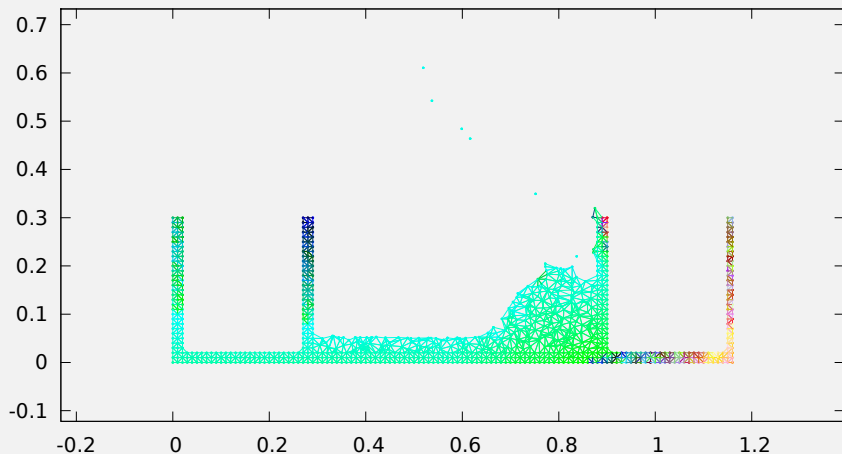
Water column collapse : pressure distribution

Time = 0.6900000000000001 sec, Color = pressure (red = 4809.25, blue = -2148.8)



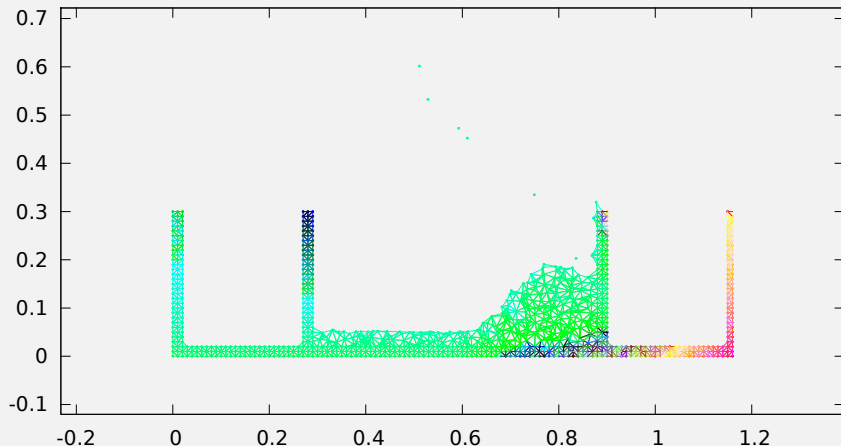
Water column collapse : pressure distribution

Time = 0.7000000000000001 sec, Color = pressure (red = 5653.29, blue = -2121.88)



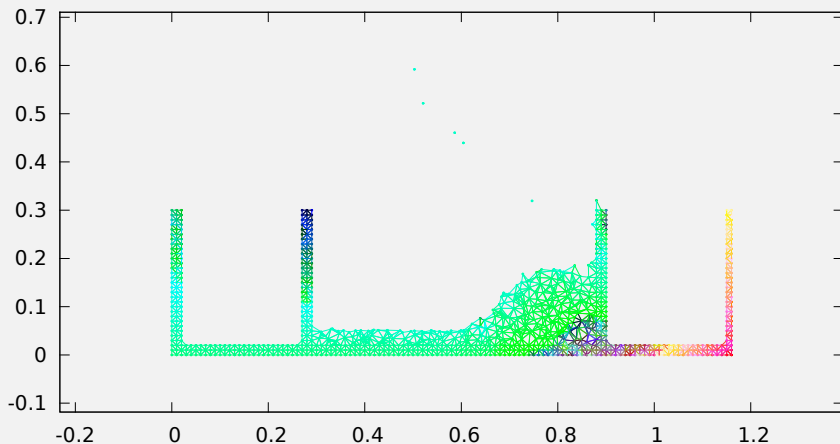
Water column collapse : pressure distribution

Time = 0.71 sec, Color = pressure (red = 4367.99, blue = -2143.9)



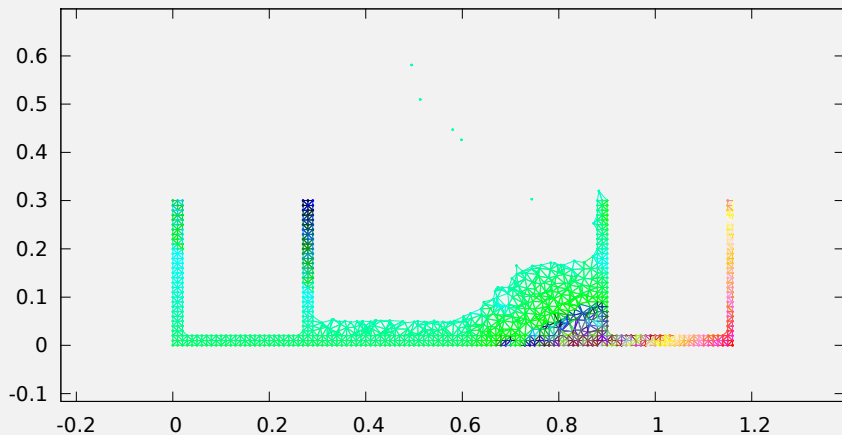
Water column collapse : pressure distribution

Time = 0.72 sec, Color = pressure (red = 4741.7, blue = -2074.46)



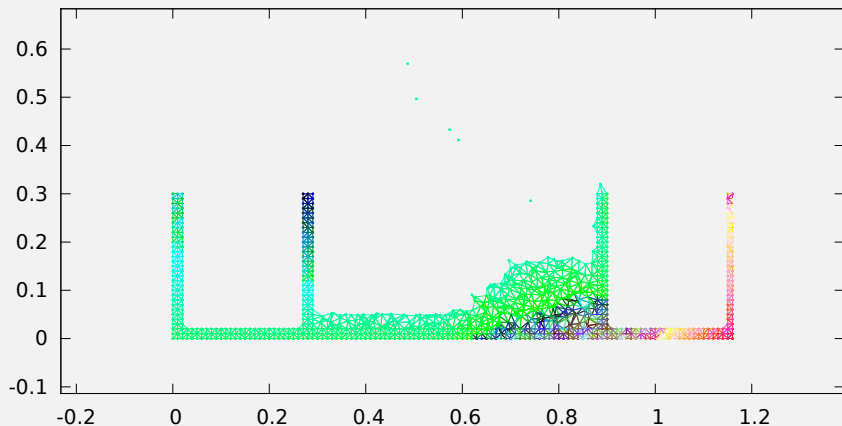
Water column collapse : pressure distribution

Time = 0.73 sec, Color = pressure (red = 4457.87, blue = -2106.75)



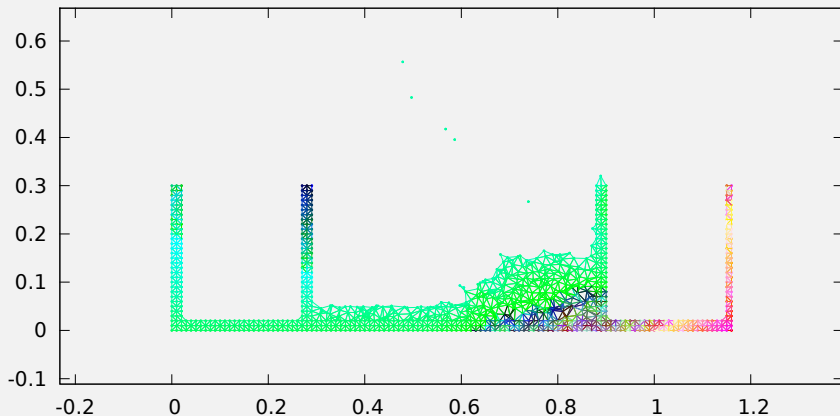
Water column collapse : pressure distribution

Time = 0.74 sec, Color = pressure (red = 4361.07, blue = -2143.31)



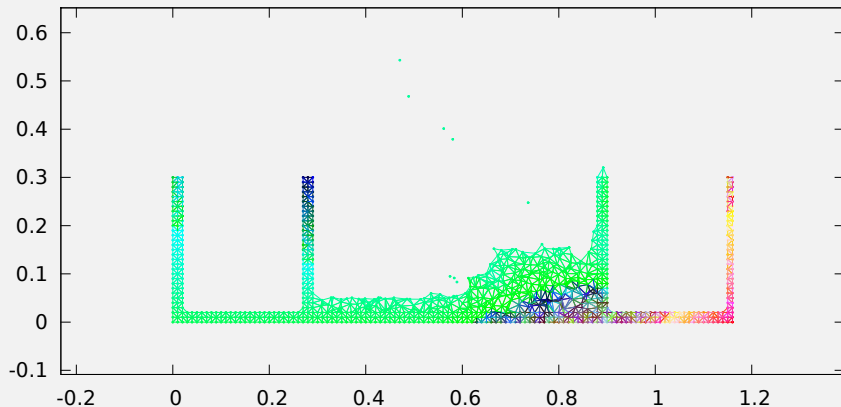
Water column collapse : pressure distribution

Time = 0.75 sec, Color = pressure (red = 4148.61, blue = -2140.47)



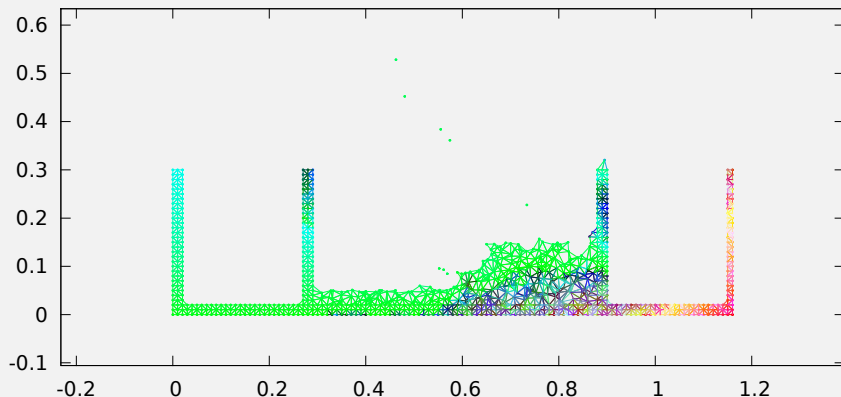
Water column collapse : pressure distribution

Time = 0.76 sec, Color = pressure (red = 3998.9, blue = -2138.83)



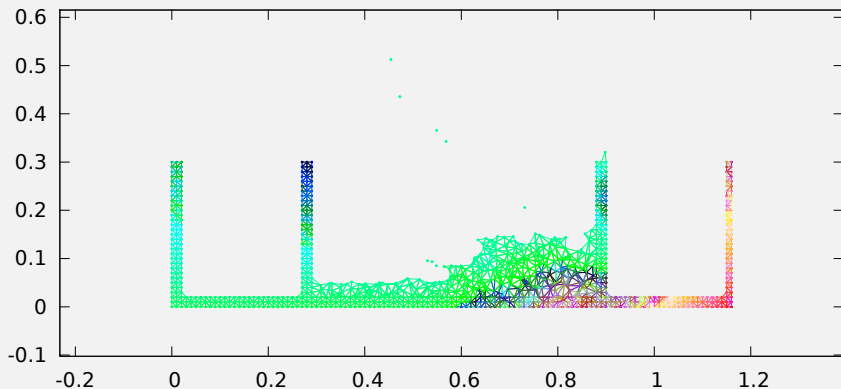
Water column collapse : pressure distribution

Time = 0.77 sec, Color = pressure (red = 3597.81, blue = -2614.37)



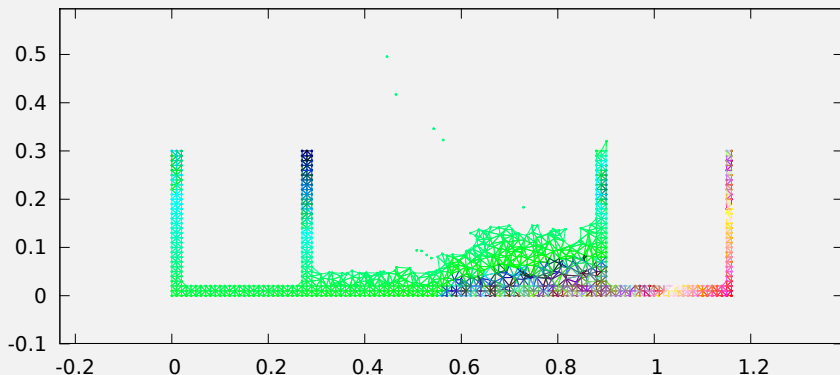
Water column collapse : pressure distribution

Time = 0.78 sec, Color = pressure (red = 3966.56, blue = -2112.79)



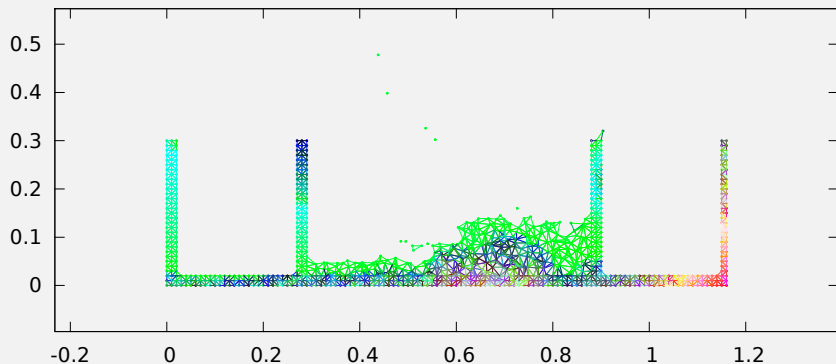
Water column collapse : pressure distribution

Time = 0.79 sec, Color = pressure (red = 3474.59, blue = -2123.63)



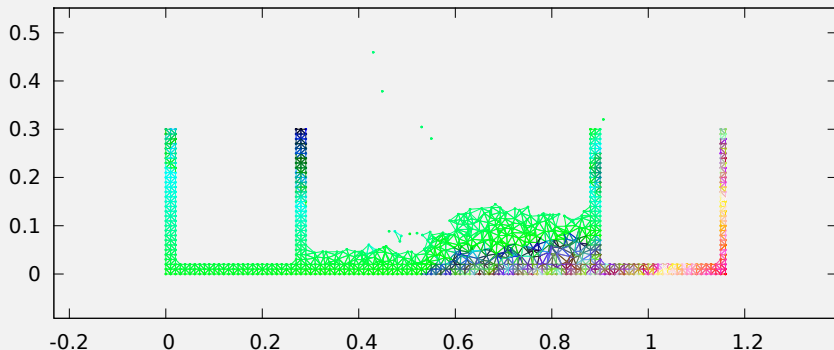
Water column collapse : pressure distribution

Time = 0.8 sec, Color = pressure (red = 2551.58, blue = -2108.55)



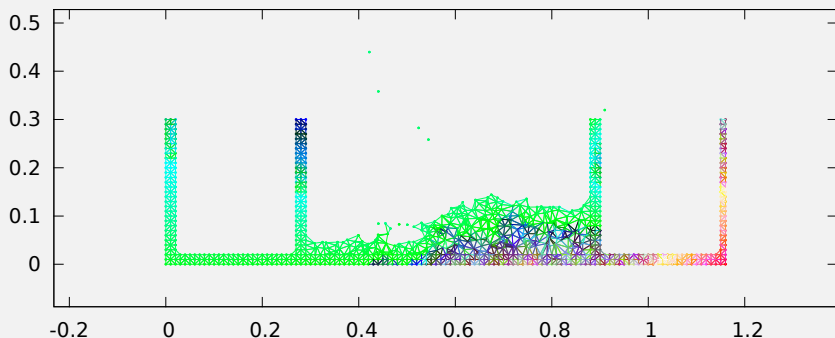
Water column collapse : pressure distribution

Time = 0.81 sec, Color = pressure (red = 3165.23, blue = -2077.78)



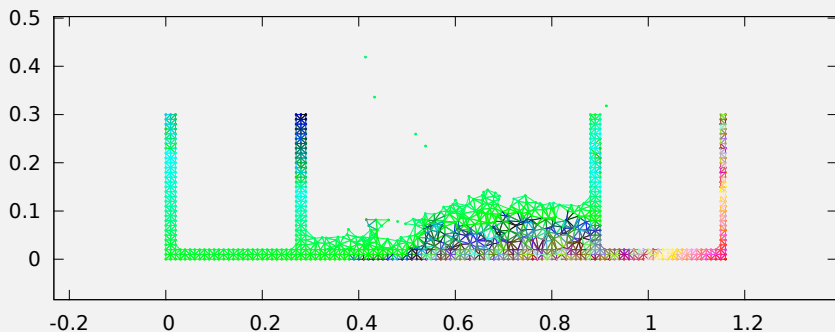
Water column collapse : pressure distribution

Time = 0.8200000000000001 sec, Color = pressure (red = 3162.99, blue = -2061.66)



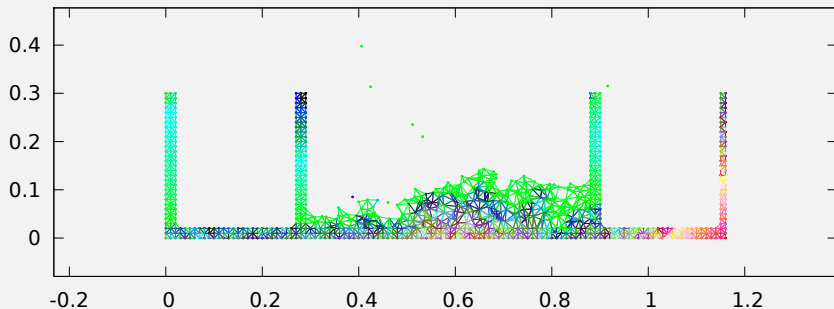
Water column collapse : pressure distribution

Time = 0.8300000000000001 sec, Color = pressure (red = 3100.48, blue = -2083.17)



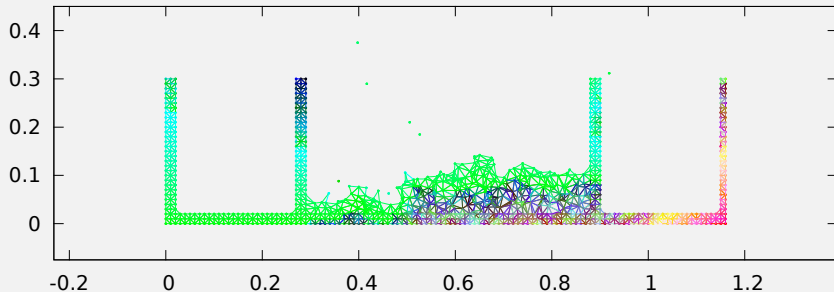
Water column collapse : pressure distribution

Time = 0.84 sec, Color = pressure (red = 2419.47, blue = -2090.35)



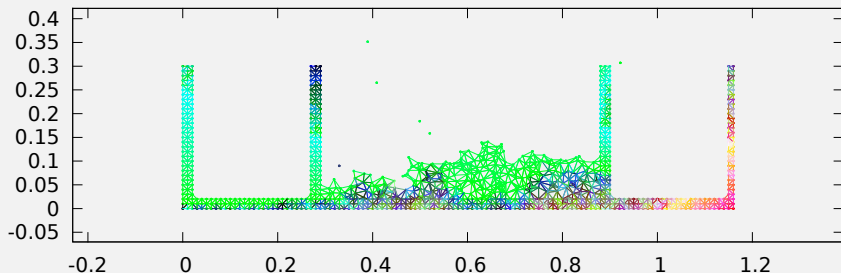
Water column collapse : pressure distribution

Time = 0.85 sec, Color = pressure (red = 3009.24, blue = -2099.45)



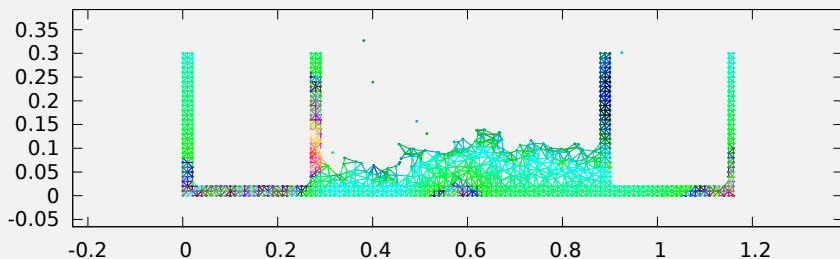
Water column collapse : pressure distribution

Time = 0.86 sec, Color = pressure (red = 2757.57, blue = -2141.59)



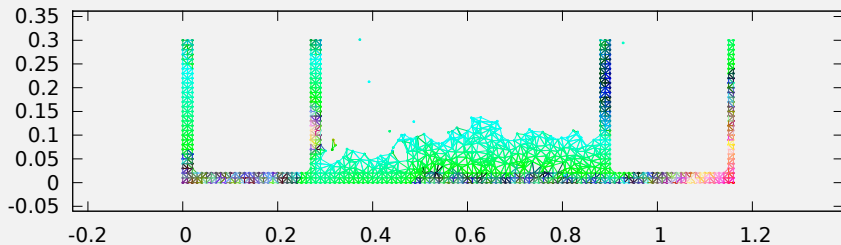
Water column collapse : pressure distribution

Time = 0.87 sec, Color = pressure (red = 4214.77, blue = -887.371)



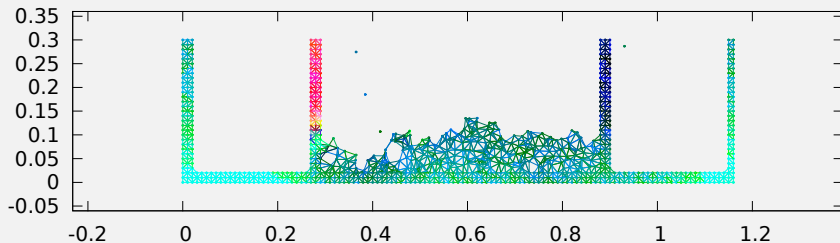
Water column collapse : pressure distribution

Time = 0.88 sec, Color = pressure (red = 2530.9, blue = -872.431)



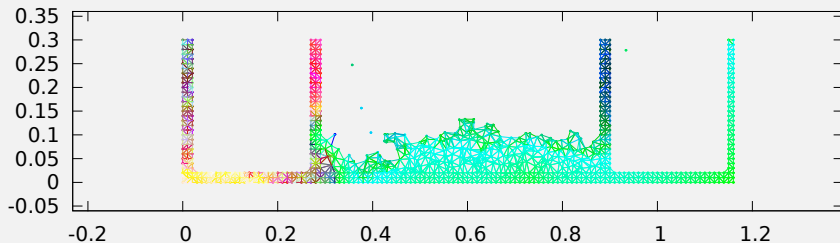
Water column collapse : pressure distribution

Time = 0.89 sec, Color = pressure (red = 12159.5, blue = -1579.14)



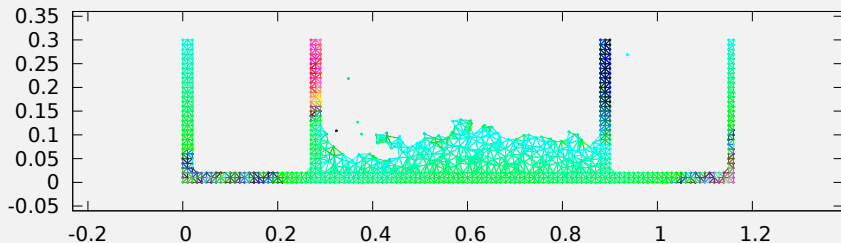
Water column collapse : pressure distribution

Time = 0.9 sec, Color = pressure (red = 6652.06, blue = -1741.57)



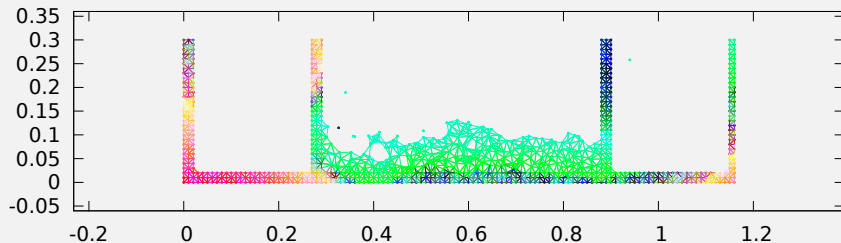
Water column collapse : pressure distribution

Time = 0.91 sec, Color = pressure (red = 3813.21, blue = -1204.05)



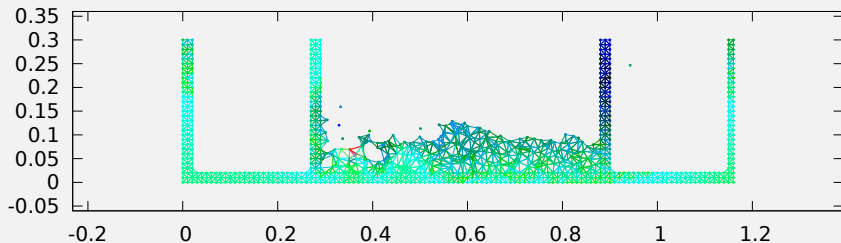
Water column collapse : pressure distribution

Time = 0.92 sec, Color = pressure (red = 2705.51, blue = -1204.18)



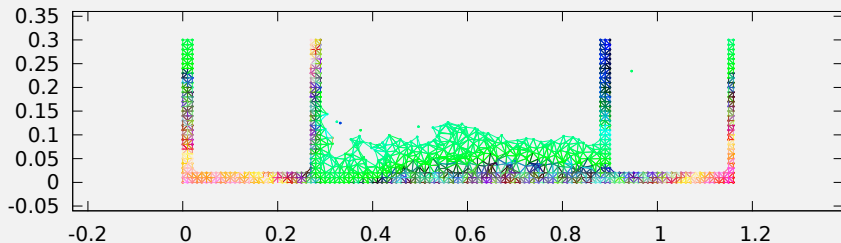
Water column collapse : pressure distribution

Time = 0.93 sec, Color = pressure (red = 7255.81, blue = -1241.52)



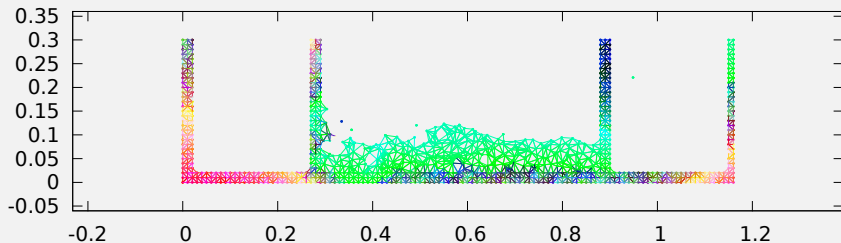
Water column collapse : pressure distribution

Time = 0.9400000000000001 sec, Color = pressure (red = 2222.01, blue = -1221.15)



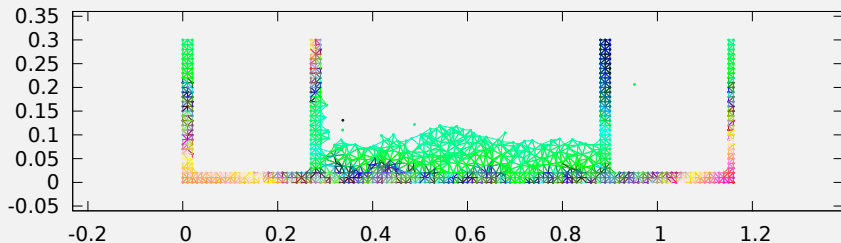
Water column collapse : pressure distribution

Time = 0.9500000000000001 sec, Color = pressure (red = 2480.51, blue = -1177.71)



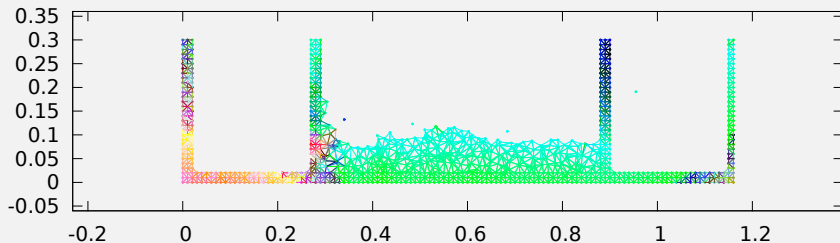
Water column collapse : pressure distribution

Time = 0.96 sec, Color = pressure (red = 2212.63, blue = -1140.38)



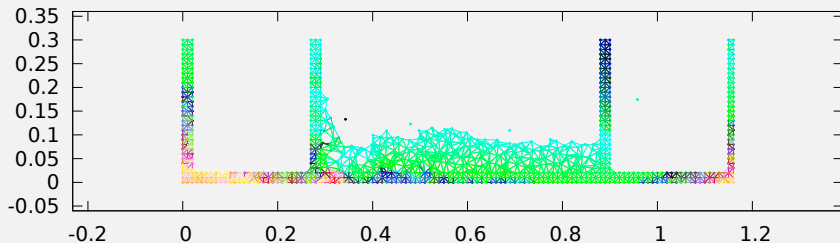
Water column collapse : pressure distribution

Time = 0.97 sec, Color = pressure (red = 3595.88, blue = -1278.48)



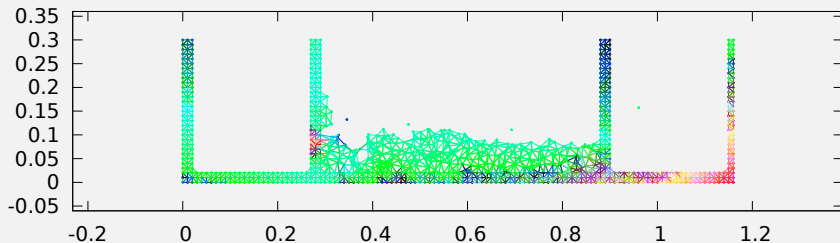
Water column collapse : pressure distribution

Time = 0.98 sec, Color = pressure (red = 2710.56, blue = -1036.94)



Water column collapse : pressure distribution

Time = 0.99 sec, Color = pressure (red = 2626.93, blue = -1282.7)



Mesh and Remesh

The mesh and remesh are based only on nodal positions and done by using the [Delaunay Triangulation](#) and [Alpha Shape](#).

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- [Alpha Shape](#) is a generalization of the convex hull of a set of nodes. Let α a real number with $0 \leq \alpha \leq \infty$. For $\alpha = \infty$, the Alpha Shape is identical to the Delaunay Triangulation. For $\alpha = 0$, the Alpha Shape shrinks to the set of nodes. Setting α to a appropriate value can remove the unnecessary triangles to recover the real boundaries.

Meshing in Tcl

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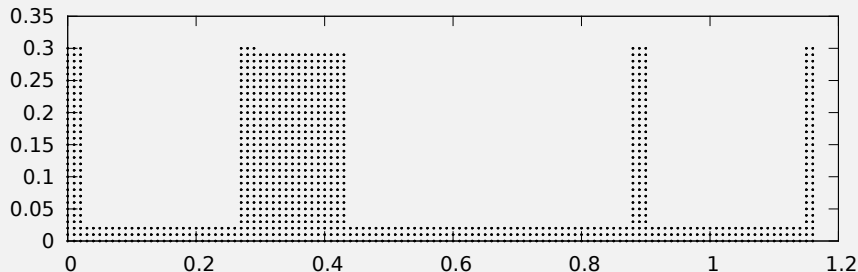
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- Remaining triangles are elements to be created in OpenSees.

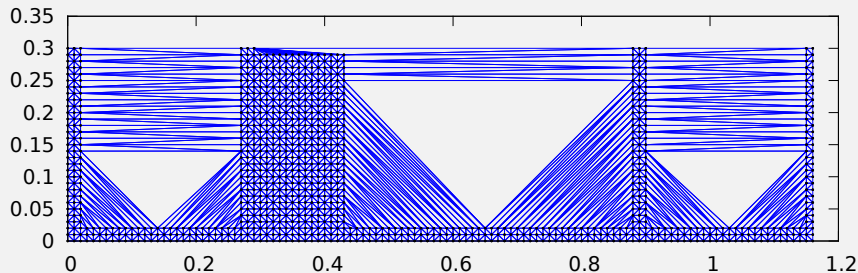
Example of meshing

At each time step, a set of nodes are given



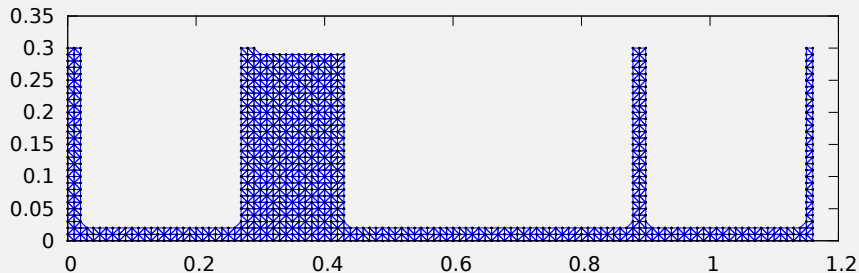
Example of meshing

Delaunay Triangulation of the set of nodes is calculated



Example of meshing

Alpha Shape of the set of nodes ($\alpha = 1.4$) is calculated



Outline

- 1 Motivation
- 2 Particle Finite Element Method
 - Introduction to PFEM
 - Basic Equations
 - Fractional Step Method (FSM)
- 3 Implementation in OpenSees
 - Problems of the implementation of FSM
 - Options and solution to the implementation of FSM
 - Example
 - Mesh and Remesh
- 4 Sensitivity Analysis of PFEM
 - Introduction to the sensitivity
 - Computing sensitivity
 - Example

Sensitivity

Definition

The effects of assumed parameter values on computed response

Sensitivity

Subsequent applications of sensitivity include

- Reliability analysis
 - Quantify system performance under multiple limit state
- Optimization
 - Find best solution for some objective functions and constraints on the system
- System identification
 - Inverse modeling, find model parameters that match observed results
- and others ...

Sensitivity

Simply speaking, a sensitivity is a gradient,

$$\frac{\partial U}{\partial \theta}(x, y, z, t)$$

where,

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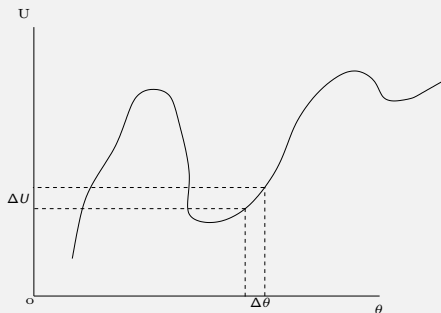
- U – response velocity, pressure, ...
- θ – modeling or physical property

Computing sensitivity

- A simple way (Finite Difference Method)

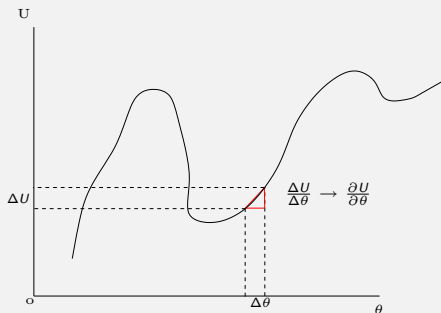
Computing sensitivity

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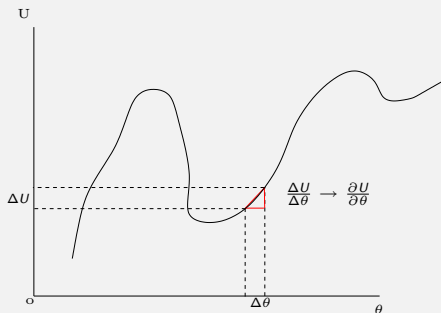
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Computing sensitivity

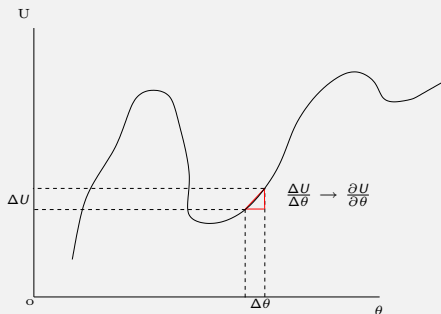
- A simple way (Finite Difference Method)



- Require to solve response equations one more time for each parameter
- Prone to round-off error for small $\Delta\theta$

Computing sensitivity

- A simple way (Finite Difference Method)



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FDM is not practical due to the computational cost

Computing sensitivity

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$$\frac{\partial \mathbf{M}}{\partial \theta} \dot{\mathbf{U}} + \mathbf{M} \frac{\partial \dot{\mathbf{U}}}{\partial \theta} + \frac{\partial \mathbf{K}}{\partial \theta} \mathbf{U} + \mathbf{K} \frac{\partial \mathbf{U}}{\partial \theta} = \frac{\partial \mathbf{F}}{\partial \theta}$$

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$$\frac{\partial \mathbf{M}}{\partial \theta} \dot{\mathbf{U}} + \mathbf{M} \frac{\partial \dot{\mathbf{U}}}{\partial \theta} + \frac{\partial \mathbf{K}}{\partial \theta} \mathbf{U} + \mathbf{K} \frac{\partial \mathbf{U}}{\partial \theta} = \frac{\partial \mathbf{F}}{\partial \theta}$$

$$\mathbf{M} \frac{\partial \dot{\mathbf{U}}}{\partial \theta} + \mathbf{K} \frac{\partial \mathbf{U}}{\partial \theta} = \frac{\partial \mathbf{F}}{\partial \theta} - \frac{\partial \mathbf{M}}{\partial \theta} \dot{\mathbf{U}} - \frac{\partial \mathbf{K}}{\partial \theta} \mathbf{U}$$

Computing sensitivity

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$$\mathbf{M}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}$$

$$\frac{\partial \mathbf{M}}{\partial \theta} \dot{\mathbf{U}} + \mathbf{M} \frac{\partial \dot{\mathbf{U}}}{\partial \theta} + \frac{\partial \mathbf{K}}{\partial \theta} \mathbf{U} + \mathbf{K} \frac{\partial \mathbf{U}}{\partial \theta} = \frac{\partial \mathbf{F}}{\partial \theta}$$

$$\mathbf{M} \frac{\partial \dot{\mathbf{U}}}{\partial \theta} + \mathbf{K} \frac{\partial \mathbf{U}}{\partial \theta} = \frac{\partial \mathbf{F}}{\partial \theta} - \frac{\partial \mathbf{M}}{\partial \theta} \dot{\mathbf{U}} - \frac{\partial \mathbf{K}}{\partial \theta} \mathbf{U}$$

Computing sensitivity

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where,

$$\frac{\partial \mathbf{F}}{\partial \theta} - \frac{\partial \mathbf{M}}{\partial \theta} \dot{\mathbf{U}} - \frac{\partial \mathbf{K}}{\partial \theta} \mathbf{U}$$

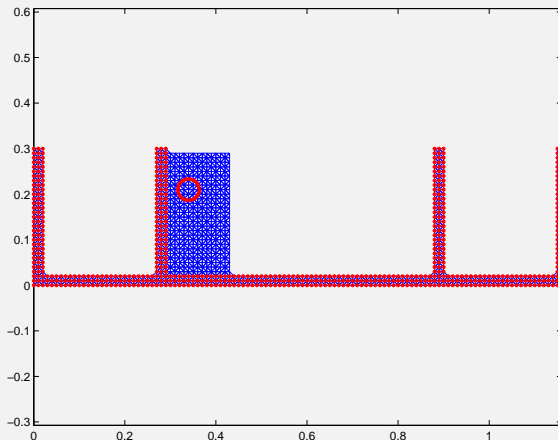
is the sensitivity “force” vector

- Only form RHS vectors and solve linear equations once for each parameter, LHS matrices don't change,
- No additional round-off error as the FDM

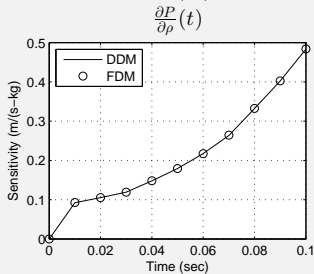
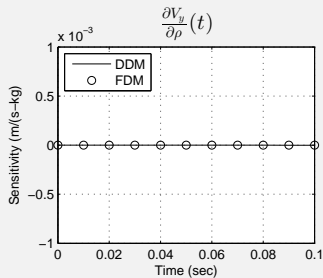
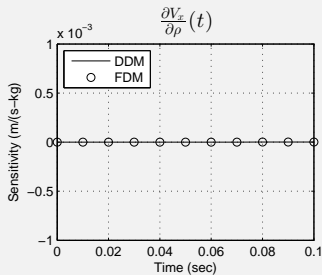
Example of sensitivity analysis

Example of sensitivity analysis

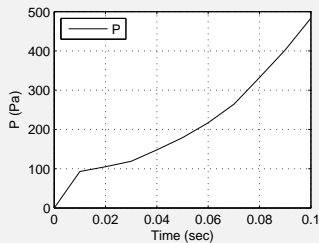
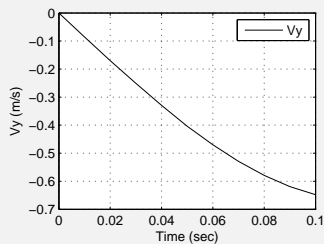
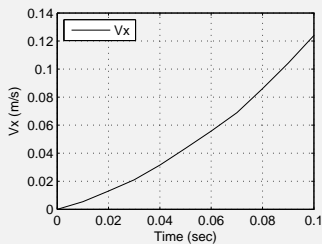
- Find $\frac{\partial V_x}{\partial \rho}(t)$, $\frac{\partial V_y}{\partial \rho}(t)$, $\frac{\partial P}{\partial \rho}(t)$ for particle 400, the sensitivity of velocities and pressure to the density of the fluid.



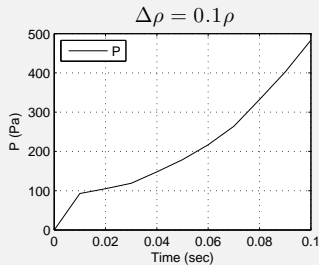
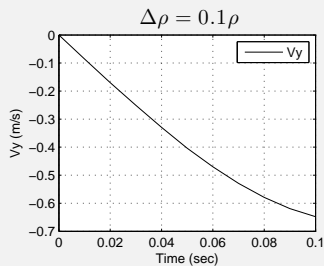
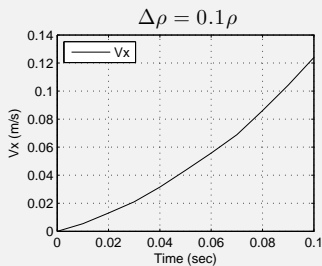
Example of sensitivity analysis



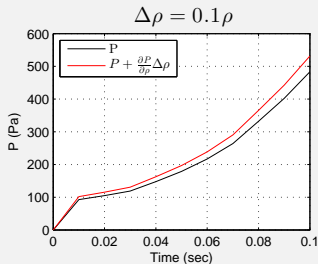
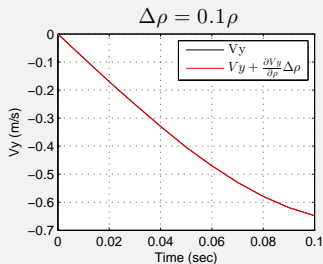
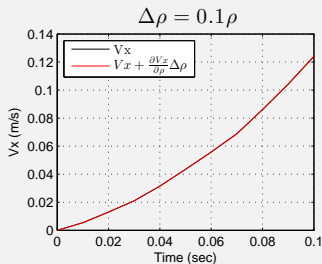
Example of sensitivity analysis



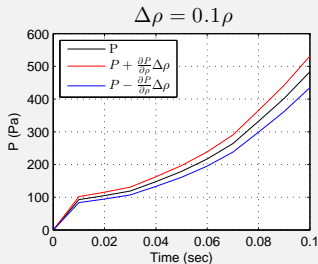
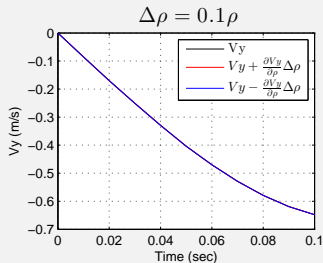
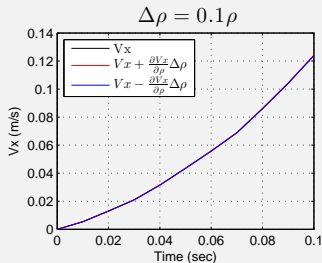
Example of sensitivity analysis



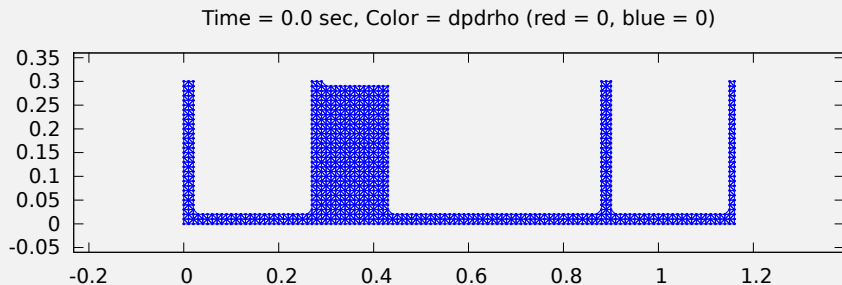
Example of sensitivity analysis



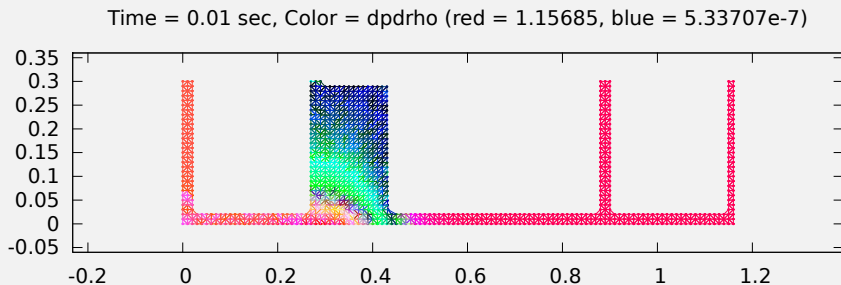
Example of sensitivity analysis



Water column collapse : pressure sensitivity distribution

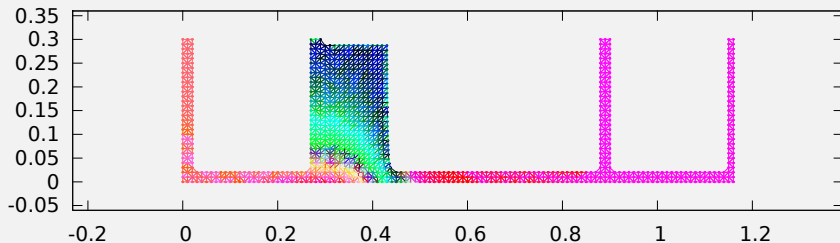


Water column collapse : pressure sensitivity distribution

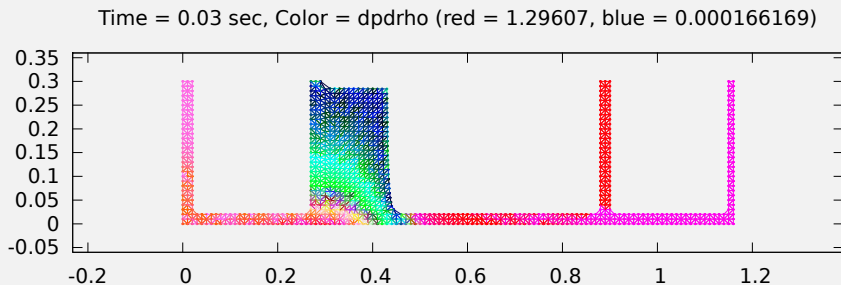


Water column collapse : pressure sensitivity distribution

Time = 0.02 sec, Color = dpdrho (red = 1.23735, blue = 0.000205394)

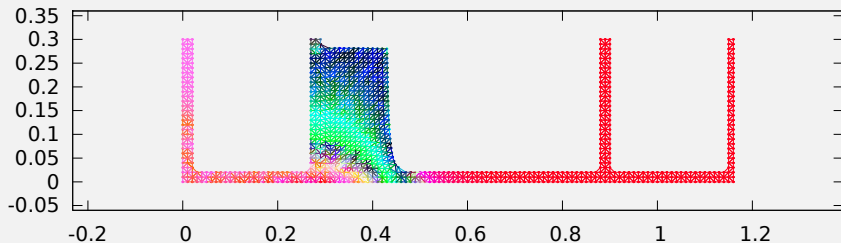


Water column collapse : pressure sensitivity distribution



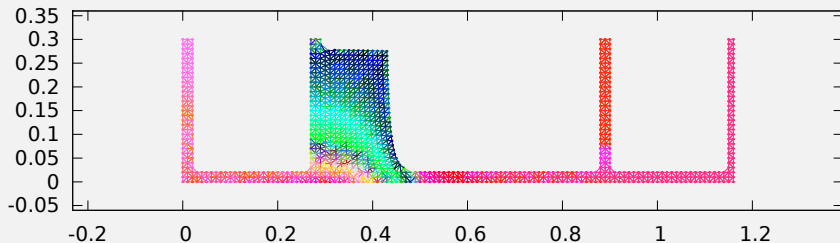
Water column collapse : pressure sensitivity distribution

Time = 0.04 sec, Color = dpdrho (red = 1.3828, blue = 0.00012243)



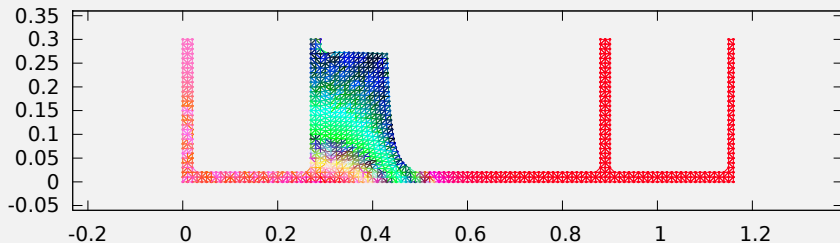
Water column collapse : pressure sensitivity distribution

Time = 0.05 sec, Color = dpdrho (red = 1.48372, blue = 0.000219554)

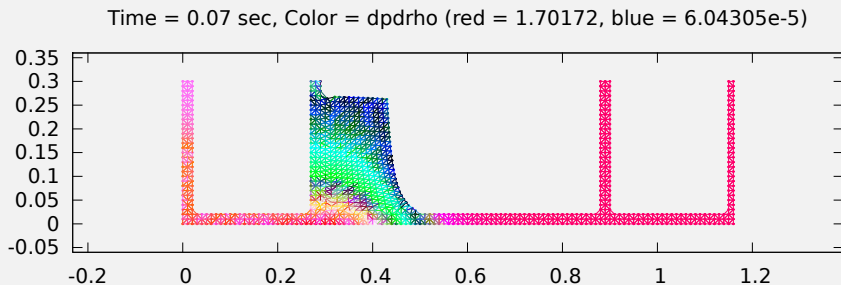


Water column collapse : pressure sensitivity distribution

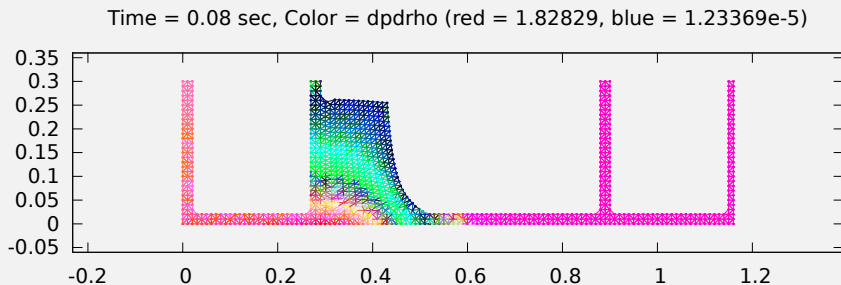
Time = 0.06 sec, Color = dpdrho (red = 1.58773, blue = 0.000122697)



Water column collapse : pressure sensitivity distribution

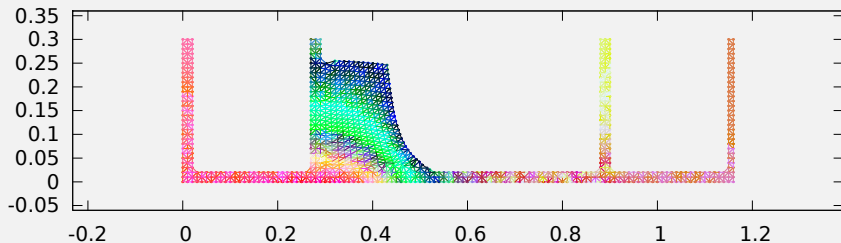


Water column collapse : pressure sensitivity distribution

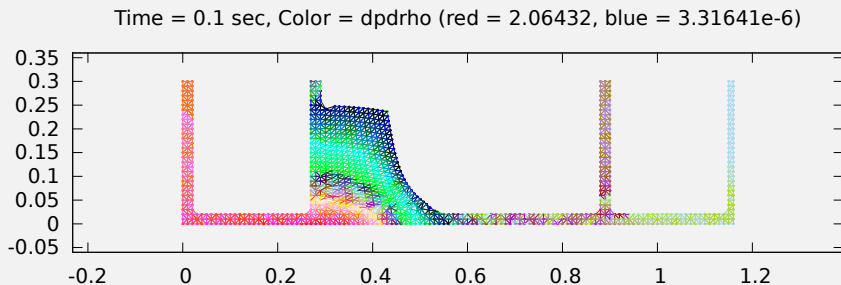


Water column collapse : pressure sensitivity distribution

Time = 0.09 sec, Color = dpdrho (red = 1.95319, blue = 2.7584e-5)

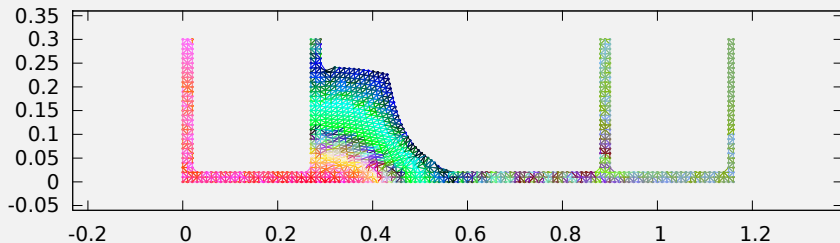


Water column collapse : pressure sensitivity distribution



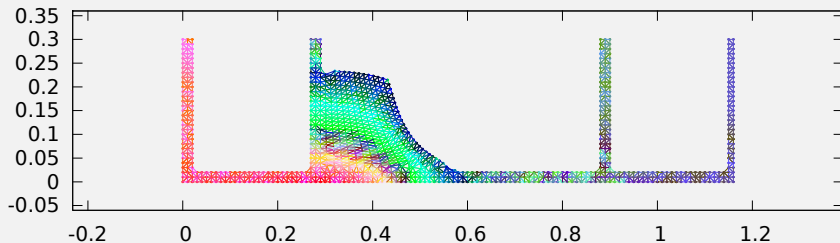
Water column collapse : pressure sensitivity distribution

Time = 0.11 sec, Color = dpdrho (red = 2.18798, blue = 3.59464e-5)



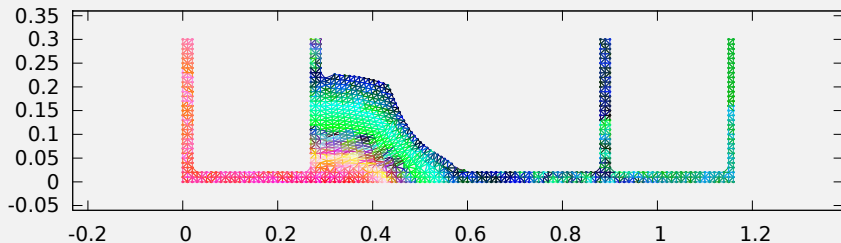
Water column collapse : pressure sensitivity distribution

Time = 0.12 sec, Color = dpdrho (red = 2.27512, blue = 5.70773e-5)



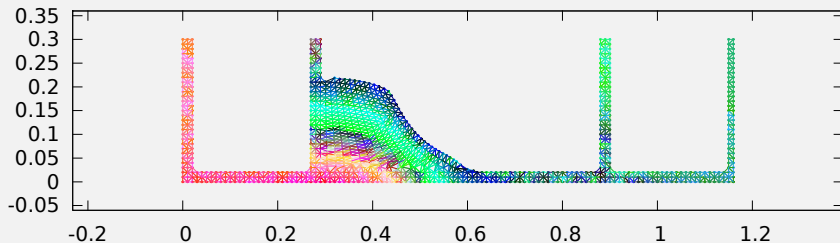
Water column collapse : pressure sensitivity distribution

Time = 0.13 sec, Color = dpdrho (red = 2.37631, blue = 8.8027e-6)



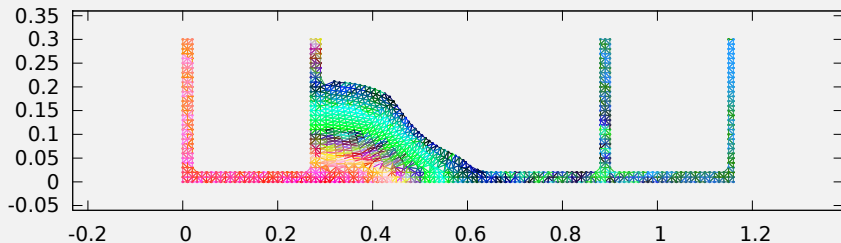
Water column collapse : pressure sensitivity distribution

Time = 0.14 sec, Color = dpdrho (red = 2.34599, blue = 0.000153807)



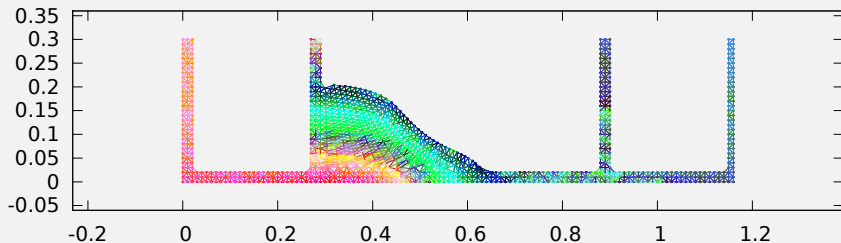
Water column collapse : pressure sensitivity distribution

Time = 0.15 sec, Color = dpdrho (red = 2.41338, blue = 0.000495244)



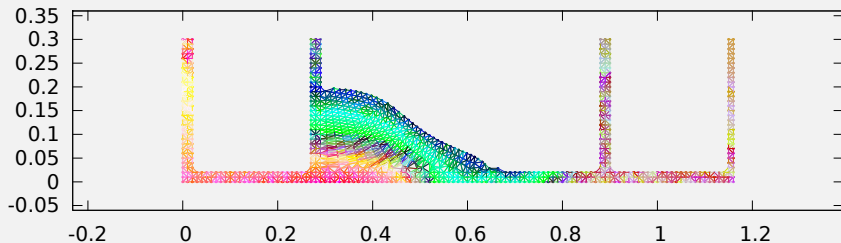
Water column collapse : pressure sensitivity distribution

Time = 0.16 sec, Color = dpdrho (red = 2.34567, blue = 0.000317595)



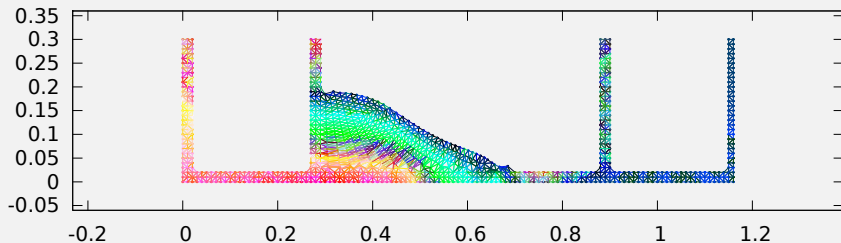
Water column collapse : pressure sensitivity distribution

Time = 0.17 sec, Color = dpdrho (red = 2.28001, blue = 0.000108735)



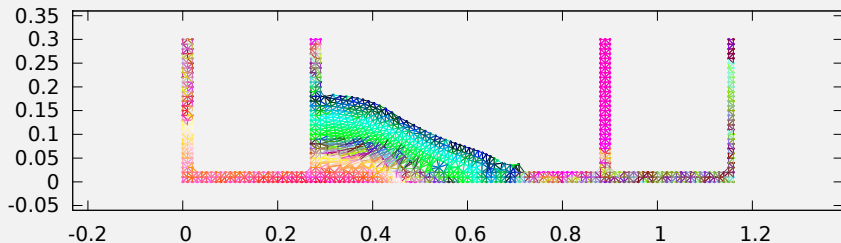
Water column collapse : pressure sensitivity distribution

Time = 0.18 sec, Color = dpdrho (red = 2.24855, blue = 1.86194e-5)



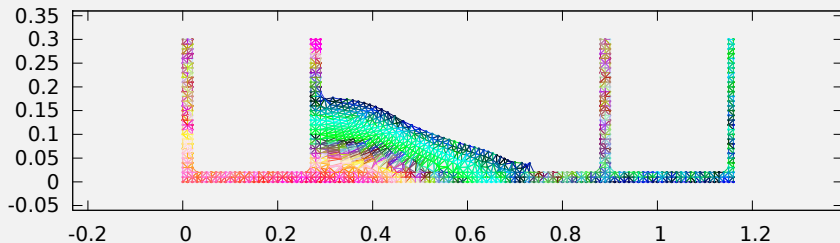
Water column collapse : pressure sensitivity distribution

Time = 0.19 sec, Color = dpdrho (red = 2.13937, blue = 3.12969e-6)



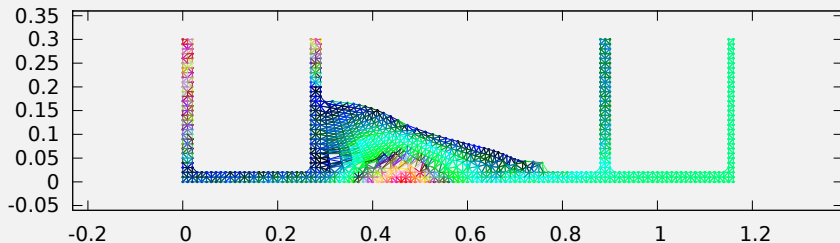
Water column collapse : pressure sensitivity distribution

Time = 0.2 sec, Color = $dpdrho$ (red = 2.04986, blue = 0.0001035)



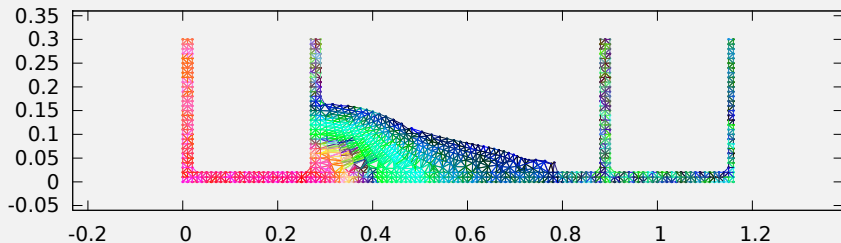
Water column collapse : pressure sensitivity distribution

Time = 0.21 sec, Color = dpdrho (red = 1.70761, blue = 1.42227e-5)



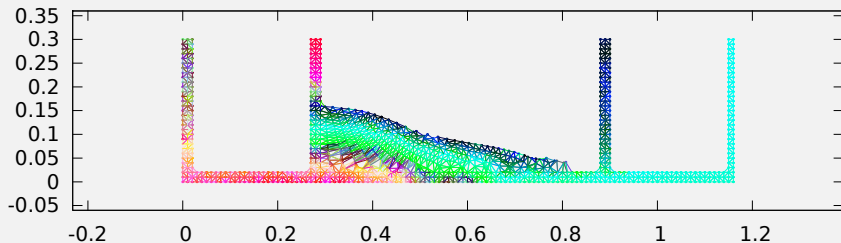
Water column collapse : pressure sensitivity distribution

Time = 0.22 sec, Color = dpdrho (red = 6.42525, blue = 0.000151324)



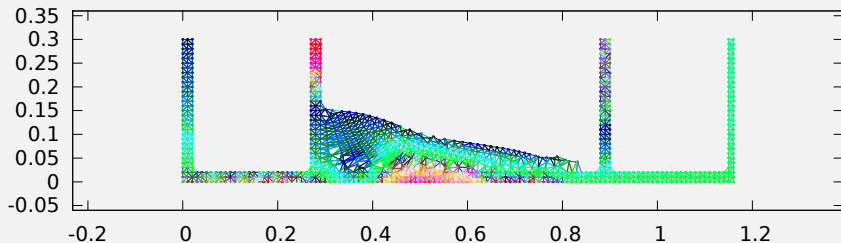
Water column collapse : pressure sensitivity distribution

Time = 0.23 sec, Color = dpdrho (red = 1.81384, blue = 2.0844e-5)



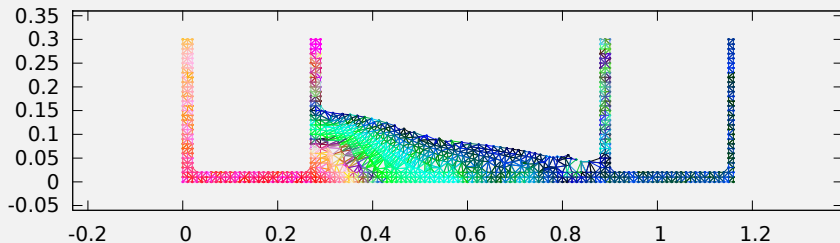
Water column collapse : pressure sensitivity distribution

Time = 0.24 sec, Color = dpdrho (red = 1.0981, blue = 7.49714e-5)



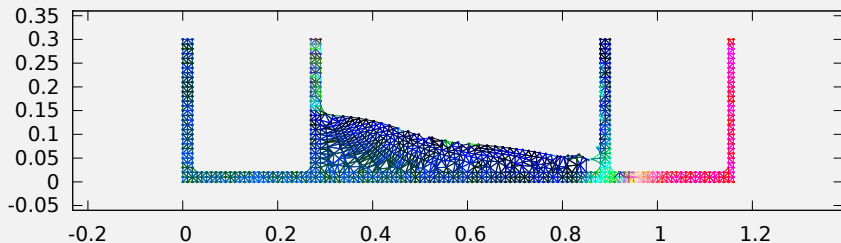
Water column collapse : pressure sensitivity distribution

Time = 0.25 sec, Color = dpdrho (red = 4.33208, blue = 0.000110583)



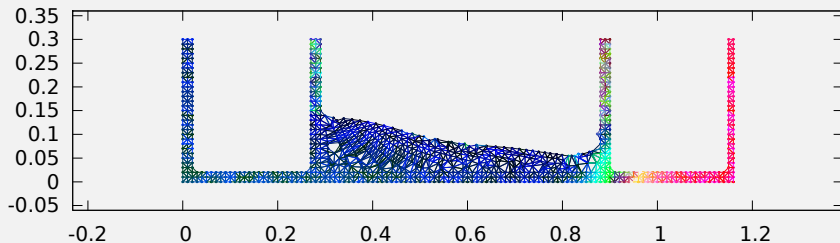
Water column collapse : pressure sensitivity distribution

Time = 0.26 sec, Color = dpdrho (red = 16.4973, blue = 0.0002105)



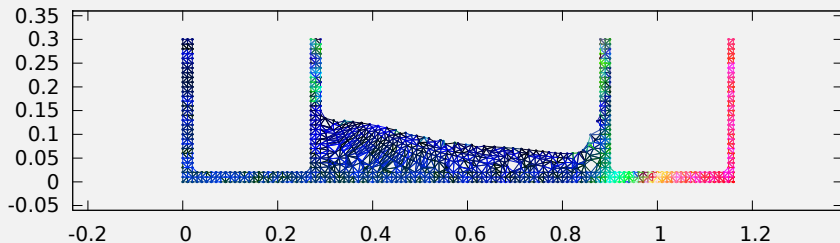
Water column collapse : pressure sensitivity distribution

Time = 0.27 sec, Color = dpdrho (red = 20.5238, blue = 1.08078e-5)



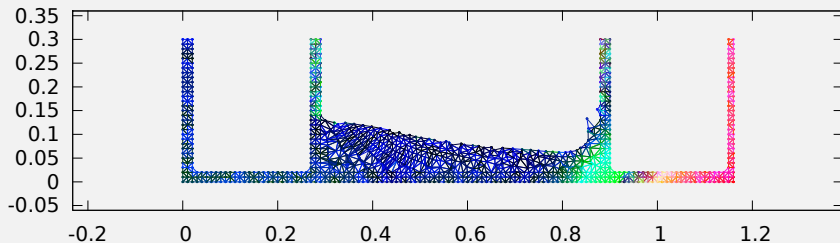
Water column collapse : pressure sensitivity distribution

Time = 0.28 sec, Color = dpdrho (red = 19.9365, blue = 1.3211e-5)



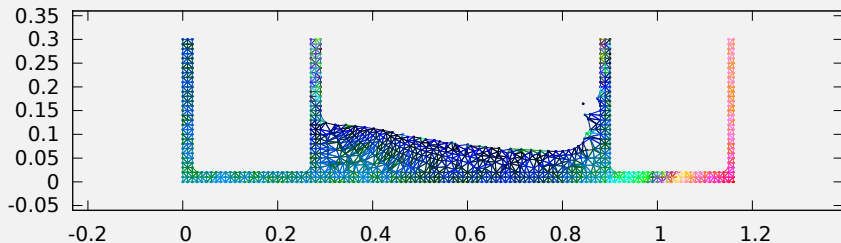
Water column collapse : pressure sensitivity distribution

Time = 0.29 sec, Color = dpdrho (red = 22.4636, blue = 1.62874e-5)



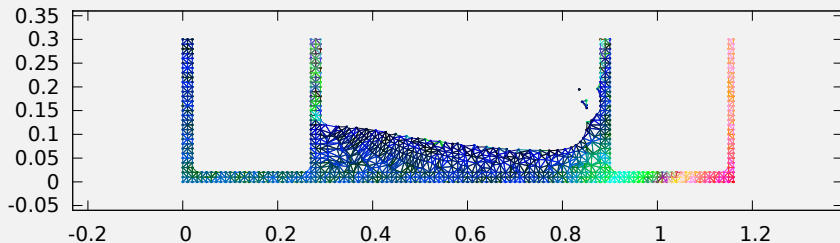
Water column collapse : pressure sensitivity distribution

Time = 0.3 sec, Color = dpdrho (red = 13.9326, blue = 5.26137e-5)



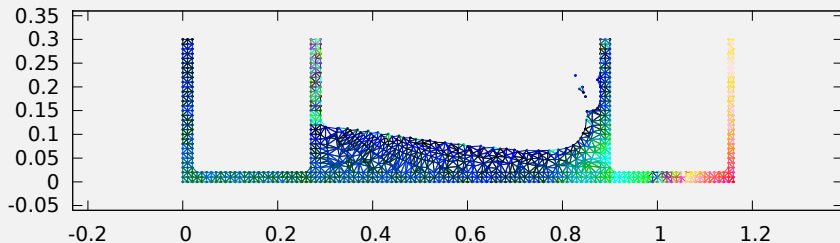
Water column collapse : pressure sensitivity distribution

Time = 0.31 sec, Color = dpdrho (red = 13.9755, blue = 0.000156356)



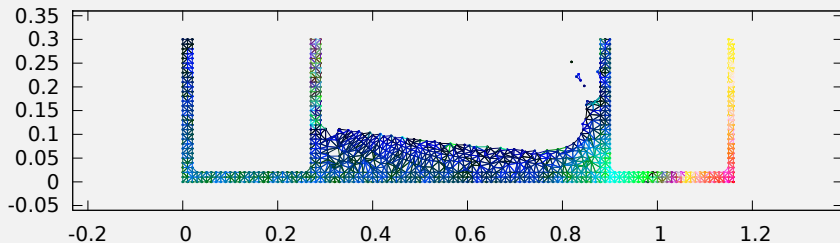
Water column collapse : pressure sensitivity distribution

Time = 0.32 sec, Color = dpdrho (red = 12.5669, blue = 0.00139861)



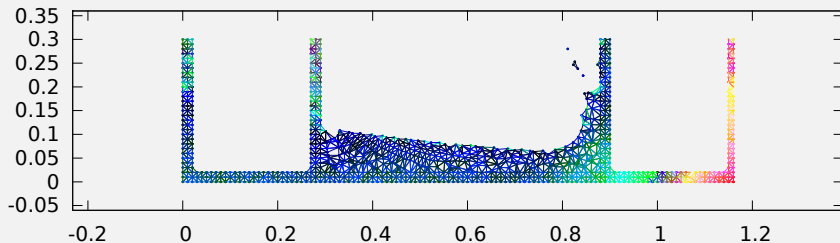
Water column collapse : pressure sensitivity distribution

Time = 0.33 sec, Color = dpdrho (red = 12.4408, blue = 0.000589244)



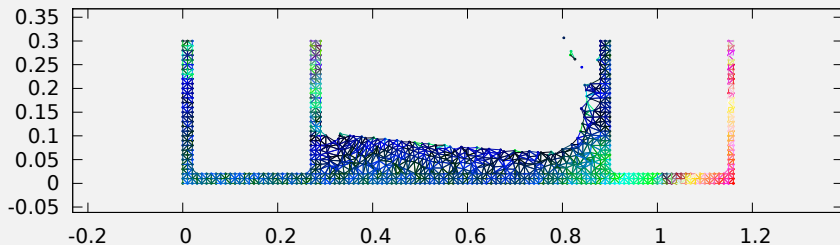
Water column collapse : pressure sensitivity distribution

Time = 0.34 sec, Color = dpdrho (red = 11.3218, blue = 0.000203923)

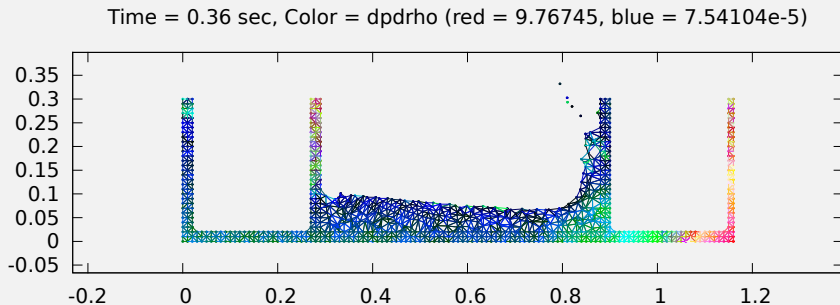


Water column collapse : pressure sensitivity distribution

Time = 0.35000000000000003 sec, Color = dpdrho (red = 10.6839, blue = 2.52956e-

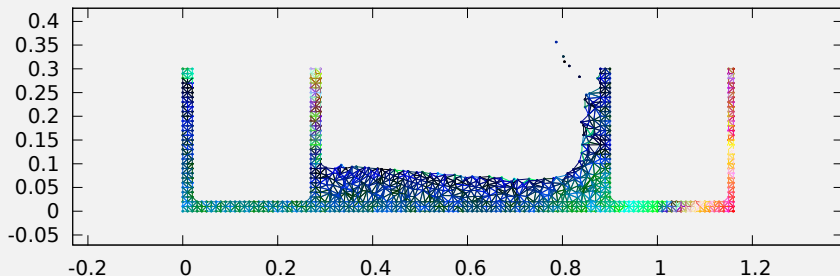


Water column collapse : pressure sensitivity distribution



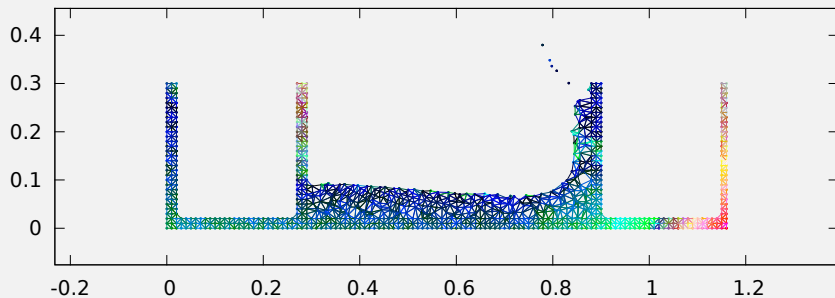
Water column collapse : pressure sensitivity distribution

Time = 0.37 sec, Color = dpdrho (red = 9.34283, blue = 0.000721572)



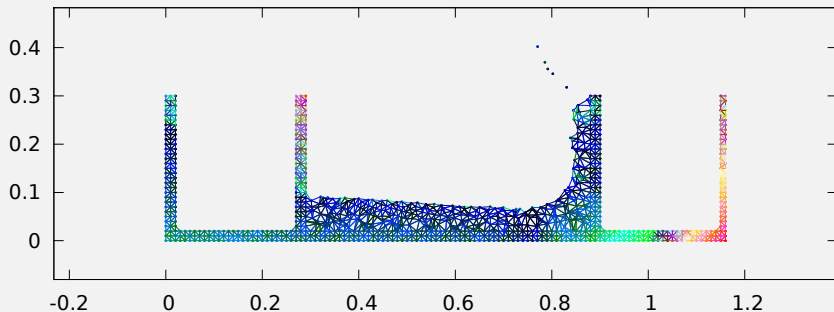
Water column collapse : pressure sensitivity distribution

Time = 0.38 sec, Color = dpdrho (red = 9.16945, blue = 0.00107358)



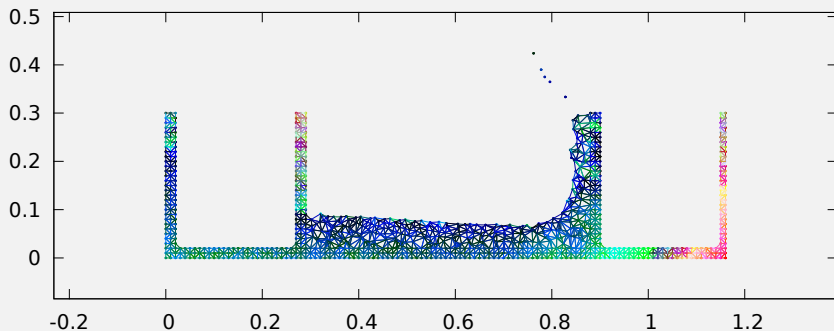
Water column collapse : pressure sensitivity distribution

Time = 0.39 sec, Color = dpdrho (red = 8.82553, blue = 0.000527362)



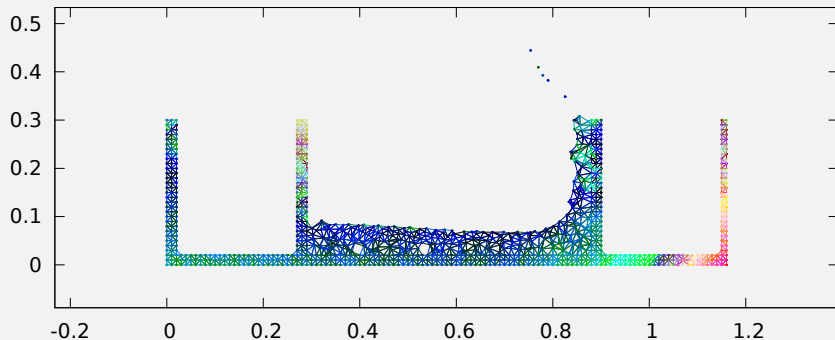
Water column collapse : pressure sensitivity distribution

Time = 0.4 sec, Color = dpdrho (red = 8.7522, blue = 1.54018e-5)



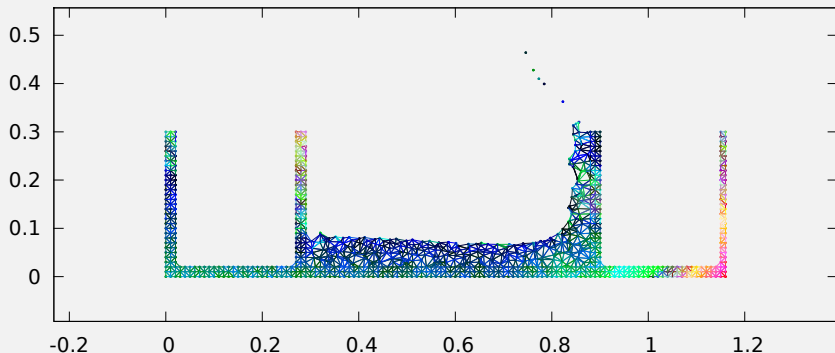
Water column collapse : pressure sensitivity distribution

Time = 0.41000000000000003 sec, Color = dpdrho (red = 8.36151, blue = 0.00026095)



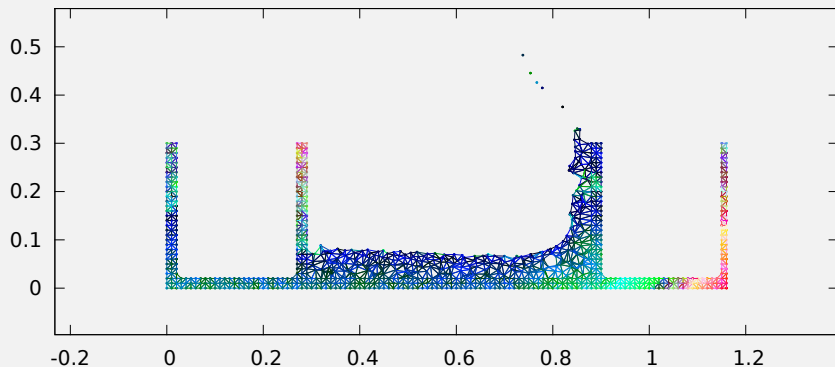
Water column collapse : pressure sensitivity distribution

Time = 0.42 sec, Color = dpdrho (red = 7.86106, blue = 0.000539944)



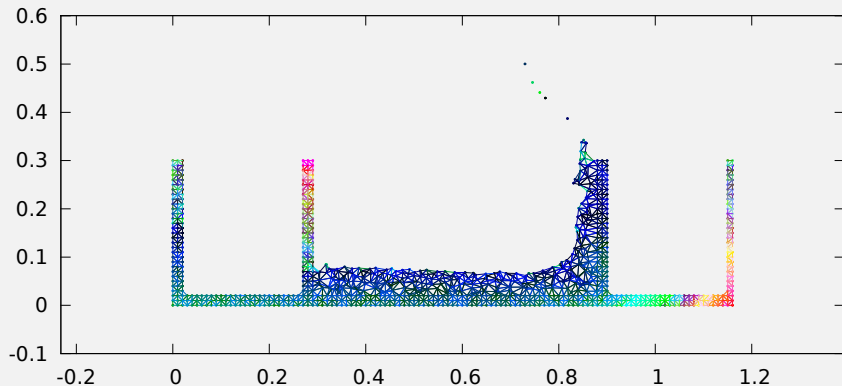
Water column collapse : pressure sensitivity distribution

Time = 0.43 sec, Color = dpdrho (red = 7.66784, blue = 5.44635e-5)



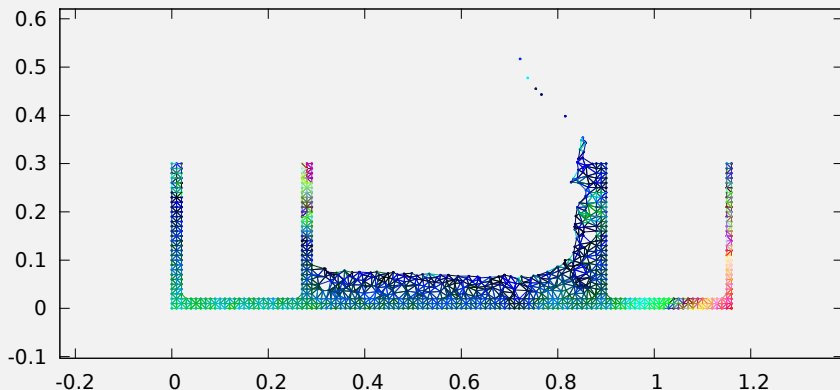
Water column collapse : pressure sensitivity distribution

Time = 0.44 sec, Color = dpdrho (red = 6.9899, blue = 0.000183941)



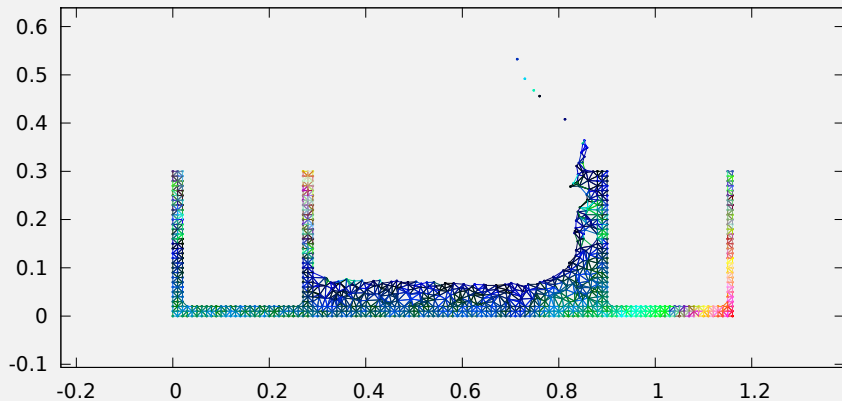
Water column collapse : pressure sensitivity distribution

Time = 0.45 sec, Color = dpdrho (red = 6.6227, blue = 4.4567e-5)



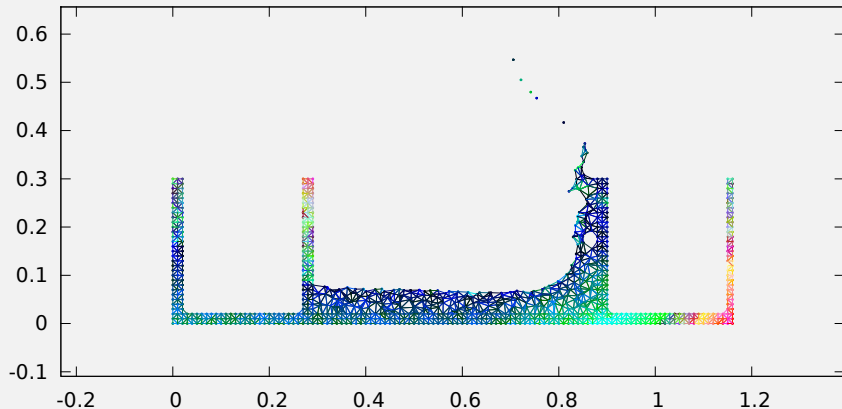
Water column collapse : pressure sensitivity distribution

Time = 0.46 sec, Color = dpdrho (red = 6.93856, blue = 8.35958e-5)



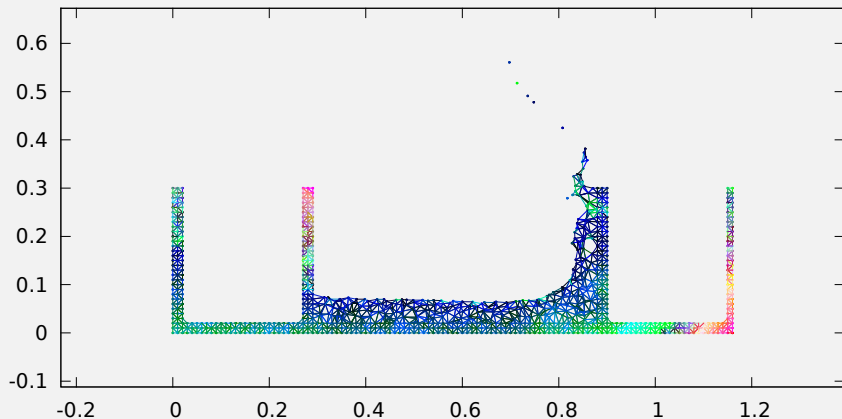
Water column collapse : pressure sensitivity distribution

Time = 0.47000000000000003 sec, Color = dpdrho (red = 7.32692, blue = 5.45995e-!



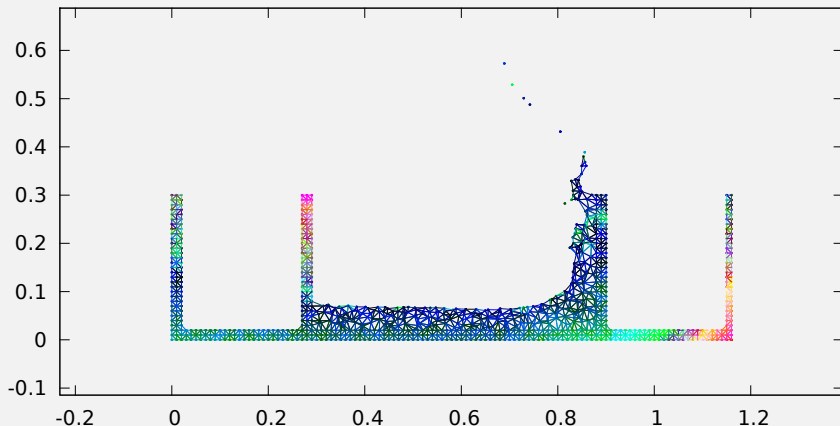
Water column collapse : pressure sensitivity distribution

Time = 0.48 sec, Color = dpdrho (red = 6.66376, blue = 0.00019052)



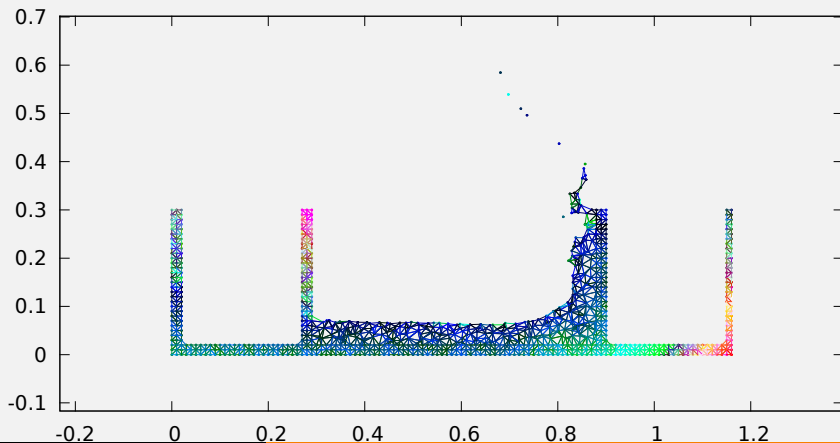
Water column collapse : pressure sensitivity distribution

Time = 0.49 sec, Color = dpdrho (red = 6.64519, blue = 4.2382e-5)



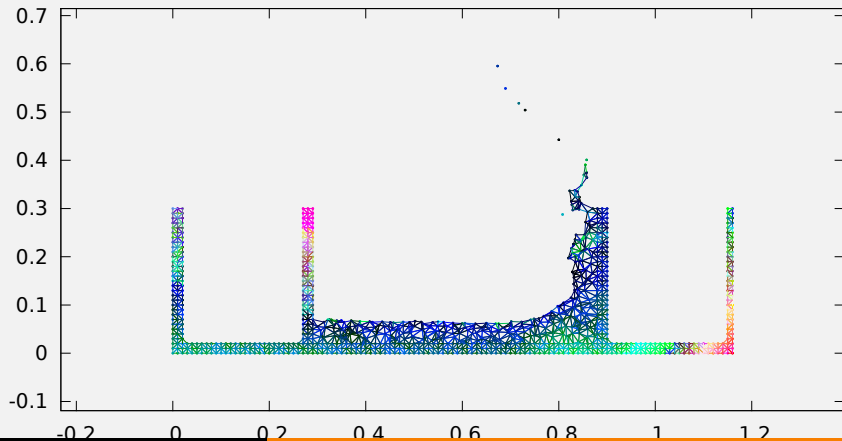
Water column collapse : pressure sensitivity distribution

Time = 0.5 sec, Color = dpdrho (red = 6.57532, blue = 5.65903e-5)



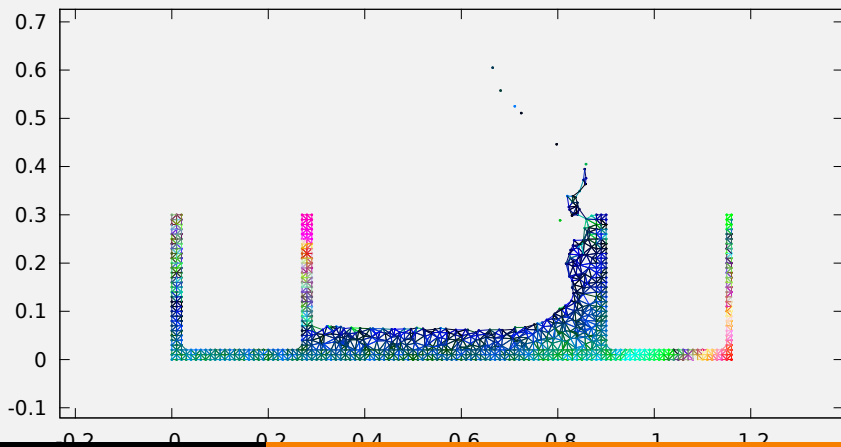
Water column collapse : pressure sensitivity distribution

Time = 0.51 sec, Color = dpdrho (red = 6.15988, blue = 6.68326e-5)



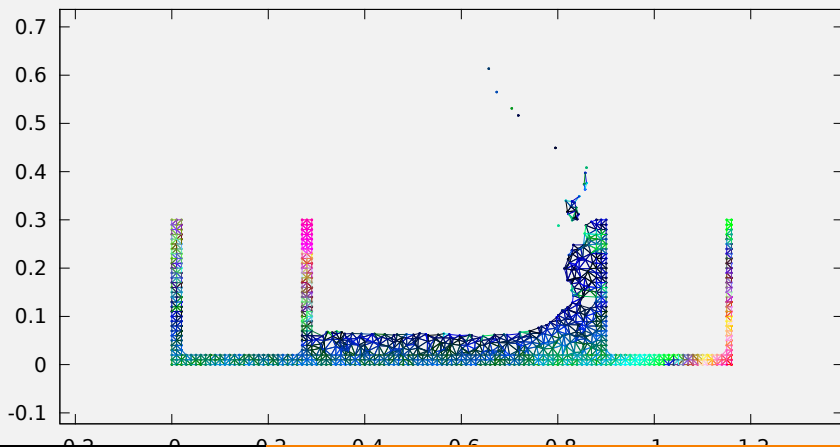
Water column collapse : pressure sensitivity distribution

Time = 0.52 sec, Color = dpdrho (red = 6.08077, blue = 6.0262e-5)



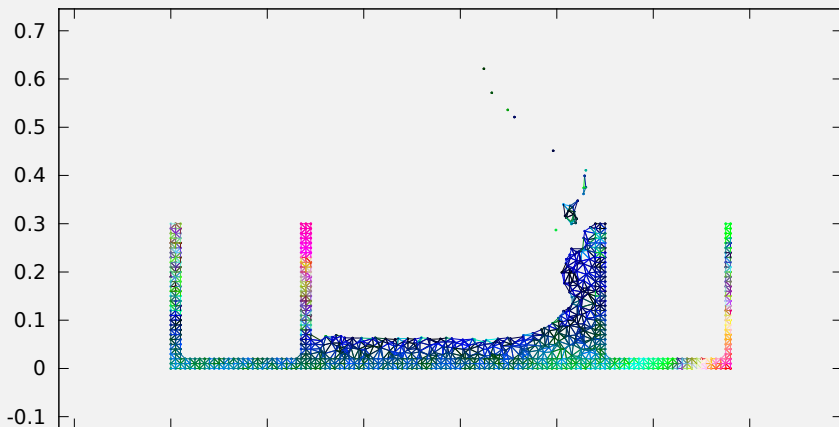
Water column collapse : pressure sensitivity distribution

Time = 0.53 sec, Color = dpdrho (red = 5.95879, blue = 7.33743e-5)



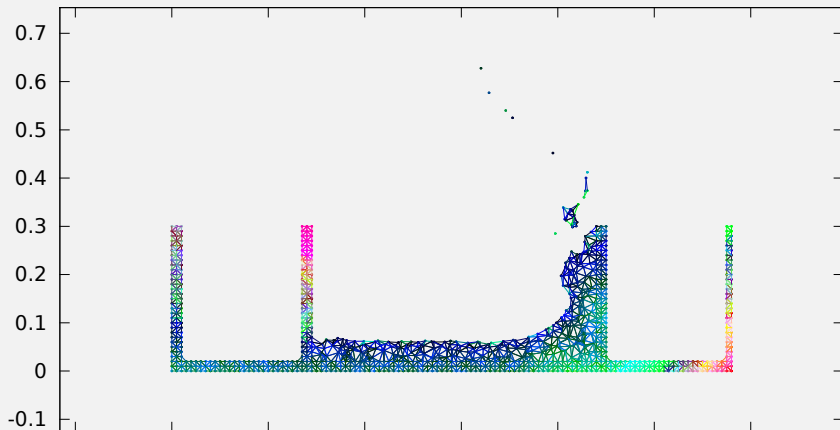
Water column collapse : pressure sensitivity distribution

Time = 0.54 sec, Color = dpdrho (red = 5.89496, blue = 1.07377e-5)



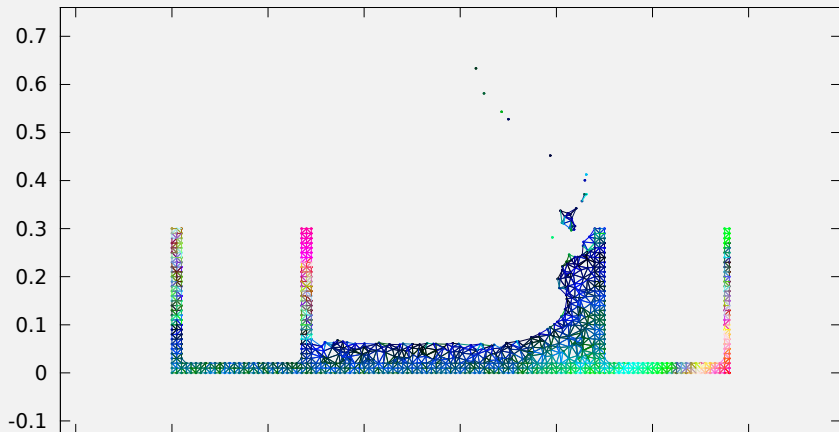
Water column collapse : pressure sensitivity distribution

Time = 0.55 sec, Color = dpdrho (red = 5.98994, blue = 0.000101665)



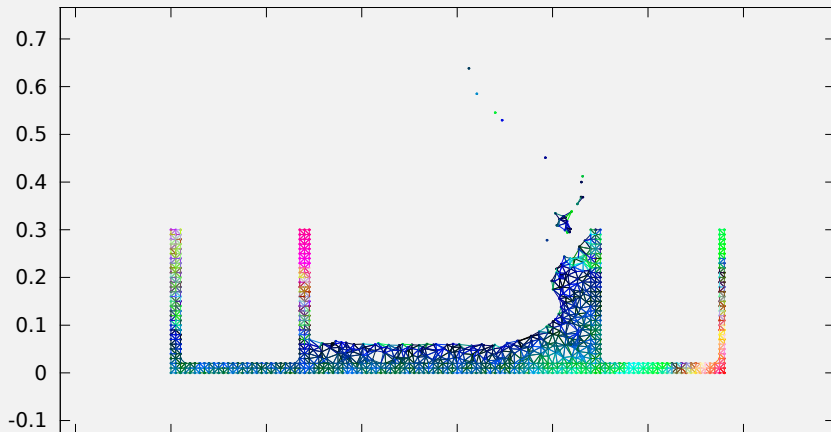
Water column collapse : pressure sensitivity distribution

Time = 0.56 sec, Color = dpdrho (red = 5.87928, blue = 0.000255144)



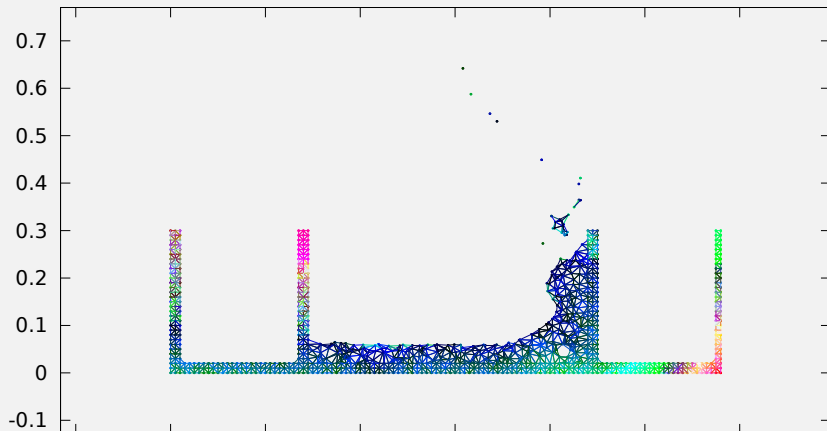
Water column collapse : pressure sensitivity distribution

Time = 0.5700000000000001 sec, Color = dpdrho (red = 5.55543, blue = 0.00011223)



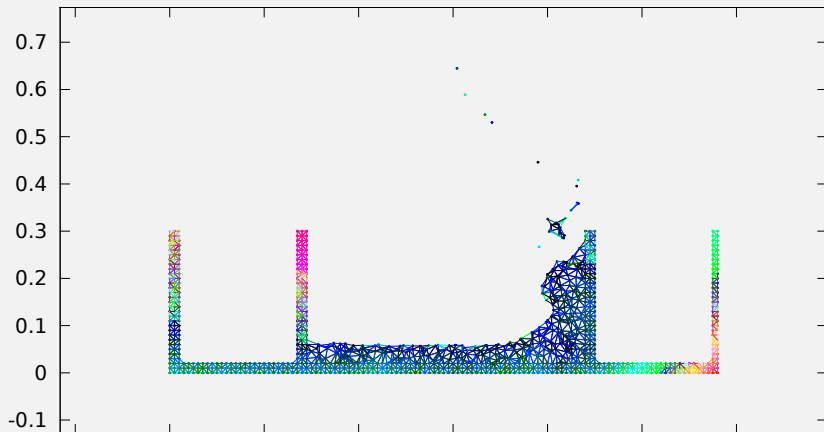
Water column collapse : pressure sensitivity distribution

Time = 0.58 sec, Color = dpdrho (red = 5.40235, blue = 0.000168255)



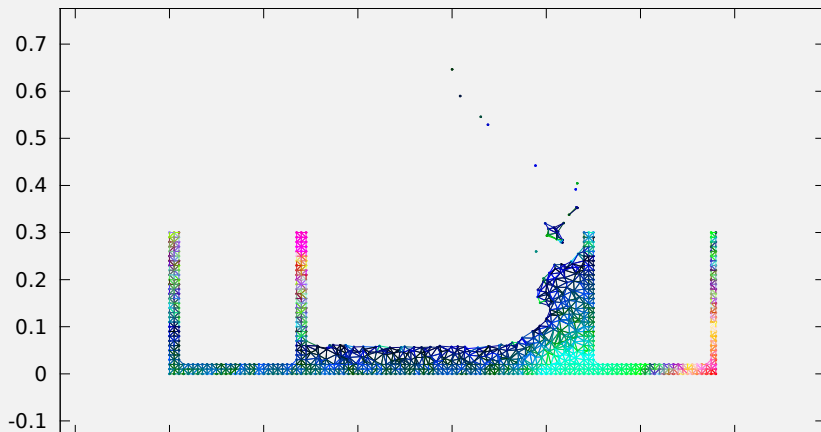
Water column collapse : pressure sensitivity distribution

Time = 0.59 sec, Color = dpdrho (red = 5.03035, blue = 0.00049999)



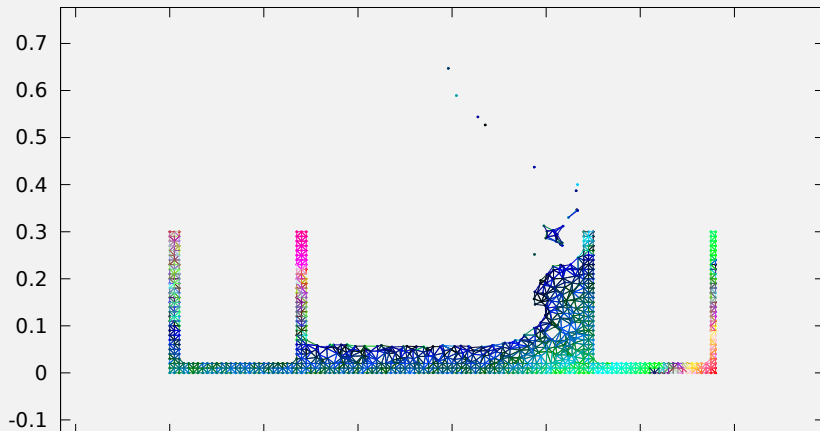
Water column collapse : pressure sensitivity distribution

Time = 0.6 sec, Color = dpdrho (red = 6.35154, blue = 1.79959e-5)



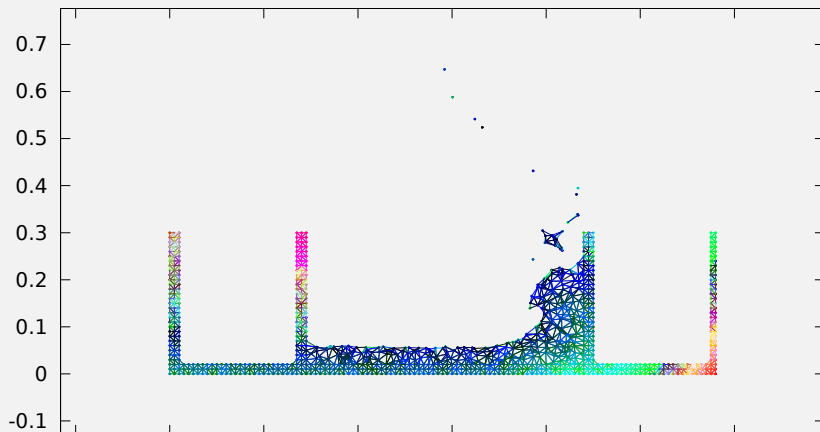
Water column collapse : pressure sensitivity distribution

Time = 0.61 sec, Color = dpdrho (red = 5.45518, blue = 0.000220668)



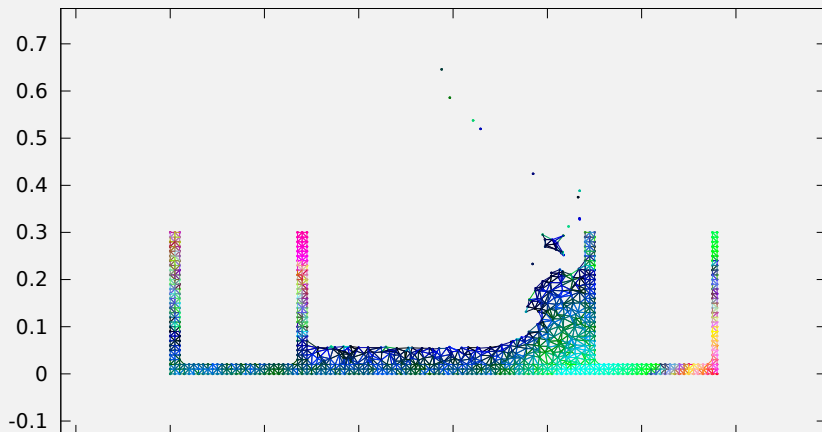
Water column collapse : pressure sensitivity distribution

Time = 0.62 sec, Color = dpdrho (red = 5.43307, blue = 0.000260069)



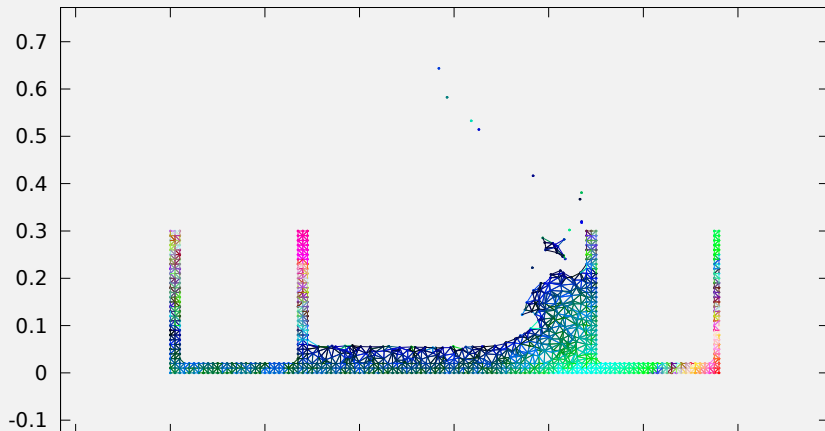
Water column collapse : pressure sensitivity distribution

Time = 0.63 sec, Color = dpdrho (red = 5.66761, blue = 2.97928e-5)



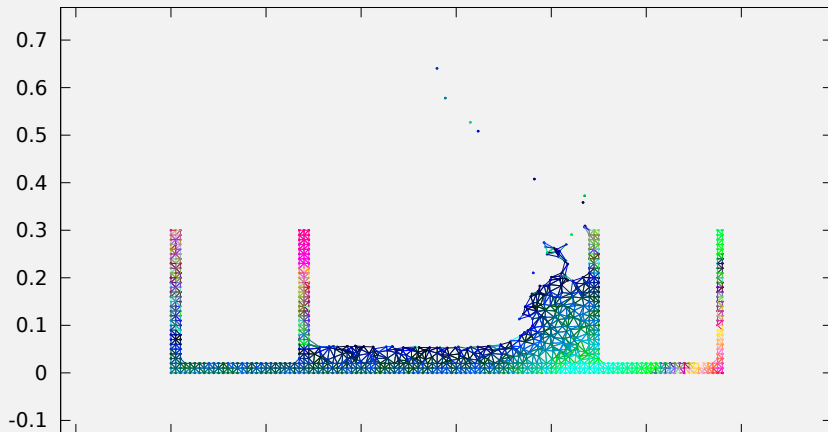
Water column collapse : pressure sensitivity distribution

Time = 0.64 sec, Color = dpdrho (red = 5.60577, blue = 2.32409e-5)



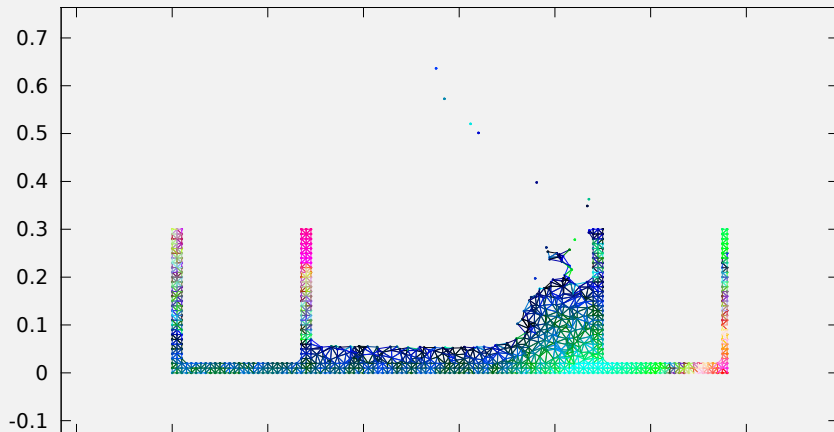
Water column collapse : pressure sensitivity distribution

Time = 0.65 sec, Color = dpdrho (red = 5.62742, blue = 1.41034e-5)



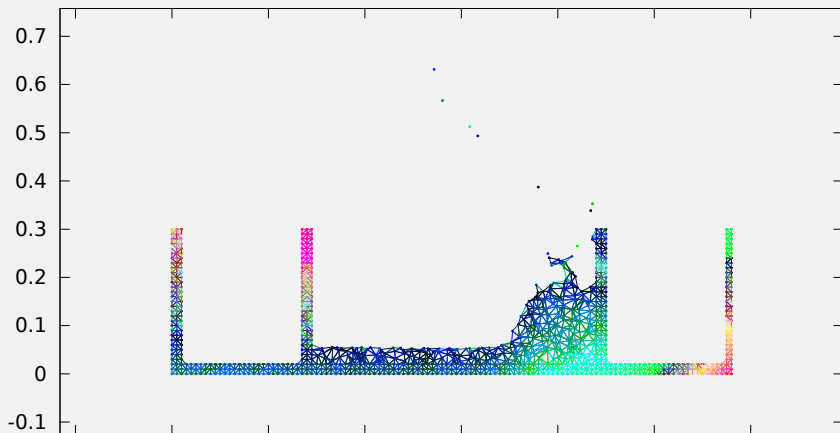
Water column collapse : pressure sensitivity distribution

Time = 0.66 sec, Color = dpdrho (red = 5.59018, blue = 0.000144775)



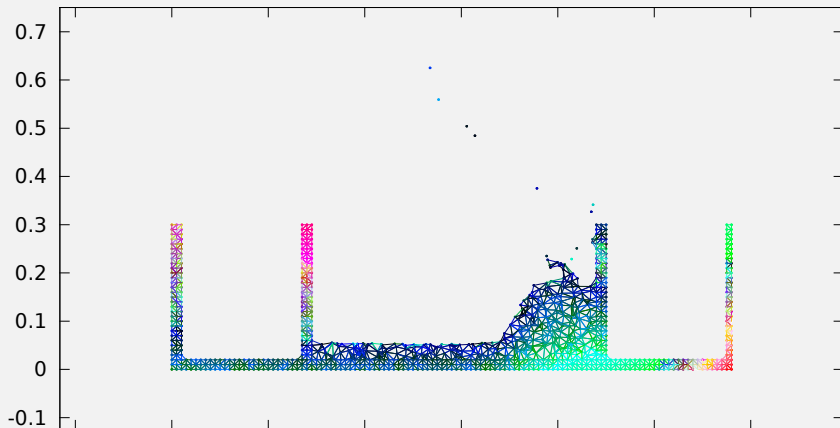
Water column collapse : pressure sensitivity distribution

Time = 0.67 sec, Color = dpdrho (red = 5.58905, blue = 0.000261916)



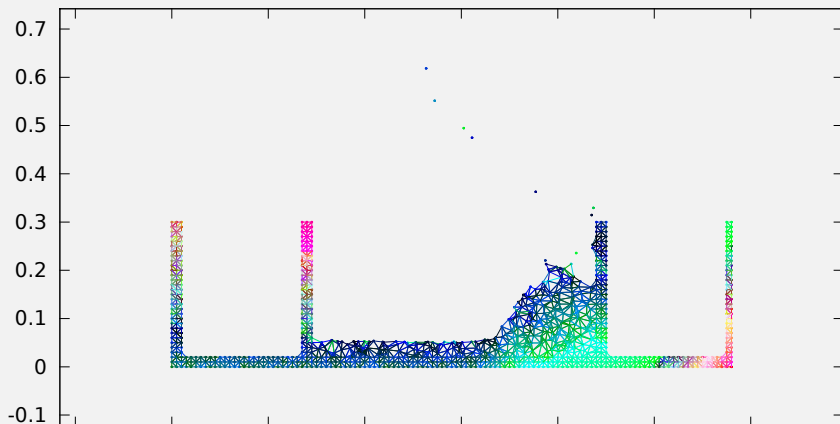
Water column collapse : pressure sensitivity distribution

Time = 0.68 sec, Color = dpdrho (red = 5.40665, blue = 0.000219889)



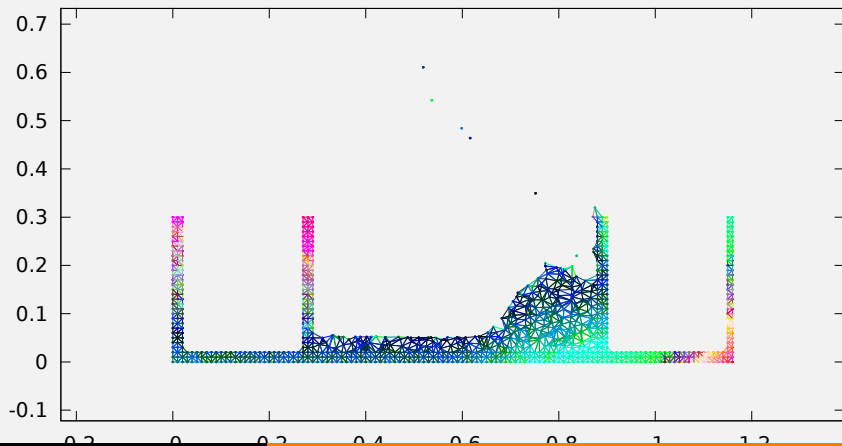
Water column collapse : pressure sensitivity distribution

Time = 0.6900000000000001 sec, Color = dpdrho (red = 5.60342, blue = 0.00029894)



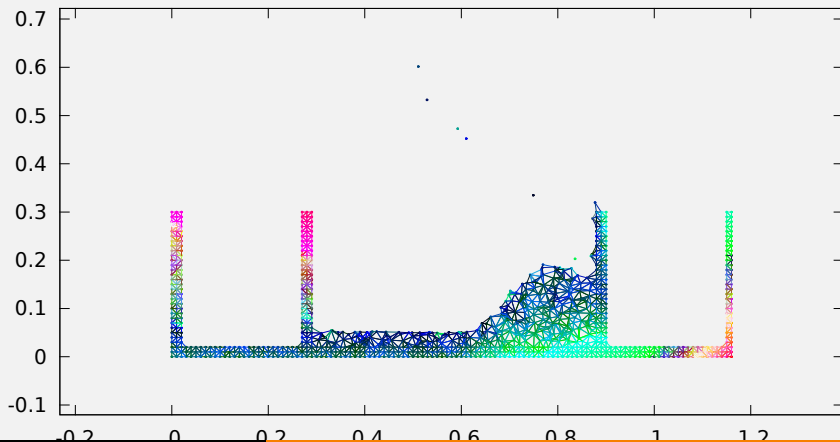
Water column collapse : pressure sensitivity distribution

Time = 0.7000000000000001 sec, Color = dpdrho (red = 5.03951, blue = 0.00035290)



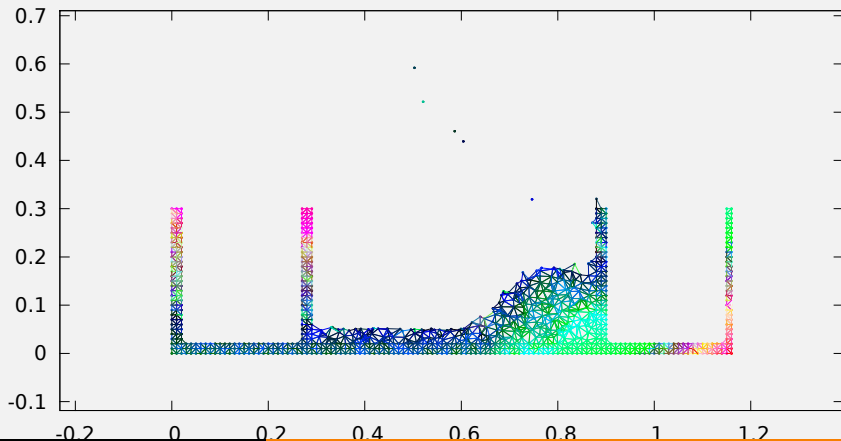
Water column collapse : pressure sensitivity distribution

Time = 0.71 sec, Color = dpdrho (red = 4.94502, blue = 0.000577185)



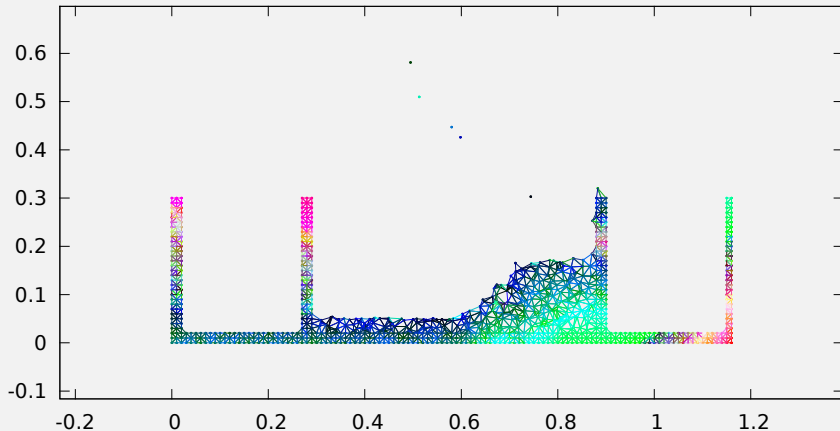
Water column collapse : pressure sensitivity distribution

Time = 0.72 sec, Color = dpdrho (red = 5.28484, blue = 0.000220029)



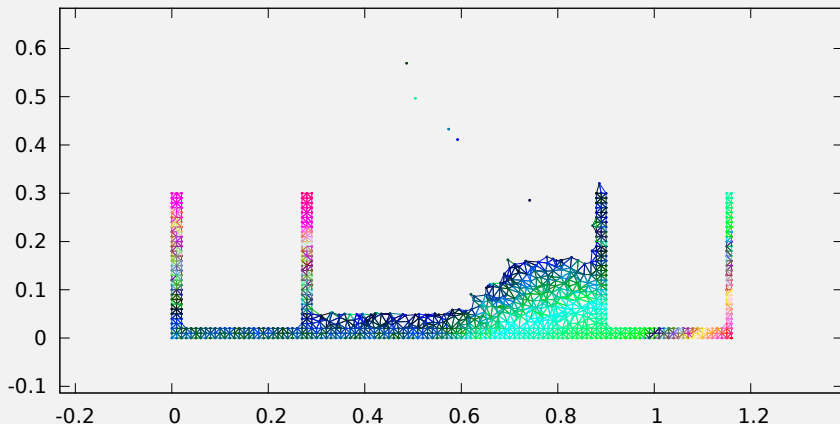
Water column collapse : pressure sensitivity distribution

Time = 0.73 sec, Color = dpdrho (red = 5.06034, blue = 0.000308997)



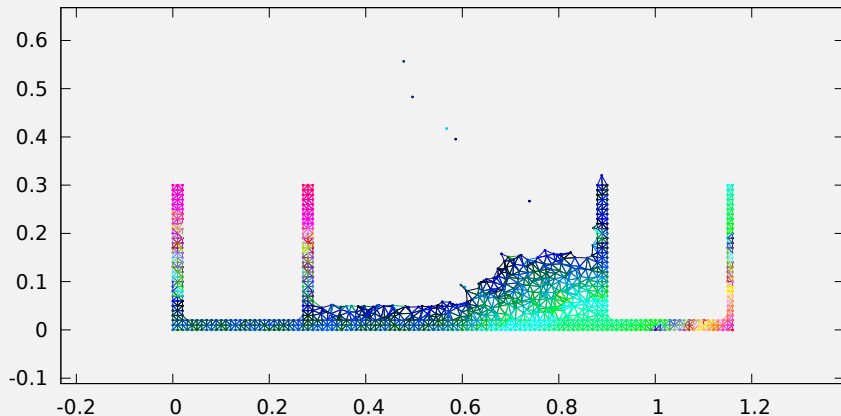
Water column collapse : pressure sensitivity distribution

Time = 0.74 sec, Color = dpdrho (red = 4.98384, blue = 5.30768e-5)



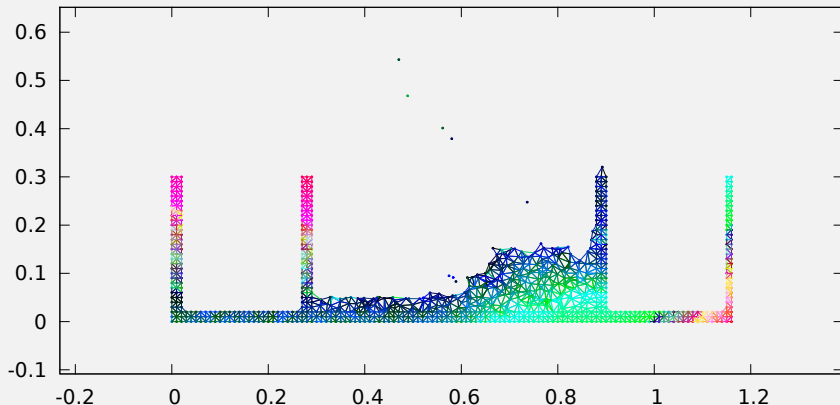
Water column collapse : pressure sensitivity distribution

Time = 0.75 sec, Color = dpdrho (red = 4.74055, blue = 0.00032761)



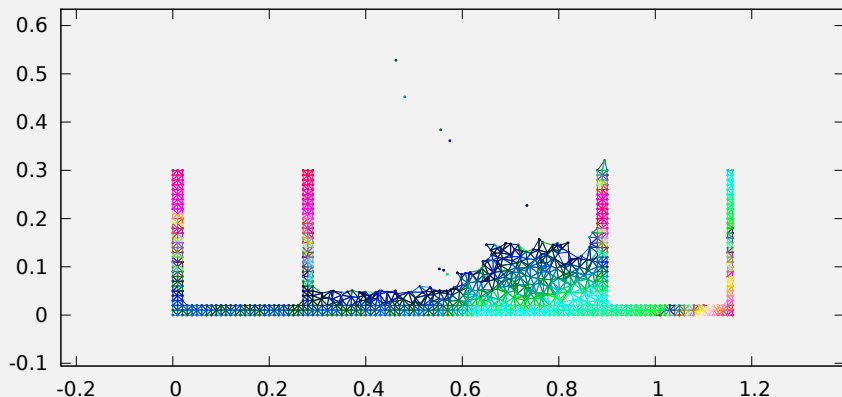
Water column collapse : pressure sensitivity distribution

Time = 0.76 sec, Color = dpdrho (red = 4.59307, blue = 0.00188903)



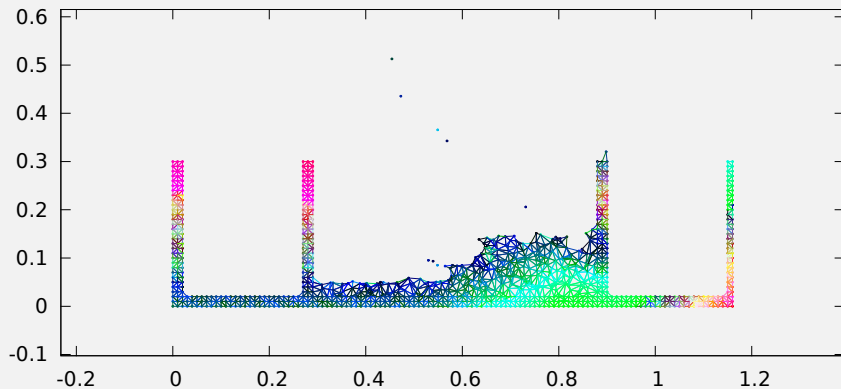
Water column collapse : pressure sensitivity distribution

Time = 0.77 sec, Color = dpdrho (red = 4.25752, blue = 2.13815e-5)



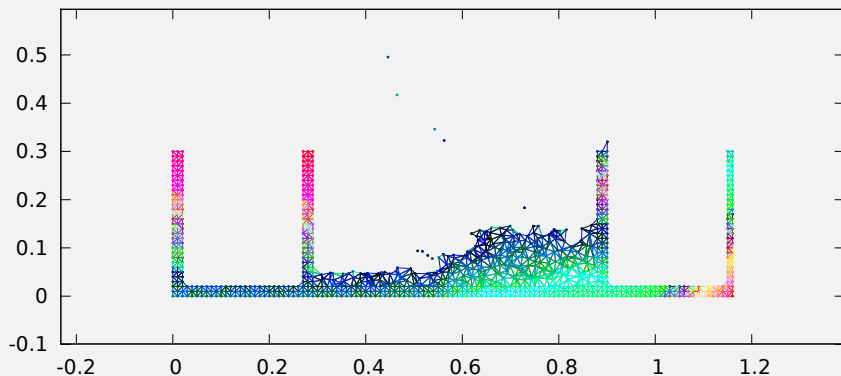
Water column collapse : pressure sensitivity distribution

Time = 0.78 sec, Color = dpdrho (red = 4.75224, blue = 0.000104296)



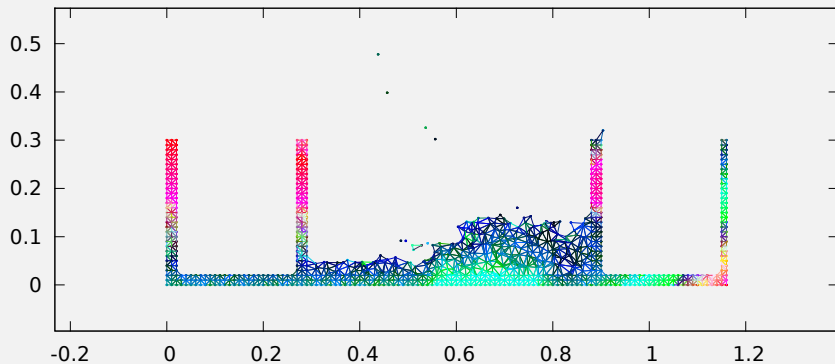
Water column collapse : pressure sensitivity distribution

Time = 0.79 sec, Color = dpdrho (red = 4.18605, blue = 8.04456e-5)



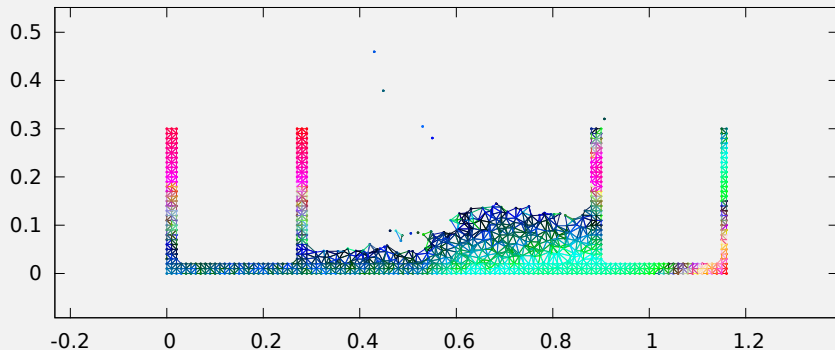
Water column collapse : pressure sensitivity distribution

Time = 0.8 sec, Color = dpdrho (red = 3.22712, blue = 0.000628374)



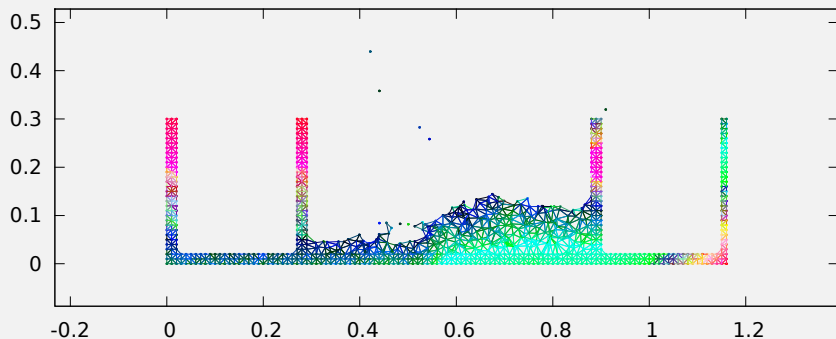
Water column collapse : pressure sensitivity distribution

Time = 0.81 sec, Color = dpdrho (red = 3.76871, blue = 0.000683035)



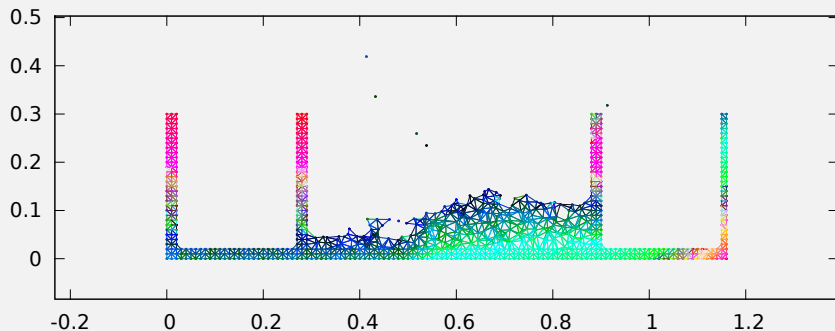
Water column collapse : pressure sensitivity distribution

Time = 0.8200000000000001 sec, Color = dpdrho (red = 3.8341, blue = 0.000480312)



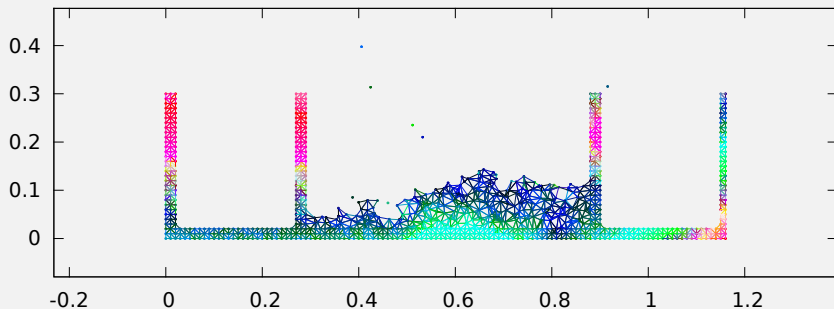
Water column collapse : pressure sensitivity distribution

Time = 0.8300000000000001 sec, Color = dpdrho (red = 3.77814, blue = 0.00026565)



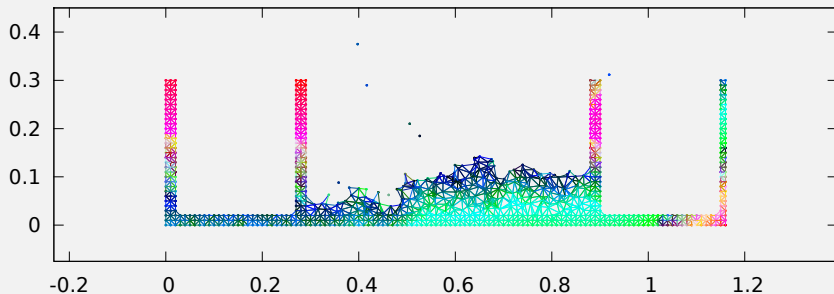
Water column collapse : pressure sensitivity distribution

Time = 0.84 sec, Color = dpdrho (red = 3.11609, blue = 0.000413546)



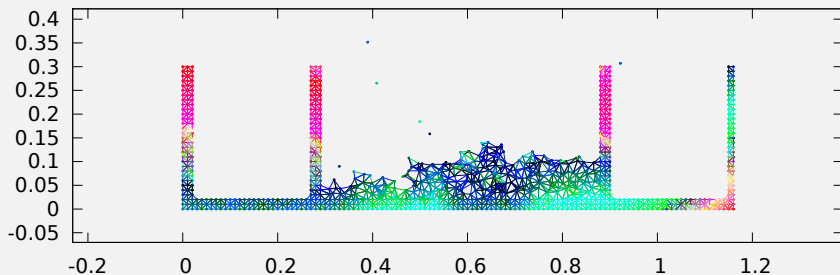
Water column collapse : pressure sensitivity distribution

Time = 0.85 sec, Color = dpdrho (red = 3.73979, blue = 0.000165726)



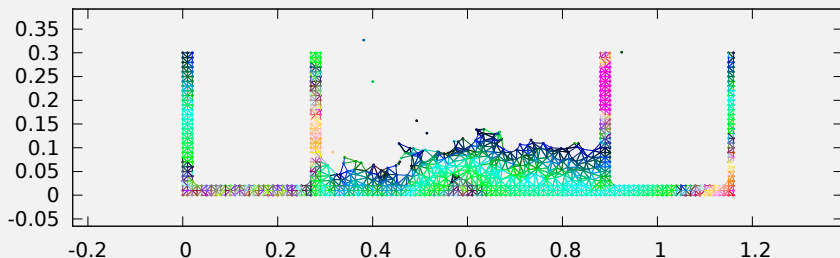
Water column collapse : pressure sensitivity distribution

Time = 0.86 sec, Color = dpdrho (red = 3.45419, blue = 0.000323471)



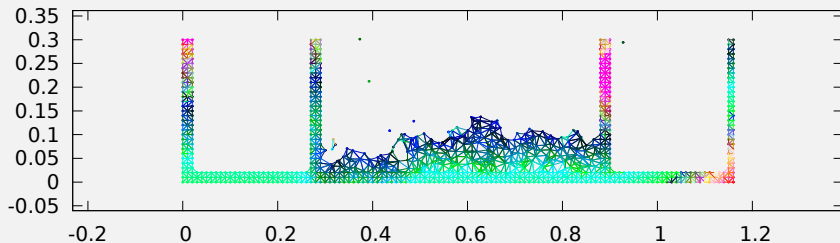
Water column collapse : pressure sensitivity distribution

Time = 0.87 sec, Color = dpdrho (red = 3.2966, blue = 0.00270993)



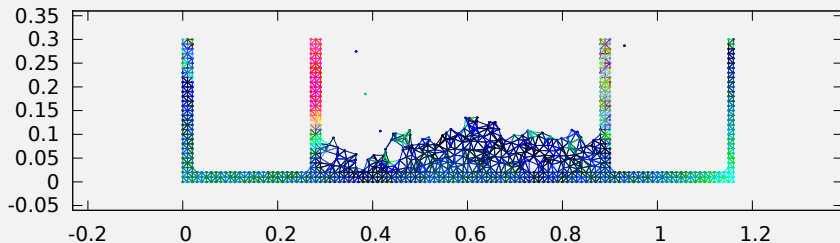
Water column collapse : pressure sensitivity distribution

Time = 0.88 sec, Color = dpdrho (red = 3.22536, blue = 0.00190607)



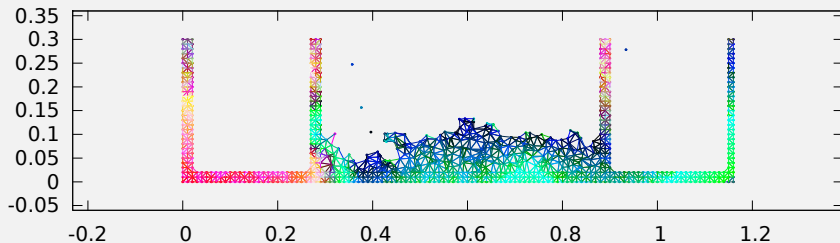
Water column collapse : pressure sensitivity distribution

Time = 0.89 sec, Color = dpdrho (red = 11.3049, blue = 0.000628666)



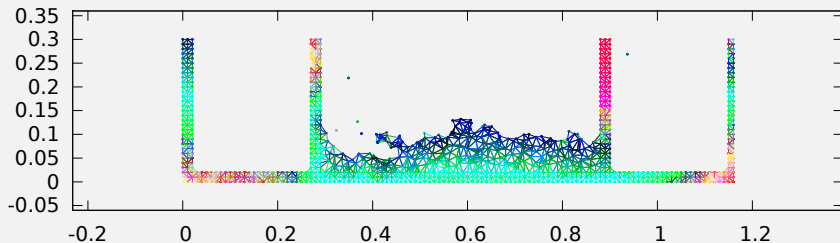
Water column collapse : pressure sensitivity distribution

Time = 0.9 sec, Color = dpdrho (red = 5.61157, blue = 4.7549e-5)



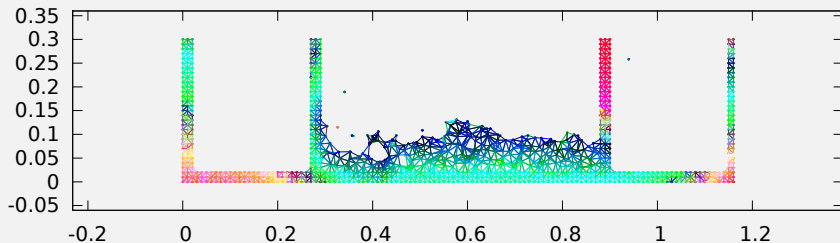
Water column collapse : pressure sensitivity distribution

Time = 0.91 sec, Color = dpdrho (red = 2.91999, blue = 6.60746e-5)

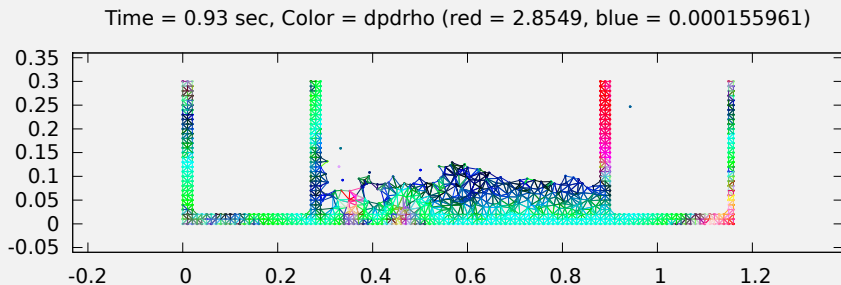


Water column collapse : pressure sensitivity distribution

Time = 0.92 sec, Color = dpdrho (red = 2.88316, blue = 0.000260147)

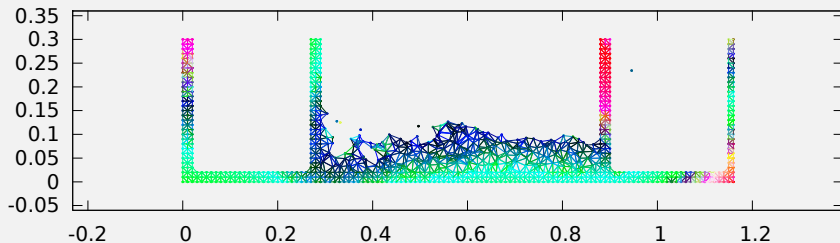


Water column collapse : pressure sensitivity distribution



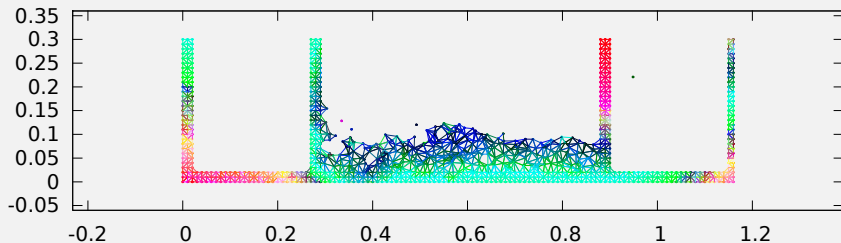
Water column collapse : pressure sensitivity distribution

Time = 0.9400000000000001 sec, Color = dpdrho (red = 2.82612, blue = 0.0002188)



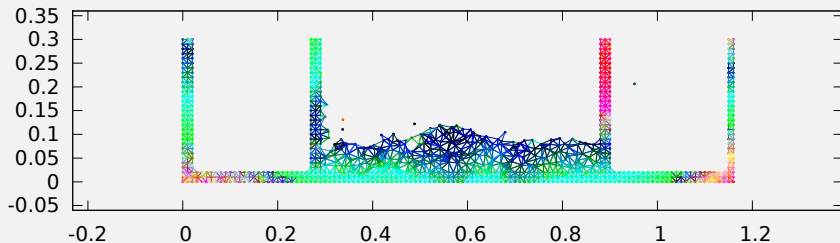
Water column collapse : pressure sensitivity distribution

Time = 0.9500000000000001 sec, Color = dpdrho (red = 3.02636, blue = 0.0006696)



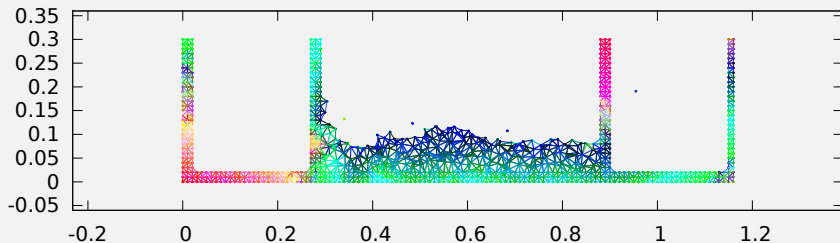
Water column collapse : pressure sensitivity distribution

Time = 0.96 sec, Color = dpdrho (red = 2.78377, blue = 0.000104537)



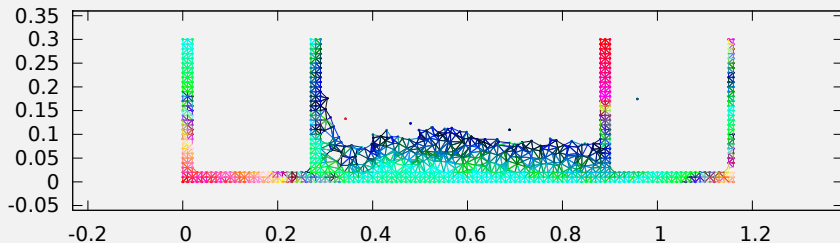
Water column collapse : pressure sensitivity distribution

Time = 0.97 sec, Color = dpdrho (red = 4.03012, blue = 0.000341349)



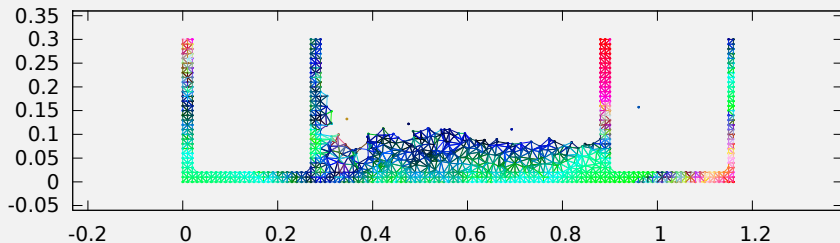
Water column collapse : pressure sensitivity distribution

Time = 0.98 sec, Color = dpdrho (red = 2.9524, blue = 4.2978e-5)



Water column collapse : pressure sensitivity distribution

Time = 0.99 sec, Color = dpdrho (red = 3.38104, blue = 0.000268518)



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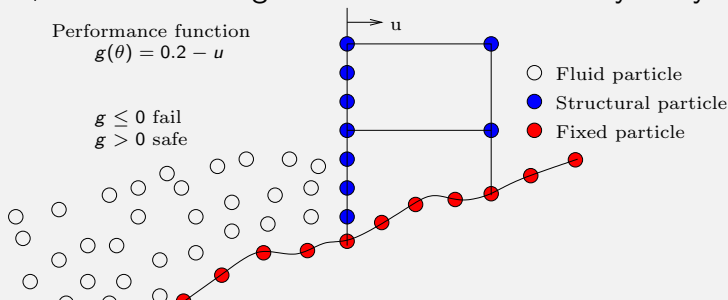
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Questions?