A one hour course on
Nonlinear Modeling of Structures

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Computational framework for earthquake simulation

at Berkeley

- OpenSees (OPEN Software for Earthquake Engineering Simulation)
  http://opensees.berkeley.edu (last release 2.2.0, August 2010)
- FEDEAS^{Mat}Lab for teaching and concept development
  http://fedeaslab.berkeley.edu (last release 3.1 July 2010)

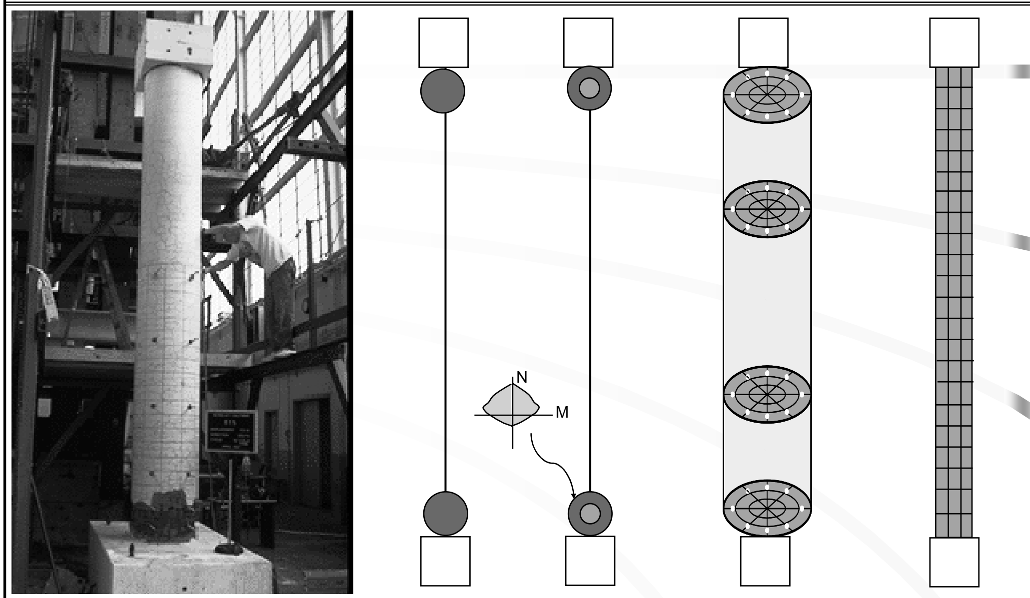
Element Selection in EQ Engineering Practice

- Criteria
  - Economy in model development and result interpretation considering parameter sensitivity and multiple ground motions
  - Knowledge and experience of analysis team
  - Detail of response (global, regional or local) and accuracy
- Selection (in decreasing popularity and increasing cost and expectations)
  - Linear elastic elements of any type (1d, 2d, 3d)
  - Nonlinear beam and column elements with “plastic hinges”
  - Nonlinear beam and column elements with material response integration (fiber, fiber-hinge, inelastic beam-column finite elements)
  - 2d and 3d finite elements (few robust constitutive models, few advanced features when compared with expectations: e.g. bond-slip, buckling of reinforcement, large discrete cracks, shear sliding, local buckling, fatigue, tearing of steel, etc)

Beam-Column Models

- Concentrated plasticity models = one rotational spring at each end + elastic element
  - Advantages: relatively simple, good for interface effects (e.g. bar pull-out)
  - Disadvantages: force-deformation of rotational spring depends on geometry and moment distribution; relation to strains requires “plastic hinge length”; interaction of axial force, moment good for “metallic” elements; more complex interaction questionable; numerical robustness difficult
- Distributed inelasticity models (FE model) = consistent integration of section response at specific “control” or monitoring points
  - Advantages: versatile and consistent; material response can be incorporated in section response; interaction of axial force and moment (and shear and torsion) can be rationally developed, thus numerical robustness is possible
  - Disadvantages: can be expensive (not clear), require good understanding of integration to determine validity of strains and local response

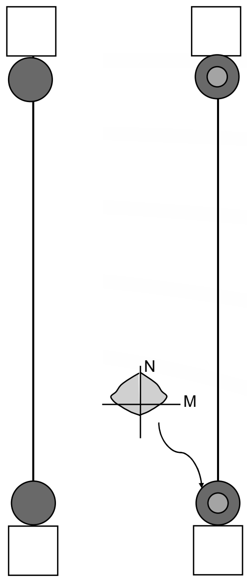
Structural Beam-Column Models



Distributed inelasticity models

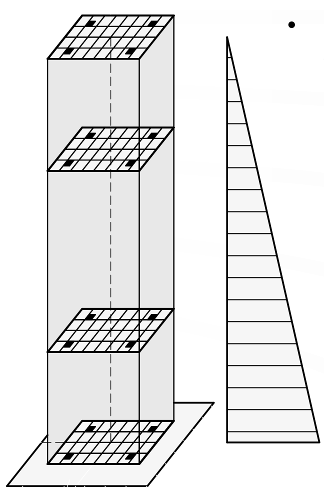
- 2 monitoring points usually at element ends = inelastic zone model
  - Good for columns and girders with low gravity loads
  - Consistent location of integration point?
  - Value of fixed length of inelastic zone?
  - Good for softening response (Fenves/Scott, ASCE 2006)
  - Variable inelastic zone element (CLLee/FCF, ASCE 2009); good for hardening response
- \>2 monitoring points
  - Good for girders with significant gravity loads
  - 4-5 integration points are advisable (“good plastic hinge length value with corresponding integration weights)
  - One element per girder -> avoid very high local deformation values

Beam-Column Models: Concentrated Inelasticity



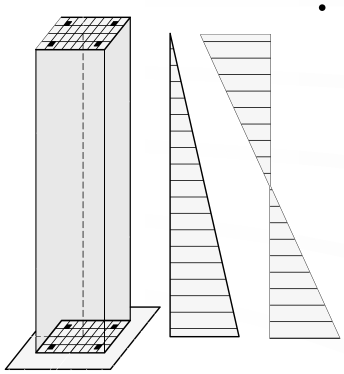
- Concentrated plasticity models = one or more rotational springs at each end + elastic element
  - Advantages:
    relatively simple, good(?) for interface effects
    (e.g. shear sliding, rotation due to bar pull-out)
  - Disadvantages:
    properties of rotational spring depend on geometry and moment distribution; relation to strains requires “plastic hinge length”;
    interaction of axial force, moment and shear ????;
    generality??? numerical robustness???

Beam-Column Models: Distributed Inelasticity



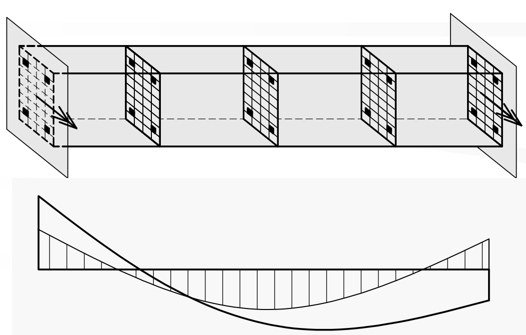
- Distributed inelasticity models (1d FE model) = consistent integration of section response at specific “control” or monitoring points
  - Advantages:
    versatile and consistent;
    section response from integration of material response
    thus N-My-Mz interaction (Bernoulli)
    (shear and torsion?? Timoshenko, …)
    thus numerical robustness is possible
  - Disadvantages:
    can be expensive (what is the price $$$$) with “wasted sections” for localized inelasticity
    inaccuracy of local response (localization)
    thus better understanding of theory for interpretation of local response and damage is necessary

“Economic” Distributed(?) inelasticity models for Columns



- 2 monitoring points at element ends = inelastic zone model
  - Good for columns and girders with low gravity loads
  - N-My-Mz interaction straightforward
  - shear and torsion ???
  - Consistent location of integration point?
  - Value of fixed length of inelastic zone?
  - Good for softening response (Fenves/Scott, ASCE 2006)
  - Hardening response → Spreading inelastic zone element SIZE (CL Lee/FCF, ASCE 2009 to appear) without N-M interaction; is generalization possible??

Good Distributed inelasticity models for Girders

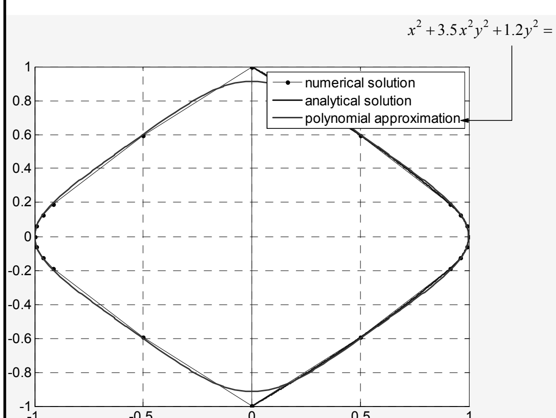


- For girders with significant gravity loads
  - 4-5 integration points are advisable (“good plastic hinge length value with corresponding integration weights)
  - One element per girder -> avoid very high local deformation values

Section Response for Distributed Inelasticity Models

- Section resultant formulation based on plasticity theory, damage, etc, etc
  - Relatively economical, effects can be "lumped" in a composite section response
  - Generalization and extension to other than the calibrated cases may not be straightforward; hardening is very difficult to incorporate let alone softening
  - Section geometry must always be accounted for (are limit capacities sufficient?)
- Integration of 1d, 1 ½ d, 2d, and 3d material response
  - For midpoint integration the name fiber model is used; but other integration methods are possible
  - For many fibers it can be expensive; how many fibers should be used?

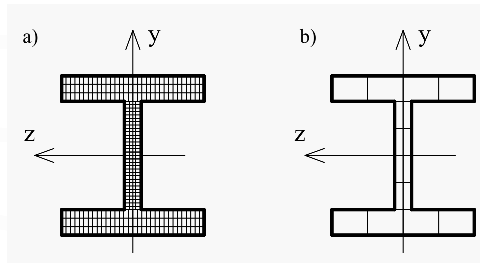
Section Response for "Distributed" Inelasticity Models

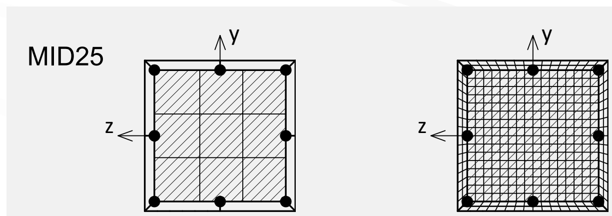


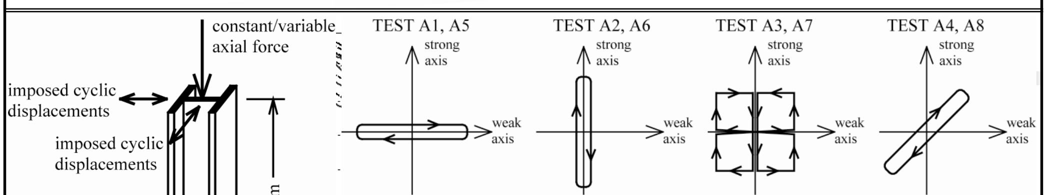
- Section resultant formulation based on plasticity theory, damage, etc, etc
  - Relatively economical, effects can be "lumped" in a composite section response
  - Generalization and extension to other than the calibrated cases may not be straightforward; hardening is difficult to incorporate, let alone softening
  - Section geometry must always be accounted for (are limit capacities sufficient?)
  - Hardening ???
  - RC section, softening ???

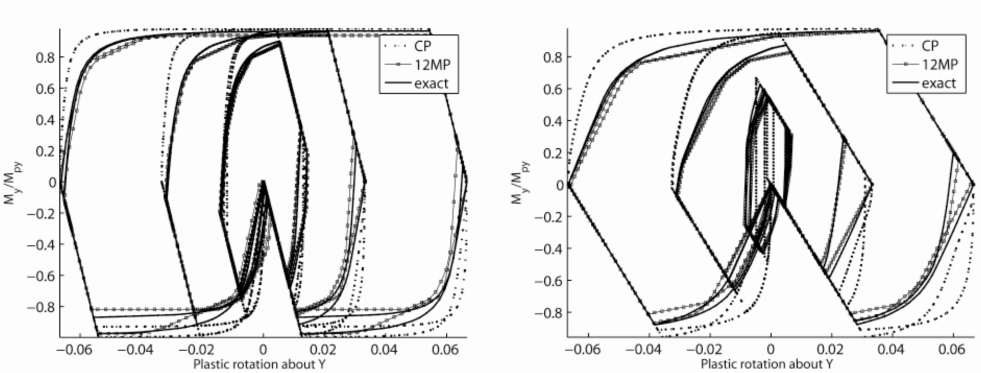
Section Response for "Distributed" Inelasticity Models

- Integration of 1d, 1 ½ d, 2d, and 3d material response
  - For midpoint integration the name fiber model is used; are other integration methods better??
  - how many fibers should be used and for what purpose?









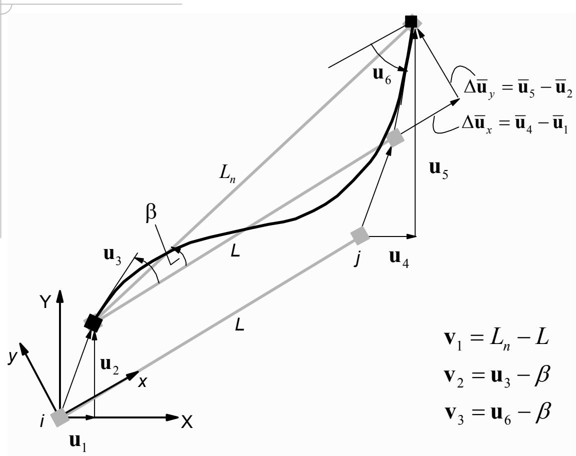
Two key ideas for beam-column elements

- How to incorporate nonlinear geometry under large displacements?
  - Formulate the element in the basic reference system without rigid body modes
  - Use corotational formulation to transform basic variables to global system
  - Keep element force-deformation relation in basic system simple: use linear geometry under small deformations
  - Use one element per structural member (see point below)
  - If second-order effects within structural member are significant, break structural member into 2 elements by inserting middle node
- How to formulate a robust frame element?
  - Use Hu-Washizu functional with independent interpolation of forces (exact under certain conditions), displacements and section deformations (Taylor/Filippou/Saritas Comp Mech, 2003)
  - No need for mesh refinement; keep integration points to 4, or 2 with variable integration weight (5 respectively 3 points for girders under large element loads)

Advantages of Mixed Formulation in Corotational Framework

- Nonlinear geometry is uncoupled from basic element response; thus, geometric transformation classes (e.g. large displacements, P-Δ, linear) can be implemented once for all user frame elements (OpenSees, FEDEASLab)
- Force interpolation functions are exact under certain conditions
- Element deformations arise in weak form and represent well the inelastic strain distribution (see next point)
- Without need for mesh and integration point refinement there is no danger of localization; local strains are relatively accurate as long as the “plastic hinge length” is accurate (= integration weight of end points) – fiber hinge is a good choice for columns under plastic or softening response
- The effect of distributed element loads can be accounted for exactly with the force interpolation functions

Corotational formulation for large displacement geometry



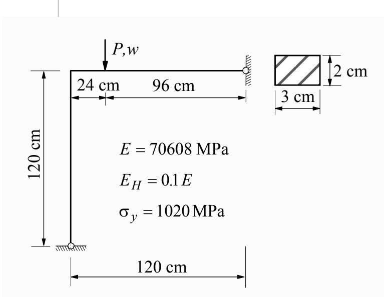
$$\frac{\partial \mathbf{v}}{\partial \mathbf{u}} = \mathbf{a}_{gu}$$

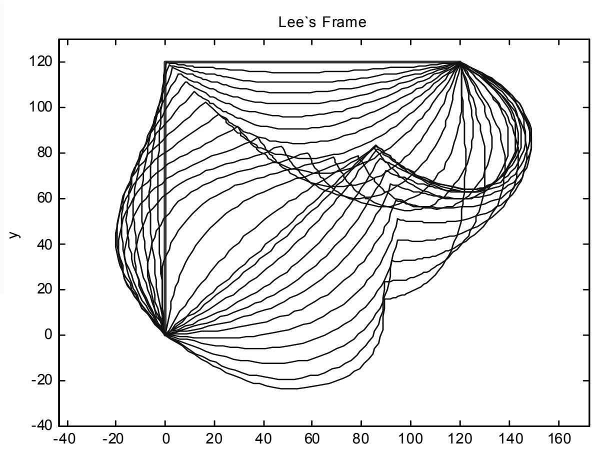
$$\mathbf{p} = \mathbf{a}_{gu}^{\mathrm{T}} \mathbf{q}$$

$$\mathbf{k}_e = \mathbf{k}_g + \mathbf{a}_{gu}^{\mathrm{T}} \mathbf{k}_t \mathbf{a}_{gu}$$

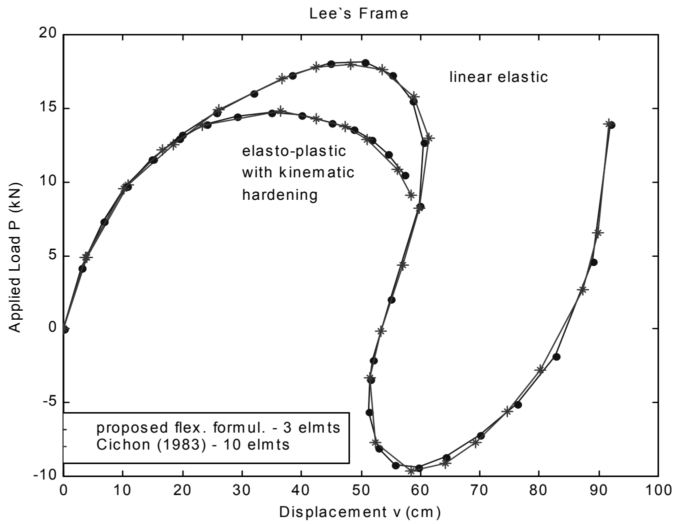
Nonlinear geometry with large displacements

Frame by Lee et al. (1968)

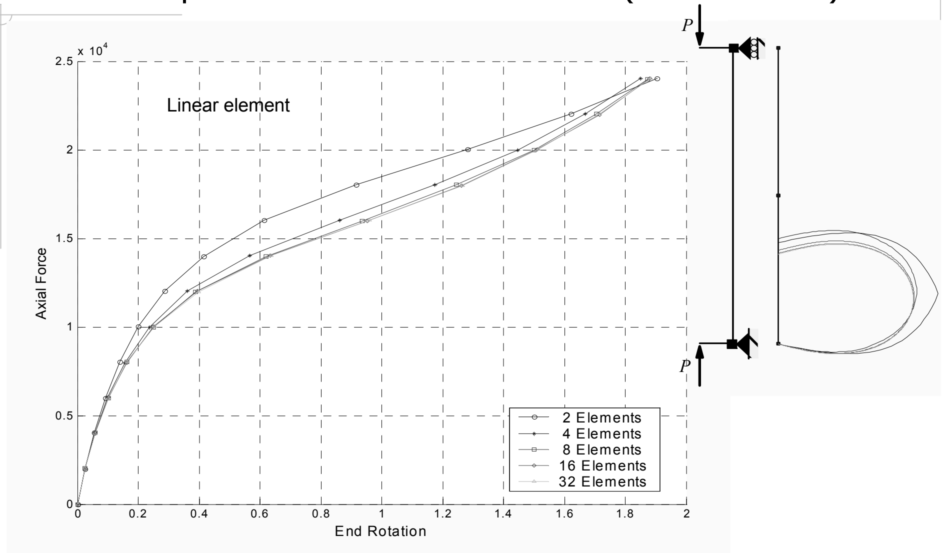




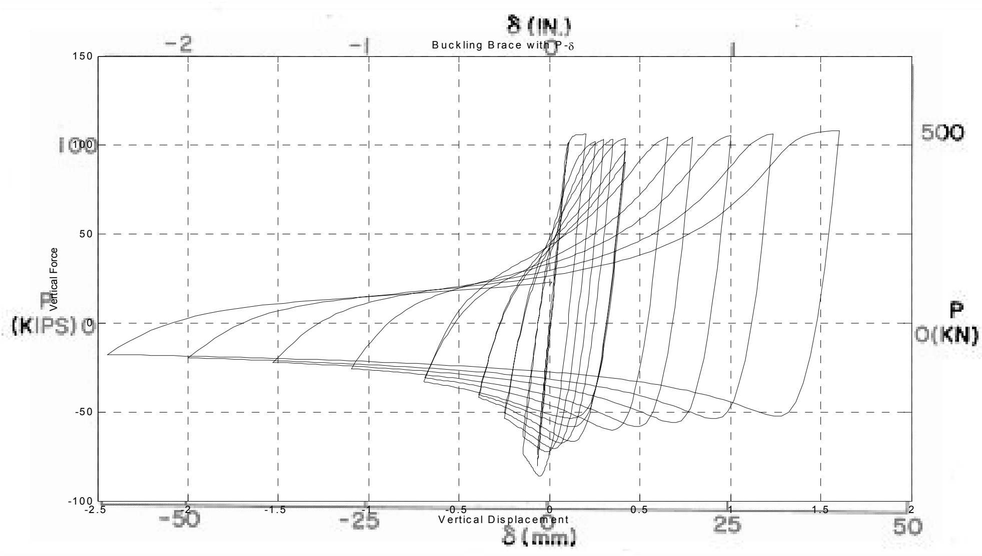
Lee Frame (Lee et al. 1968)



Force-displacement of eccentric column (basic element)

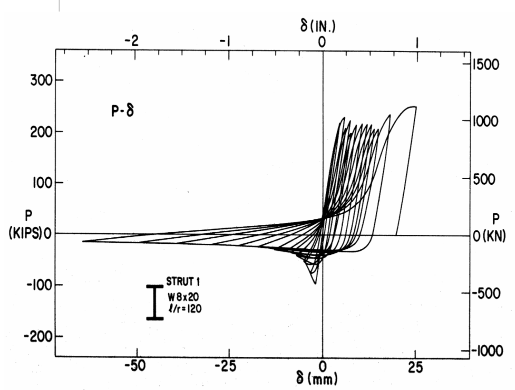


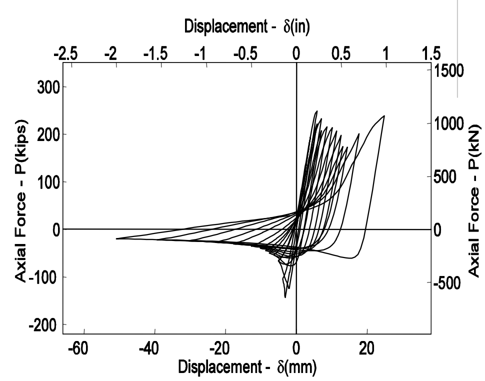
Brace buckling (Black and Popov 1980)



PEER 2004 Annual Meeting

Brace buckling (Black and Popov 1980)





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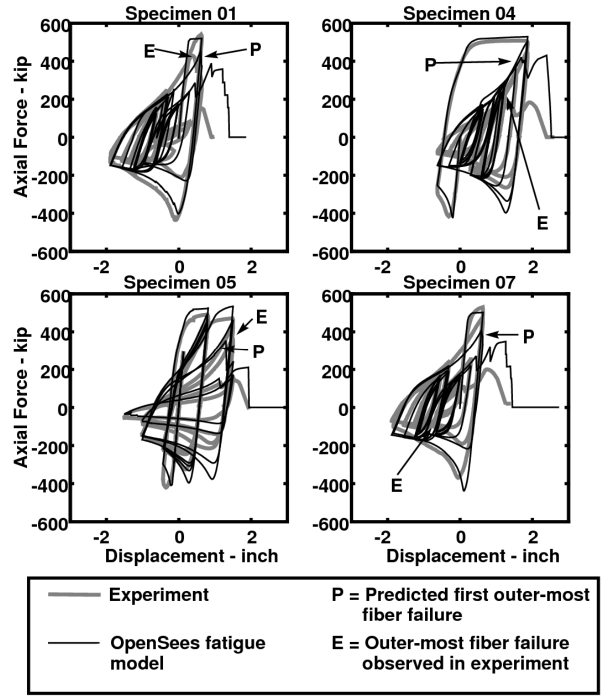
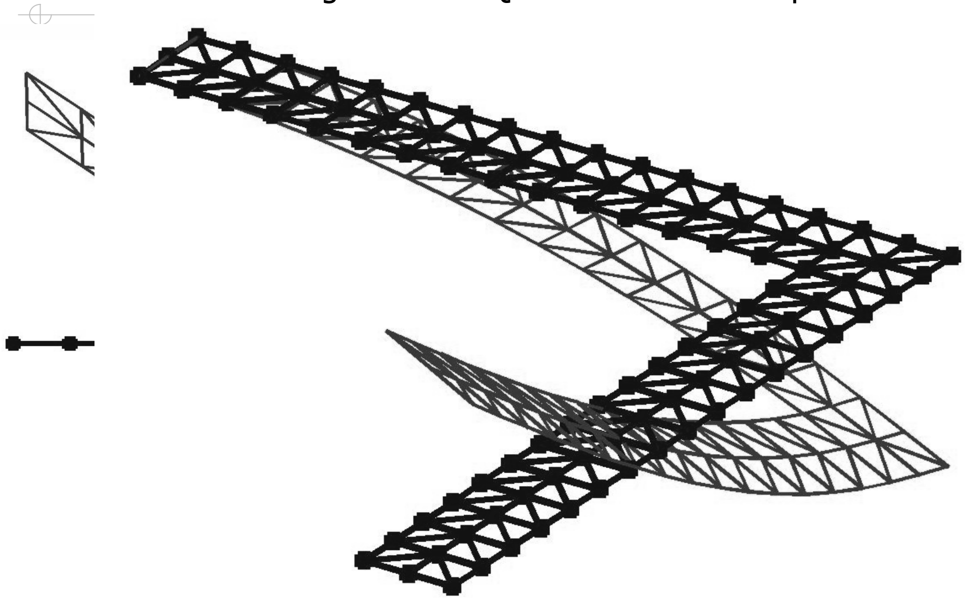
P. Uriz and S. Mahin
UC Berkeley



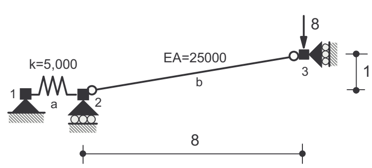
Lateral Buckling: CST and Quad element in 3d space



Newton-Raphson algorithm and its variants

- The Newton-Raphson algorithm requires that a new tangent stiffness matrix be assembled at every iteration of every load step. This means that the linear system of equations for the displacement correction needs to be solved from scratch at every iteration of every load step. For very large structural models this is a very expensive proposition.
- In the modified Newton-Raphson method the tangent matrix is not updated at every iteration, but only once at the beginning of each load step. In the initial stiffness method, the initial stiffness is used throughout the incremental analysis. Alternative strategies that update the stiffness matrix every so often are also possible. If the stiffness matrix is not updated, the last decomposition of the stiffness matrix is used for the solution of the linearized equilibrium equations and only the load changes at each iteration.
- Finally, quasi-Newton methods do not use the tangent stiffness matrix of the structure but obtain secant stiffness approximations of the inverse of the stiffness matrix from the displacement vectors of previous iterations. Among the best known quasi-Newton methods is the BFGS method, which was originally developed for nonlinear optimization problems. For a brief description of the method consult Bathe's 1982 book pp. 759-761.

Response of 2-dof truss under 2 load increments with $\Delta\lambda = 0.5$




```
no_step  = 2;
Dlam0    = 0.5;
tol      = 1.e-16;
maxiter  = 10;
S_Initialize_State
Pf = zeros(nf,1);
S_SimpInitialize
for k=1:no_step
    StifUpdt = 'yes';
    S_SimpIncrement
    StifUpdt = 'yes';
    S_SimpIterate
    if (ConvFlag)
        S_Update_State
    end
end
```

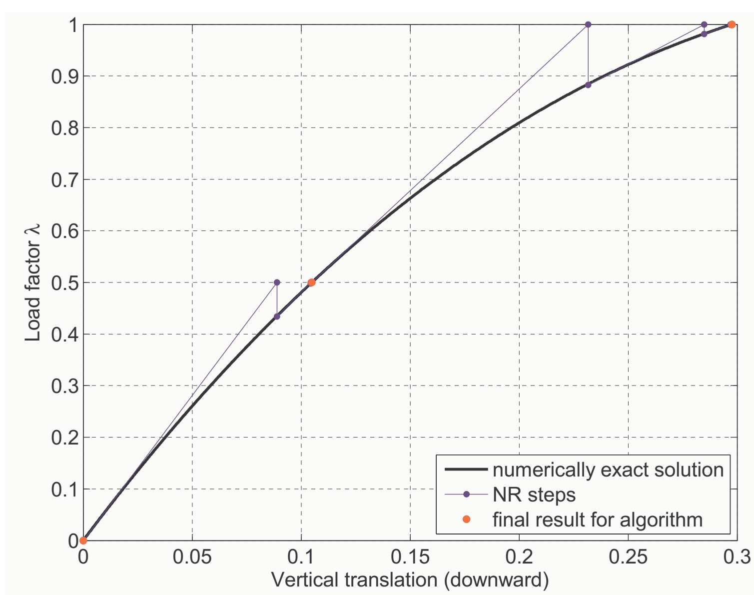
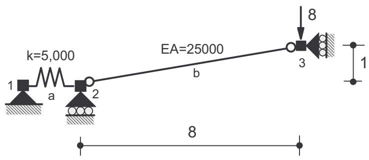


Figure: Newton-Raphson method

Response of 2-dof truss under 2 load increments with $\Delta\lambda = 0.5$




```
no_step  = 2;
Dlam0    = 0.5;
tol      = 1.e-16;
maxiter = 10;
S_Initialize_State
Pf = zeros(nf,1);
S_SimpInitialize
for k=1:no_step
    StifUpdt = 'yes';
    S_SimpIncrement
    StifUpdt = 'no';
    S_SimpIterate
    if (ConvFlag)
       S_Update_State
    end
end
```

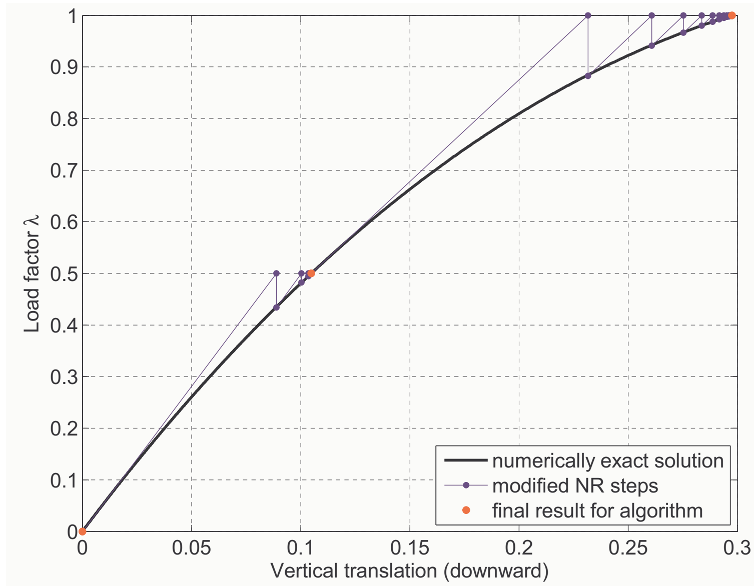
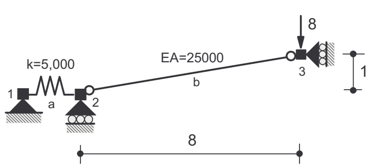


Figure: Modified Newton-Raphson method

Response of 2-dof truss under 2 load increments with $\Delta\lambda = 0.5$




```
no_step  = 2;
Dlam0    = 0.5;
tol      = 1.e-16;
maxiter = 10;
S_Initialize_State
Pf = zeros(nf,1);
S_SimpInitialize
StifUpdt = 'yes';
for k=1:no_step
    S_SimpIncrement
    StifUpdt = 'no';
    S_SimpIterate
    if (ConvFlag)
       S_Update_State
    end
end
```

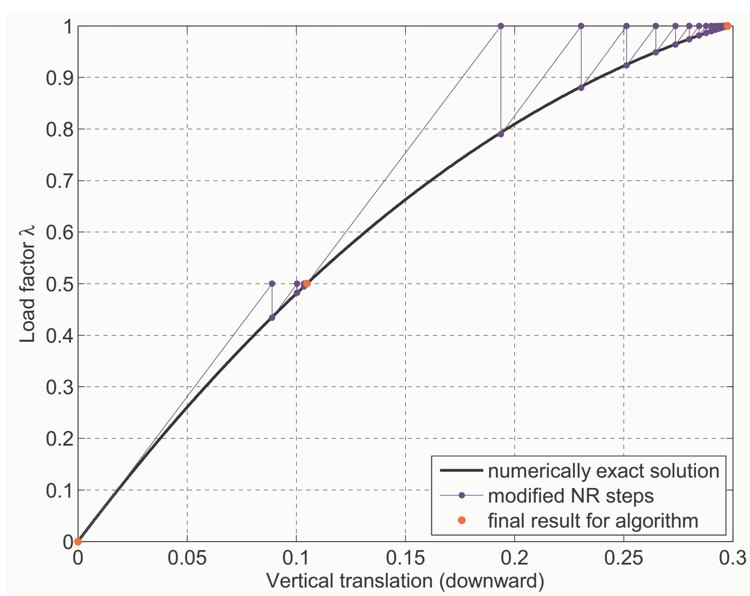
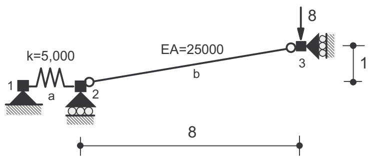


Figure: Initial stiffness method

Response of 2-dof truss under 2 load increments with $\Delta\lambda = 0.5$




```
no_step  = 2;
Dlam0    = 0.5;
tol      = 1.e-16;
maxiter  = 10;
S_Initialize_State
Pf = zeros(nf,1);
S_SimpInitialize
for k=1:no_step
    StifUpdt = 'yes';
    S_SimpIncrement
    S_Update_State
end
```

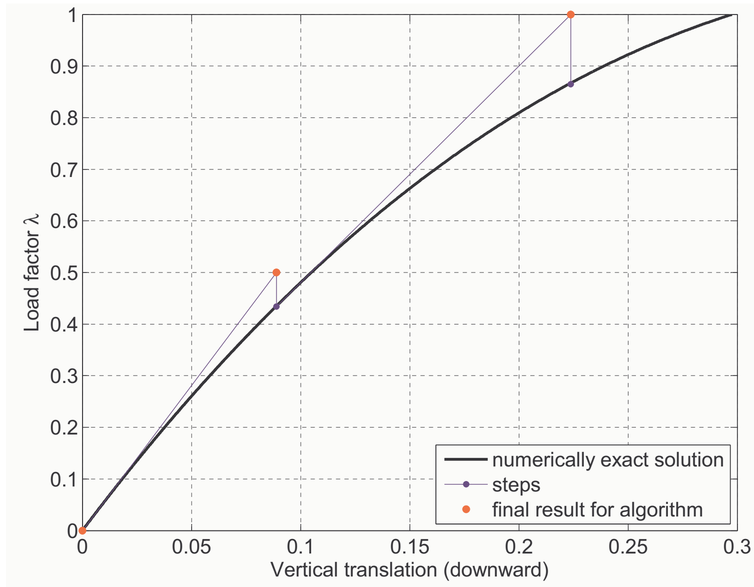
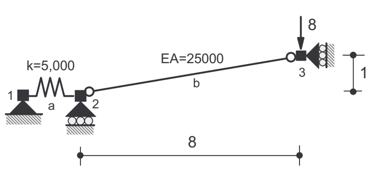


Figure: Incrementation without correction

Response of 2-dof truss under 5 load increments with $\Delta\lambda = 0.2$




```
no_step  = 5;
Dlam0    = 0.2;
tol      = 1.e-16;
maxiter  = 10;
S_Initialize_State
Pf = zeros(nf,1);
S_SimpInitialize
for k=1:no_step
    StifUpdt = 'yes';
    S_SimpIncrement
    S_Update_State
end
```

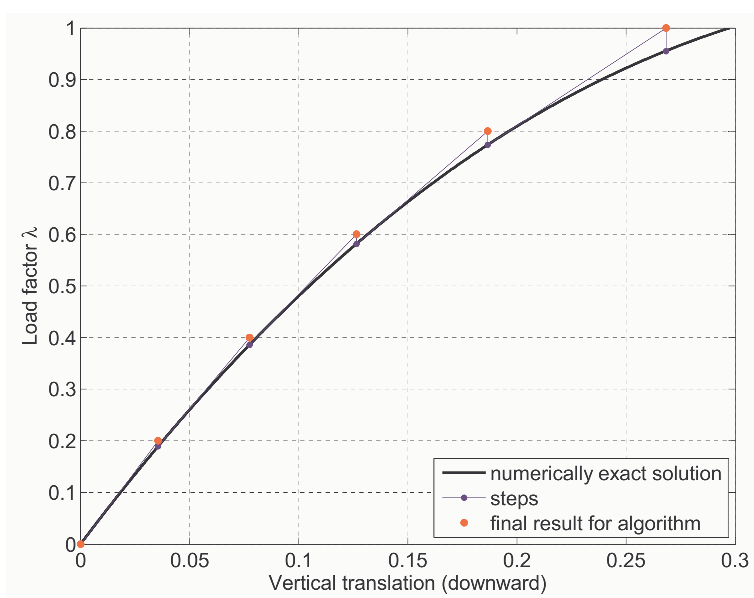
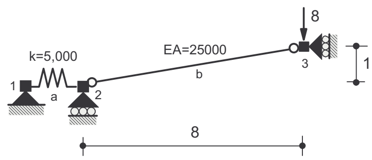


Figure: Incrementation without correction

Response of 2-dof truss under 10 load increments with $\Delta\lambda = 0.1$




```
no_step  = 10;
Dlam0    = 0.1;
tol      = 1.e-16;
maxiter  = 10;
S_Initialize_State
Pf = zeros(nf,1);
S_SimpInitialize
for k=1:no_step
    StifUpdt = 'yes';
    S_SimpIncrement
    S_Update_State
end
```

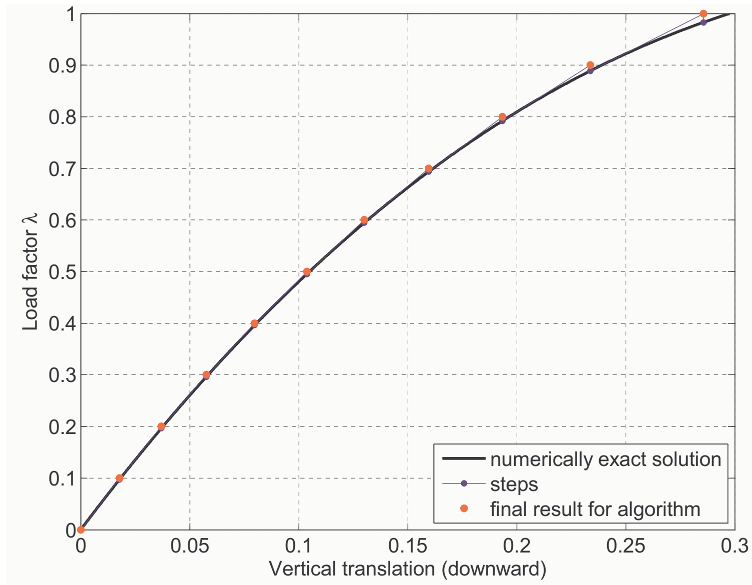
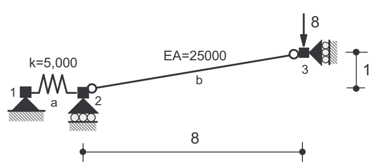


Figure: Incrementation without correction

Response of 2-dof truss under 22 load increments with $\Delta\lambda = 0.1$




```
no_step  = 23;
Dlam0    = 0.10;
tol      = 1.e-16;
maxiter  = 10;
S_Initialize_State;
Pf = zeros(nf,1);
S_Initialize
for k=1:no_step
    StifUpdt = 'yes';
    LoadCtrl = 'yes';
    S_Increment
    StifUpdt = 'yes';
    S_SimpIterate
    if (ConvFlag)
        S_Update_State
    end
end
```

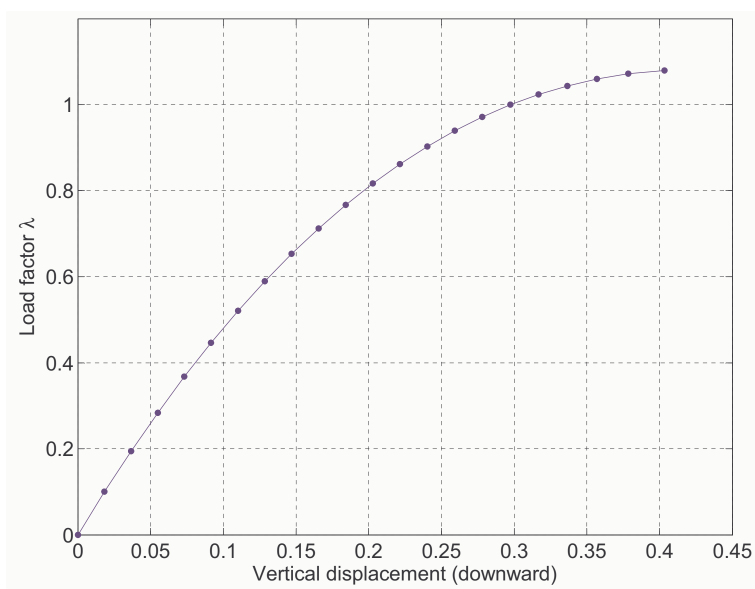
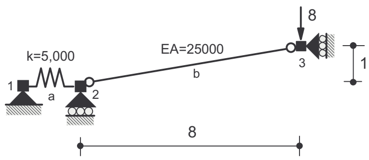


Figure: Load factor control during incrementation; only 21 steps are able to complete; the last step "flip-flops"

Response of 2-dof truss under 120 load increments with $\Delta\lambda = 0.1$




```
no_step  = 120;
Dlam0    = 0.10;
tol      = 1.e-16;
maxiter  = 10;
S_Initialize_State;
Pf = zeros(nf,1);
S_Initialize
for k=1:no_step
    StifUpdt = 'yes';
    LoadCtrl = 'yes';
    S_Increment
    StifUpdt = 'yes';
    LoadCtrl = 'yes';
    S_Iterate
    if (ConvFlag)
        S_Update_State
    end
end
```

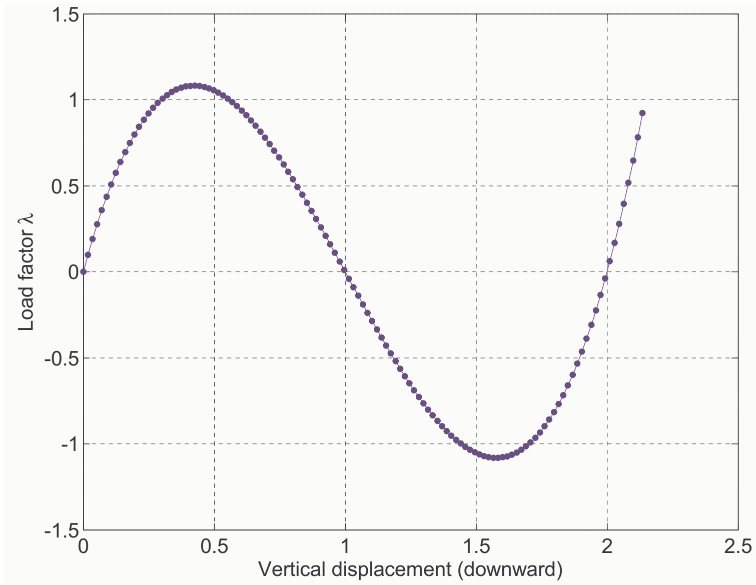


Figure: Response of 2-dof truss with load factor control

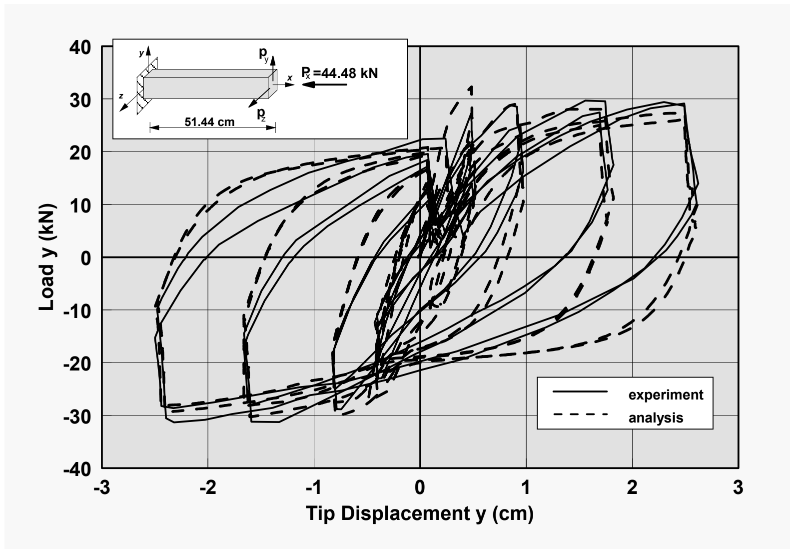
Correlation studies of analysis with experiment: important but …

- Many tests have been conducted and more are under way
  - Before understanding the behavior of assemblies one should understand the behavior of the constituent parts; not always possible or available
  - Reduced scale models require attention to scaling laws (e.g. weld fractures, bond-slip)
  - “Older” tests are not complete either for lack of enough channels of measurement or for lack of reporting (lost data); it is hard to obtain funding to “repeat” old tests
  - Tests may have experimental errors (these are not reported always)
  - Success or failure can be decided by looking at all experimental data, not a suitable subset of them
  - We can learn from failure as much as we learn from success, even though this is not accepted practice in research publications; better paradigm is necessary

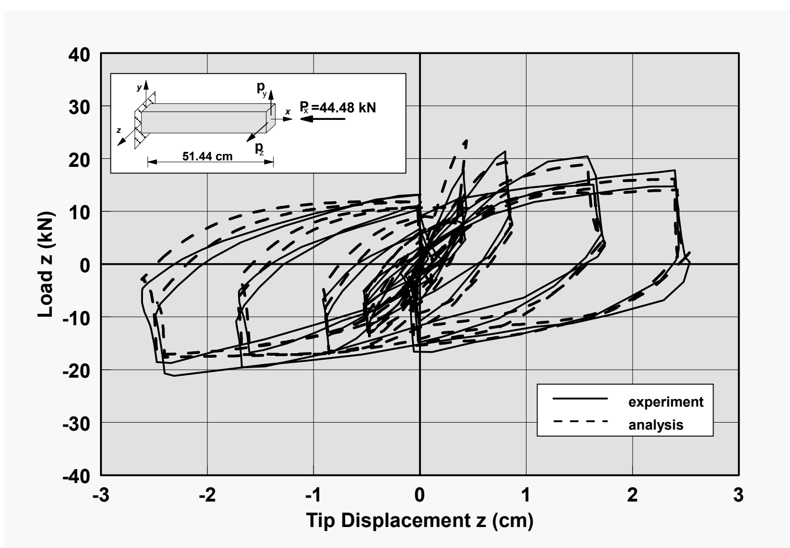
A simple start …

RC columns with biaxial bending and variable axial force

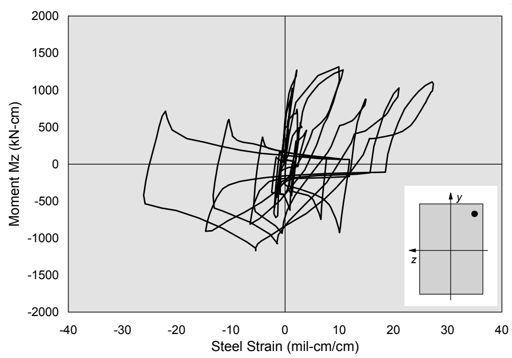
Low-Moehle Specimen 5: Load-Displacement Response in y

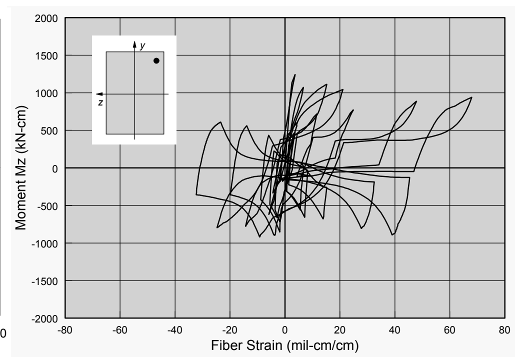


Low-Moehle Specimen 5: Load-Displacement Response in z



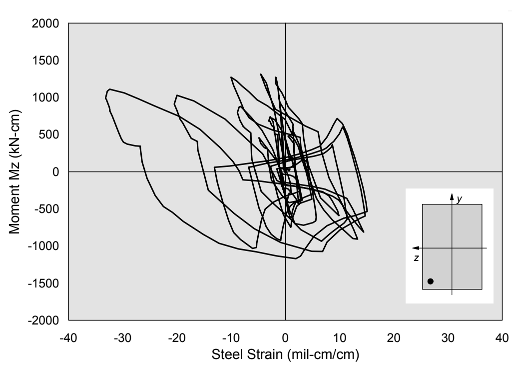
Low-Moehle Specimen 5: Reinforcing Steel Strain History

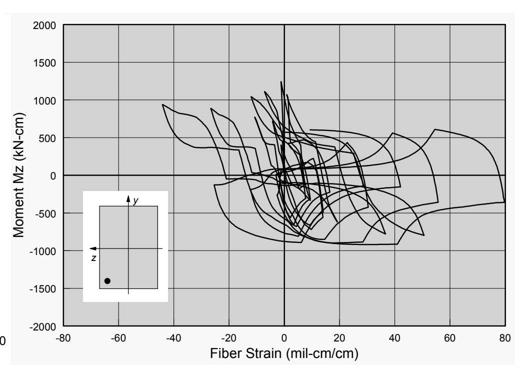




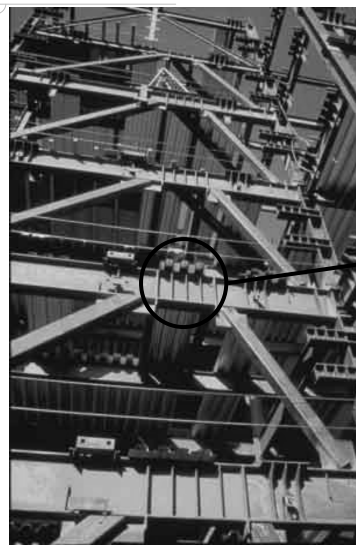
Effect of reinforcing bar pull-out from base

Low-Moehle Specimen 5: Reinforcing Steel Strain History

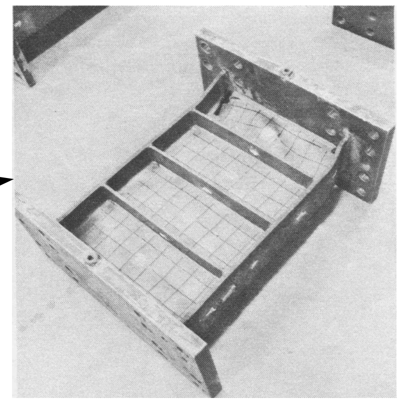




Eccentrically braced steel frames



Eccentrically Braced Frame



Shear Link

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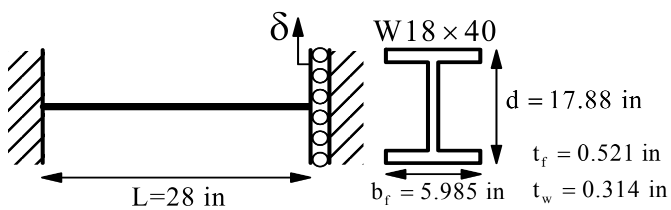
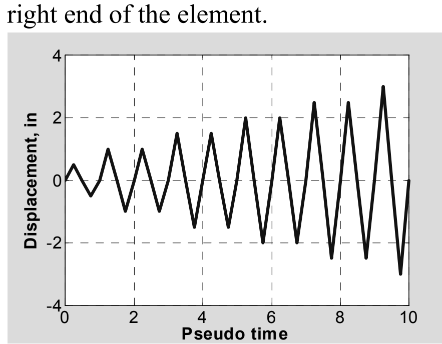
Shear Link Experiment (Hjelmstad/Popov 1983)

Equal moments at member ends



Imposed vertical displacement at the right end of the element.

Loading History



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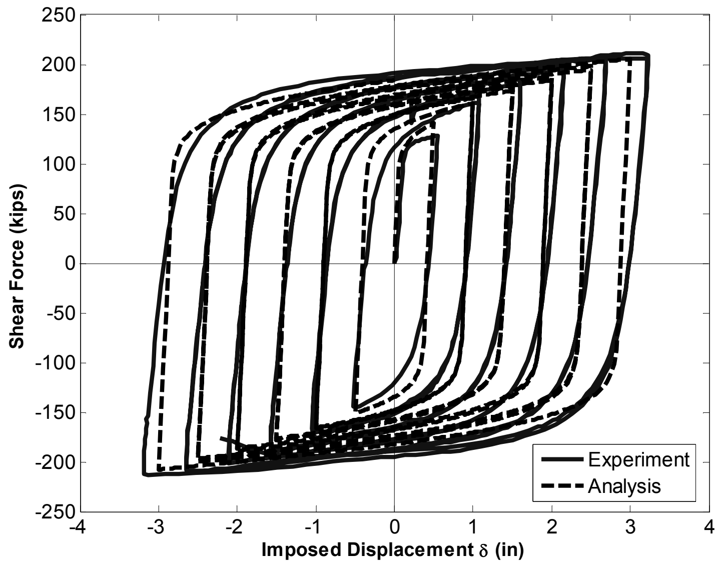
Shear Link Experiment (Hjelmstad/Popov 1983)



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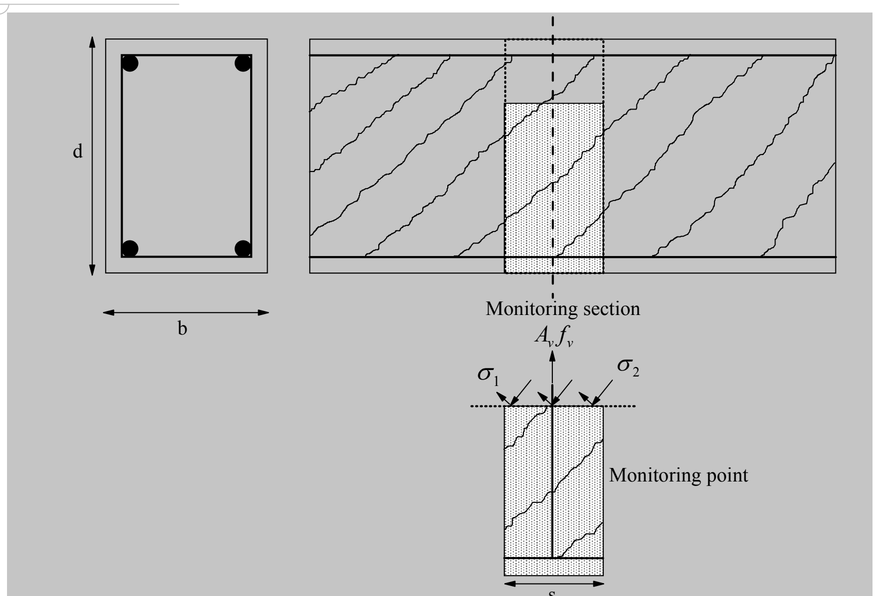
Concrete shear model



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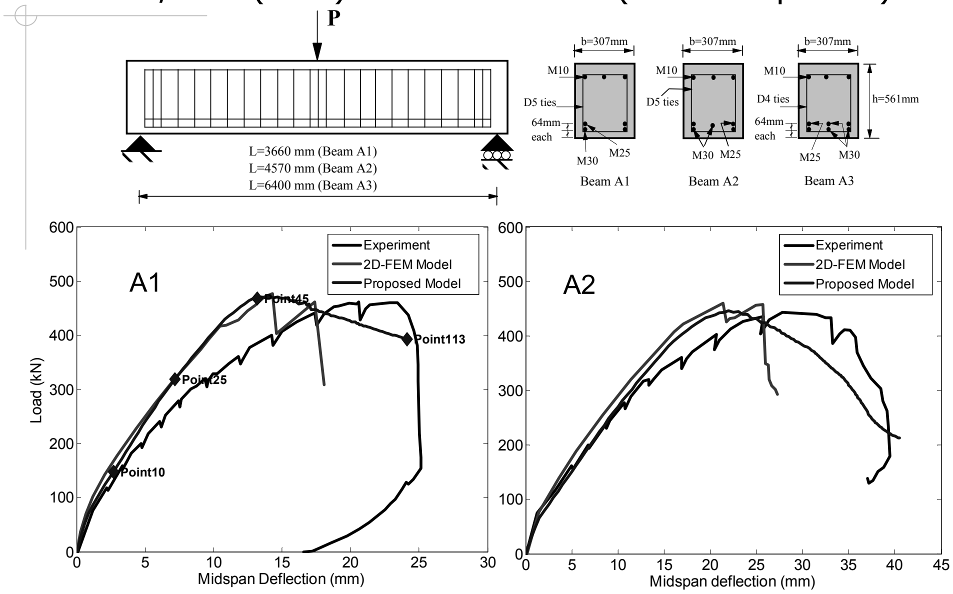
Vecchio/Shim (2004) Beams A1 and A2 (shear-compresion)



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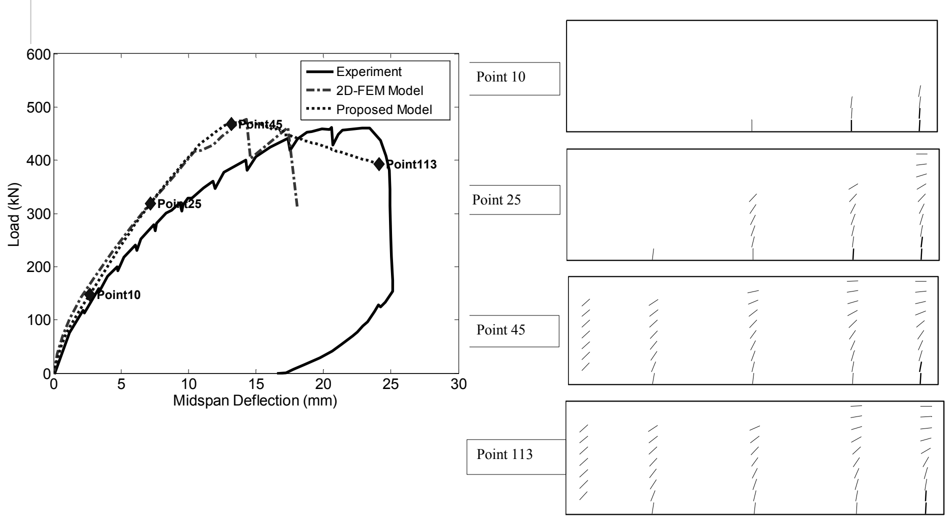
Vecchio/Shim (2004) Beam A1 (shear-compresion failure)



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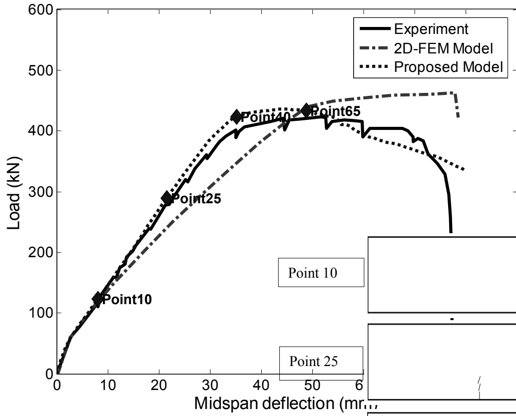
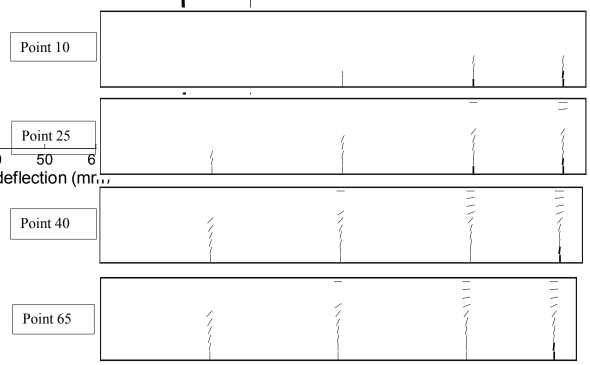
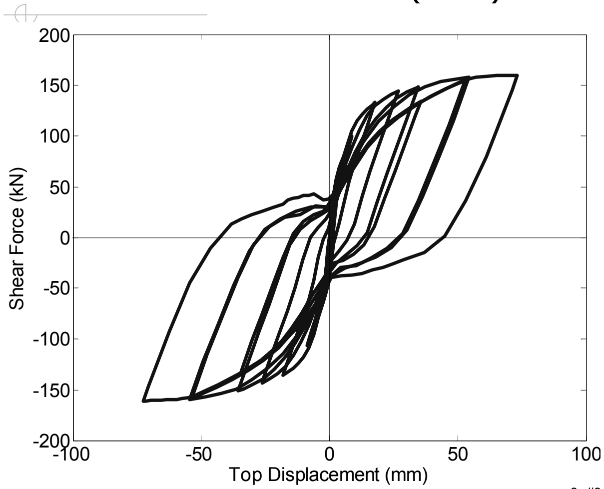
Vecchio/Shim (2004) Beam A3 (compression failure)

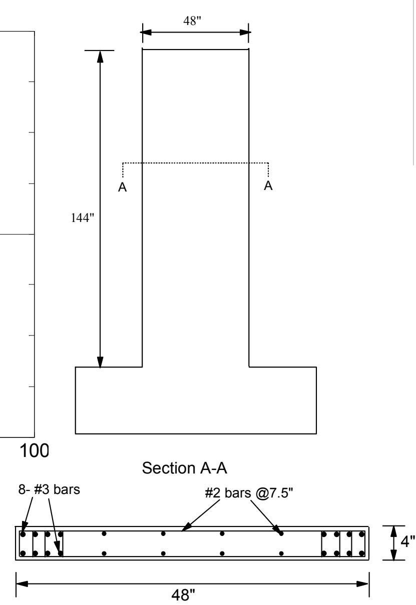


One element per half span with 5 Lobatto integration points is used
The cross-section is divided into 10 midpoint layers
Basic concrete material parameters are taken from Vecchio-Shim



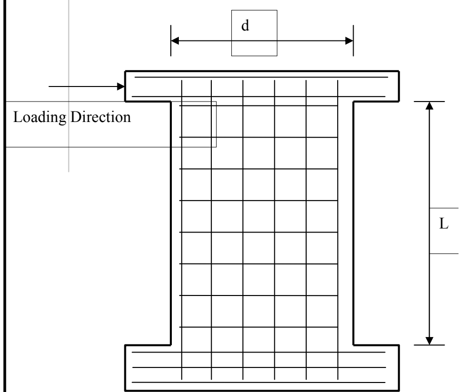
Thomsen-Wallace (2004) Slender Shear Walls

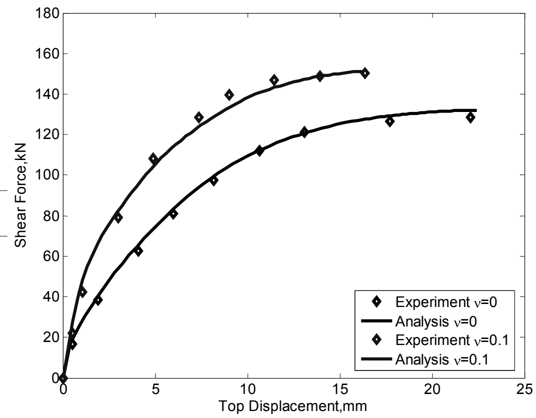




Lefas-Kotsovos-Ambraseys (1990) Shear Walls

•Shear walls SW21 and SW22 have aspect ratio L/h=2.0





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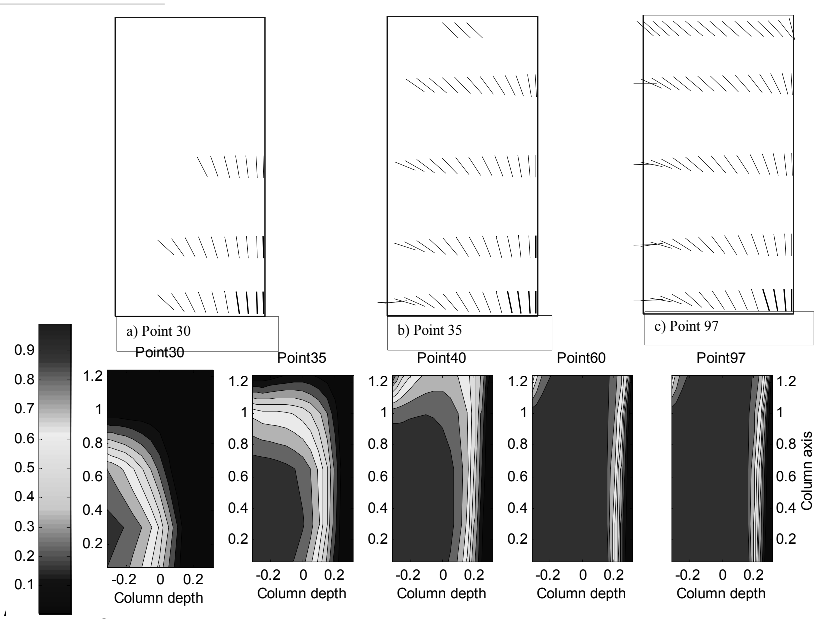
Evolution of shear wall cracking – Tensile damage



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Analytical Model of Reinforced Concrete Walls

Paolo Martinelii, Politecnico di Milano, Italy
FCF, University of California, Berkeley

Full scale 7 story wall building (Panagiotou, Restrepo, Conte, UCSD)

Flange Wall

Web Wall

Gravity Column

Height ≈ 20 m

Weight ≈ 226 tons

Precast Column

Table motion direction

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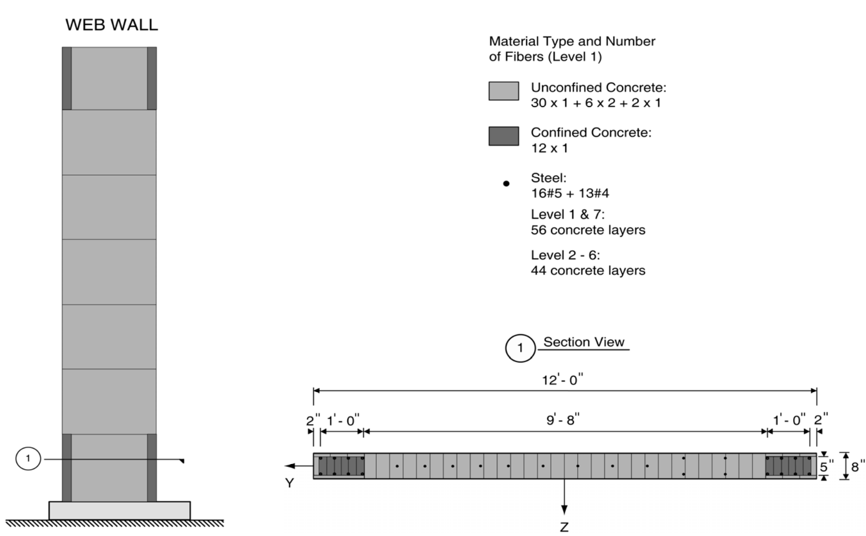
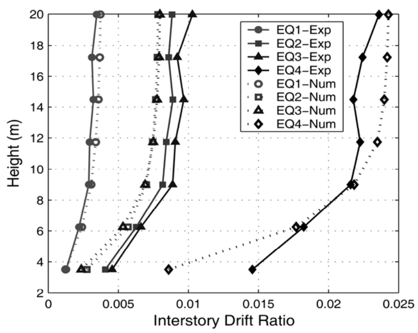
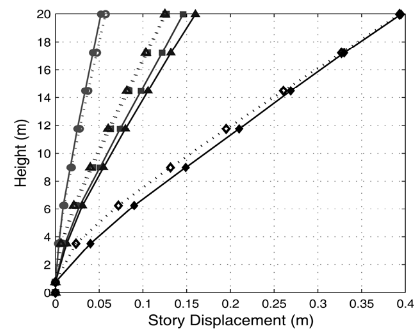
Confined and unconfined zones in the web wall



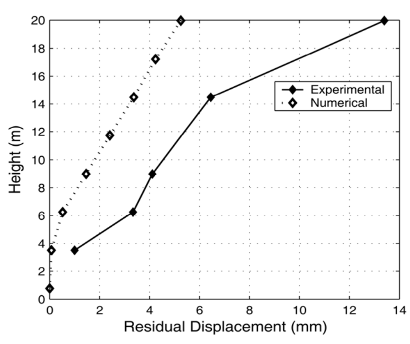
Interstory drift and displacement envelopes

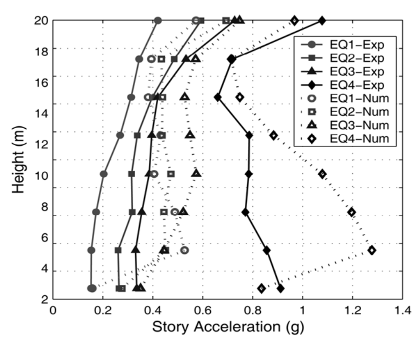




- Excellent agreement in terms of interstory drift and displacement
- Increasing damage due to increasing intensity motions is visible

Residual displacement and floor acceleration envelopes





- Good agreement in terms of residual displacement
- Satisfactory results in terms of floor acceleration

Analytical Model of

Reinforced Concrete Walls

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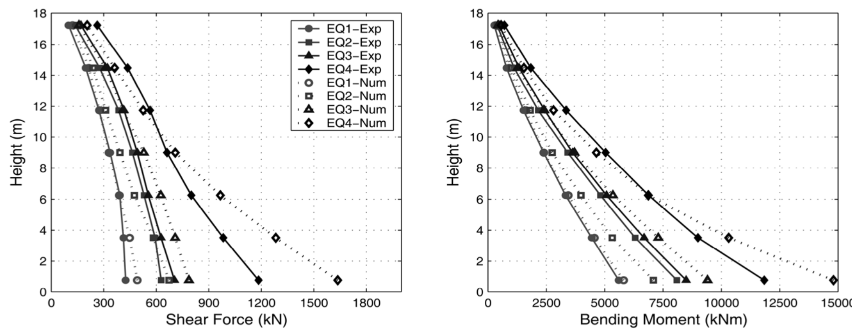
Story shear and overturning moment envelopes



- Good agreement also in terms of internal forces
- Maximum discrepancy during last motion at shear wall base

Analytical Model of

Reinforced Concrete Walls

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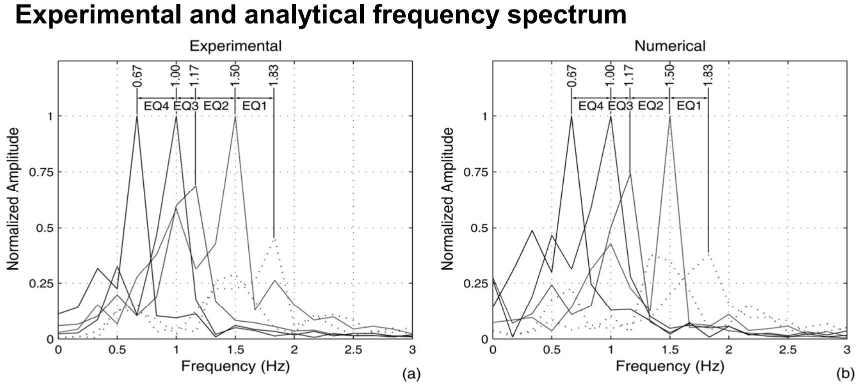
Experimental and analytical frequency spectrum



f_1 = 1.83 Hz (0.55 s)
Beginning of EQ1

f_1 = 0.67 Hz (1.43 s)
End of EQ4

- Significant lengthening of fundamental period of specimen (> than 2.5 x)
- Damage evolution tracked with remarkable accuracy

Analytical Model of Reinforced Concrete Walls

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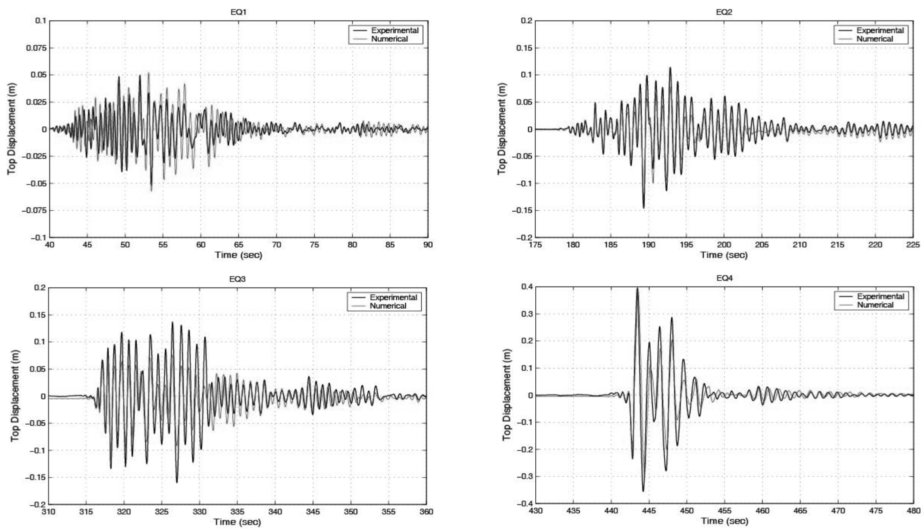
Top displacement time history



Conclusions

- Nonlinear Analysis is gradually going to become a designer's tool for the evaluation of existing structures and the design of new important structures
- It can offer significant insights into the global, but particularly into the local response of structures and, thus serve for the identification of local failure mechanisms
- The current state of the art permits the simulation of the hysteretic behavior of structural elements with limited success; deeper understanding of material behavior under cyclic loading is, however, indispensable in order to arrive at failure mode prediction
- Thorough correlations between numerical models and specimens of increasing complexity are indispensable; limited progress has been made to date, particularly regarding 3d response

Continuing Challenges

- Effect for shear, torsion and interaction with axial force and bending moment (3d and not just 2d analysis for shear)
- Effect of bond-slip, pull-out of reinforcing steel
- 3d beam-column joint model that is robust and efficient
- 3d constitutive model for concrete under large inelastic strains (damage, dilatation, …)
- Buckling of reinforcing steel (global and not local)
- Low cycle fatigue of structural steel; fracture
- Simulation of structural subassemblies and full-scale structures
- Many more: partitions, slab-wall-column interactions, cladding, infills …

Future outlook

- Much work needs to be done before nonlinear analysis can have widespread use, because of the complexity of nonlinear solution algorithms and the lack of training of modern engineers; thus researchers and educators need to redouble their efforts in clarifying the concepts
- I hope that with the contributions of all those present in an open forum (open platform) we will be able to experience significant progress in the nonlinear simulation of concrete structures in the future